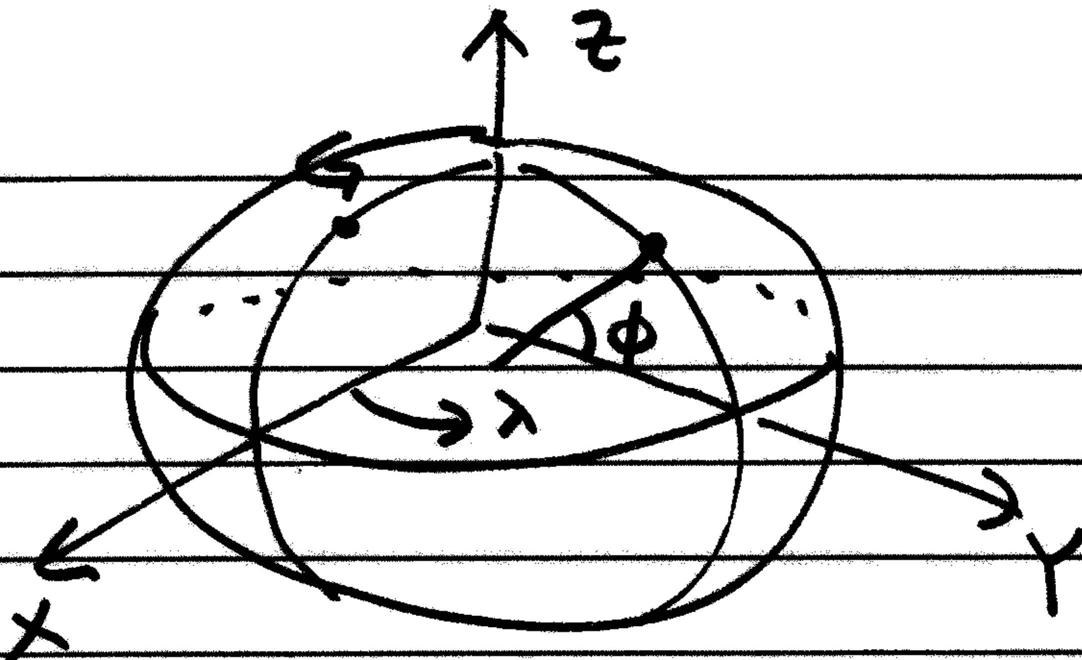


$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \longleftrightarrow \begin{pmatrix} \varphi \\ \lambda \\ h \end{pmatrix}$$

ϕ latitude

λ longitude

h ellipsoid

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{bmatrix} (N+h)\cos\phi\cos\lambda \\ (N+h)\cos\phi\sin\lambda \\ ((1-e^2)N+h)\sin\phi \end{bmatrix}$$

N: radius of curvature in prime vertical

$$N = \frac{a}{(1-e^2\sin^2\phi)^{1/2}}$$

inverse is iterative, need ϕ

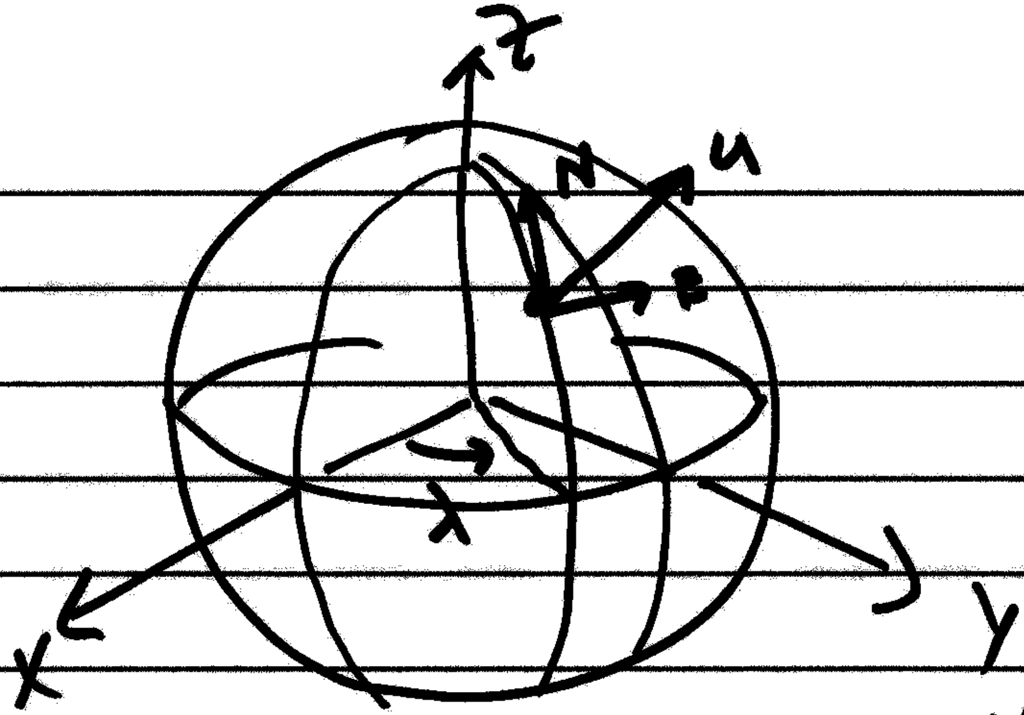
$$\phi = \tan^{-1}\left[\frac{Z}{(X^2+Y^2)^{1/2}}\left(1-e^2\left(\frac{N}{N+h}\right)\right)^{-1}\right]$$

