fitting 5 pts. to parabola $y = \underline{a_0} + \underline{a_1} x + \underline{a_2} x^2$

$n = 5$ indirect observations

$n_0 = 3$

$r = 2$ choose $u = n_0 = 3$ parameters

~~x_i~~ y_i : σ_i $\quad w_i = \dfrac{\sigma_0^2}{\sigma_i^2}$

$$Y_1 + V_1 = a_0 + a_1 X_1 + a_2 X_1^2$$

$$Y_2 + V_2 = a_0 + a_1 X_2 + a_2 X_2^2$$

$$\vdots$$

$$Y_5 + V_5 = a_0 + a_1 X_5 + a_2 X_5^2$$

$$v_1 - a_0 - a_1 x_1 - a_2 x_1^2 = -y_1$$
$$v_2 - a_0 - a_1 x_2 - a_2 x_2^2 = -y_2$$
$$\vdots$$
$$v_5 - a_0 - a_1 x_5 - a_2 x_5^2 = -y_5$$

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_5 \end{bmatrix} + \begin{bmatrix} -1 & -x_1 & -x_1^2 \\ -1 & -x_2 & -x_2^2 \\ & \vdots & \\ -1 & -x_5 & -x_5^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -y_1 \\ -y_2 \\ \vdots \\ -y_5 \end{bmatrix}$$

condition equations

$$V + B \Delta = f$$

$(n,1)$ (n,u) $(u,1)$ $(n,1)$

$(5,1)$ $(5,3)$ $(3,1)$ $(5,1)$

$$V + B\Delta = f \qquad \Phi = \sum v_i^2 = \sum w_i v_i^2$$

$$= V^T W V$$

$$\Delta = (B^T W B)^{-1} B^T W f$$

$$V = f - B\Delta$$

$$\hat{\ell} = \ell + v$$

$$x = \underline{a_0} + \underline{a_1}X + \underline{a_2}Y$$
$$y = \underline{b_0} + \underline{b_1}X + \underline{b_2}Y$$

6 parameter transf.
5 points xy obs
XY constant

$$x_i + v_{x_i} = a_0 + a_1 X_i + a_2 Y_i$$
$$y_i + v_{y_i} = b_0 + b_1 X_i + b_2 Y_i$$

$n = 10$
$n_0 = 6$
$r = 4$

$u = n_0 = 6$
$c = n = 10$

$$v_{x_i} - a_0 - a_1 x_i - a_2 y_i = -x_i$$
$$v_{y_i} - b_0 - b_1 x_i - b_2 y_i = -y_i$$

$$\begin{bmatrix} v_{x_1} \\ v_{y_1} \\ v_{x_2} \\ v_{y_2} \\ \vdots \\ v_{x_5} \\ v_{y_5} \end{bmatrix}_{n,1} + \begin{bmatrix} -1 & -x_1 & -y_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -x_1 & -y_1 \\ -1 & -x_2 & -y_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -x_2 & -y_2 \\ \vdots & & & & & \\ -1 & -x_5 & -y_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -x_5 & -y_5 \end{bmatrix}_{n,u} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}_{u,1} = \begin{bmatrix} -x_1 \\ -y_1 \\ -x_2 \\ -y_2 \\ \vdots \\ -x_5 \\ -y_5 \end{bmatrix}_{n,1}$$

$$\Delta = (B^T W B)^{-1} B^T W f$$

$$v = f - B\Delta$$

$$\hat{\ell} = \ell + v$$

$$x = \frac{a_0 + a_1 X + a_2 Y}{c_1 X + c_2 Y + 1} \qquad y = \frac{b_0 + b_1 X + b_2 Y}{c_1 X + c_2 Y + 1}$$

6 points $n = 12$ Indirect Obs.

$n_0 = 8$ $u = 8$

$r = 4$ $c = 12$

$$v_{x_i} - a_0 - a_1 X_i - a_2 Y_i - x_i c_1 X_i - x_i c_2 Y_i = -x_i$$

$$v_{y_i} - b_0 - b_1 X_i - b_2 Y_i - y_i c_1 X_i - y_i c_2 Y_i = -y_i$$

8- parameter transformation

$$\begin{bmatrix} v_{x_1} \\ v_{y_1} \\ v_{x_2} \\ v_{y_2} \\ \vdots \\ v_{x_6} \\ v_{y_6} \end{bmatrix} + \begin{bmatrix} -1 & -x_1 & -y_1 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 \\ 0 & 0 & 0 & -1 & -x_1 & -y_1 & -y_1X_1 & -y_1Y_1 \\ -1 & -x_2 & -y_2 & 0 & 0 & 0 & -x_2X_2 & -x_2Y_2 \\ 0 & 0 & 0 & -1 & -x_2 & -y_2 & -y_2X_2 & -y_2Y_2 \\ & & & & \vdots & & & \\ -1 & -x_6 & -y_6 & 0 & 0 & 0 & -x_6X_6 & -x_6Y_6 \\ 0 & 0 & 0 & -1 & -x_6 & -y_6 & -y_6X_6 & -y_6Y_6 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -X_1 \\ -Y_1 \\ -X_2 \\ -Y_2 \\ \vdots \\ -X_6 \\ -Y_6 \end{bmatrix}$$

$$V + B \cdot \Delta = f$$

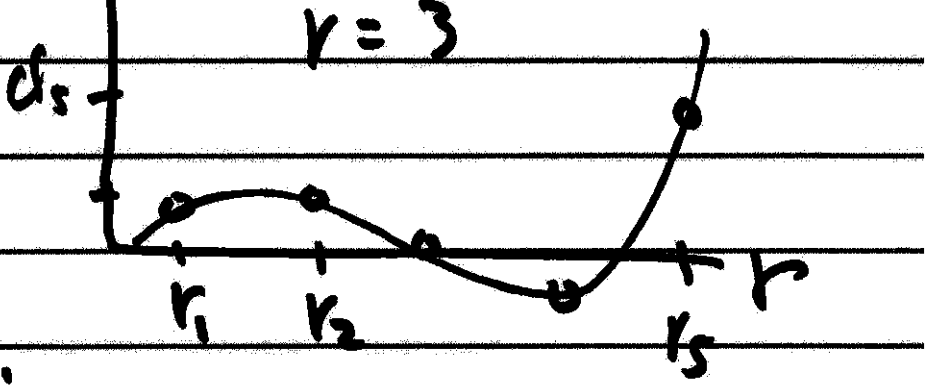
Lens Distortion

$n = 5 \quad m = 2$

$n_0 = 2 \quad c = 5$

$r = 3$

$$d = \underbrace{K_1 r^3 + K_2 r^5}_{} + K_3 r^7$$



$$d_i + v_i = K_1 r_i^3 + K_2 r_i^5$$

$$v_i - K_1 r_i^3 - K_2 r_i^5 = -d_i$$

$$\Delta = (B^T W B)^{-1} B^T W$$

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_5 \end{bmatrix} + \begin{bmatrix} -r_1^3 & -r_1^5 \\ -r_2^3 & -r_2^5 \\ \vdots & \vdots \\ -r_5^3 & -r_5^5 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} -d_1 \\ -d_2 \\ \vdots \\ -d_5 \end{bmatrix}$$

5,1 5,2 2,1 5,1