$$\lambda = \tan^{-1} Y/X \; *$$

$$h = \frac{(X^2+Y^2)^{1/2}}{\cos\phi} - N$$

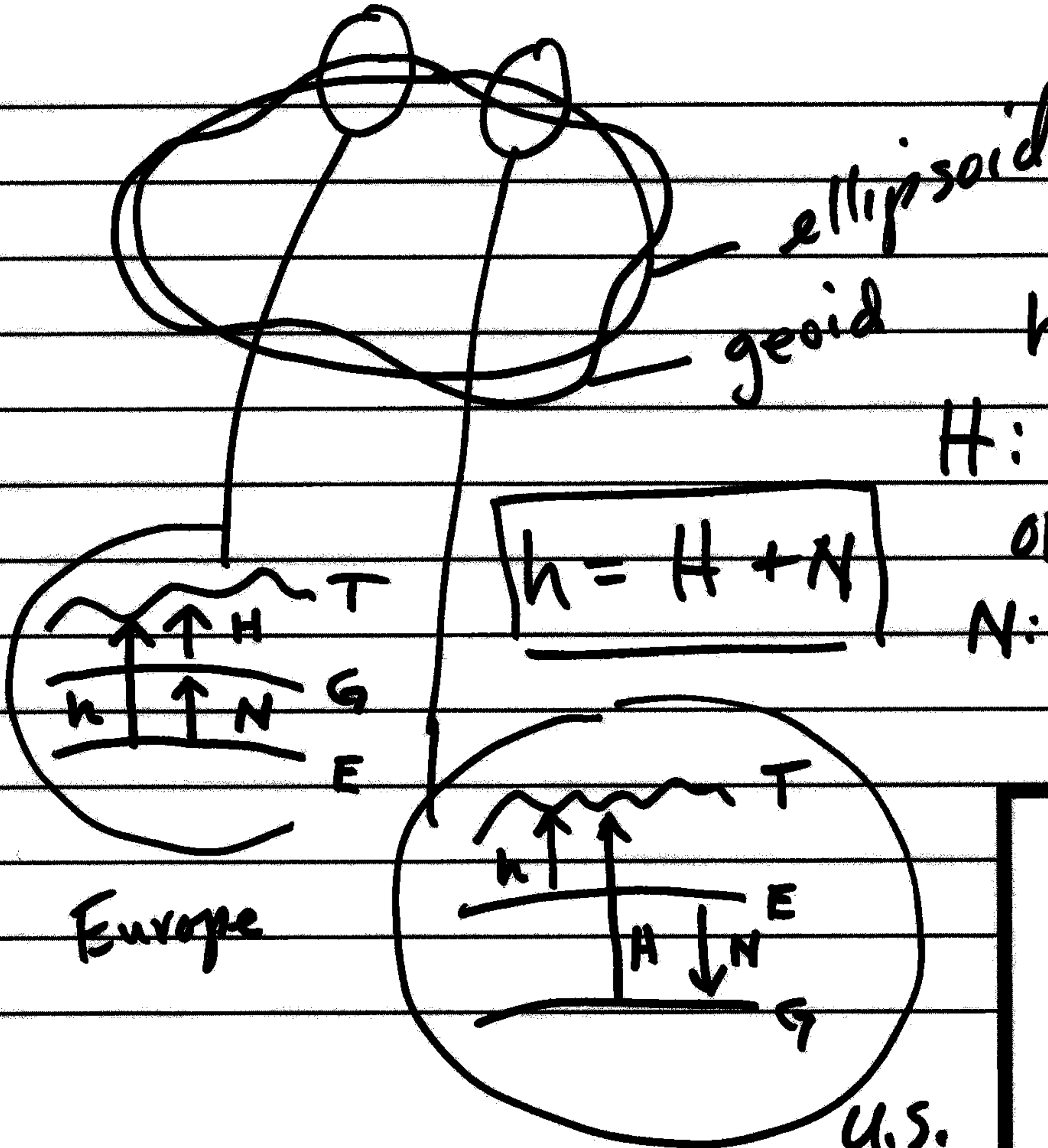
iterate until no change

* use 2 argument atan2(Y, X) to get correct quadrant



$$\begin{pmatrix} E \\ N \\ u \end{pmatrix}$$

$$\begin{pmatrix} E \\ N \\ u \end{pmatrix} = \underbrace{M_x(90^\circ - \phi)\, M_z(\lambda + 90^\circ)}_{R_3} \left[\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} - \begin{pmatrix} (N+h)\cos\phi\cos\lambda \\ (N+h)\cos\phi\sin\lambda \\ ((1-e^2)N+h)\sin\phi \end{pmatrix} \right]$$



$$\boxed{h = H + N}$$

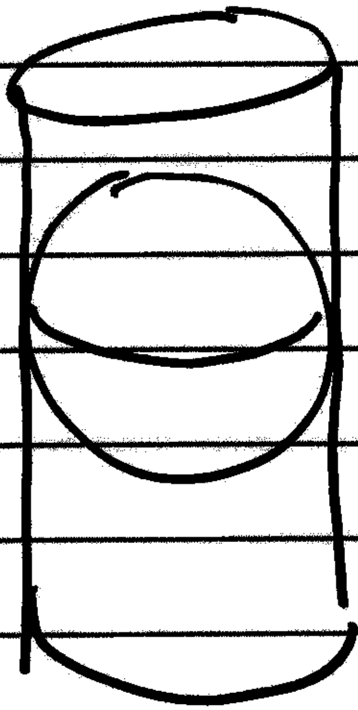
h: ellipsoid height

H: sea level height
orthometric height

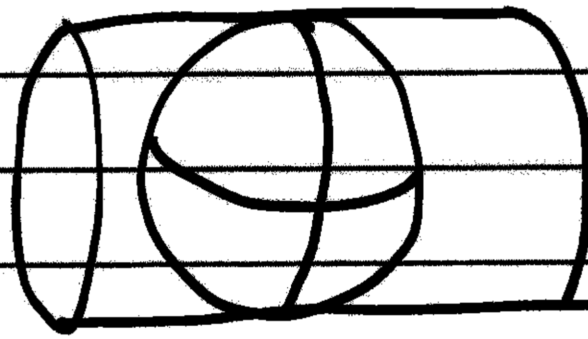
N: Geoid undulation

$\phi\lambda \rightarrow XY$ conformal

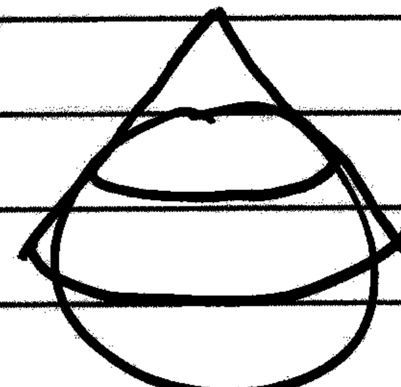
shapes are preserved locally



cylindrical mercator

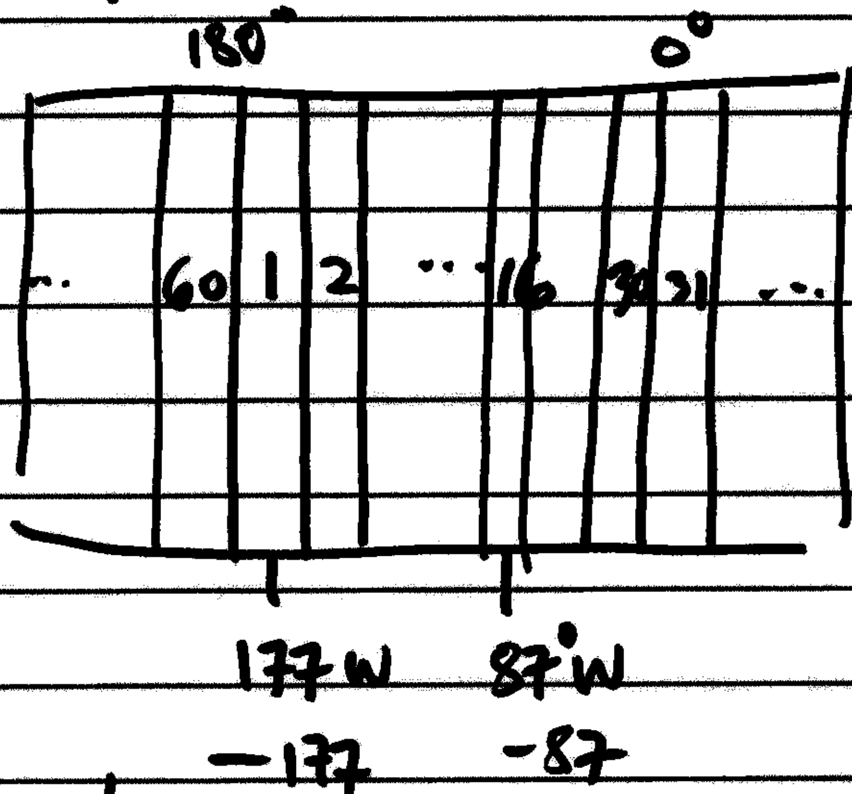


transverse mercator



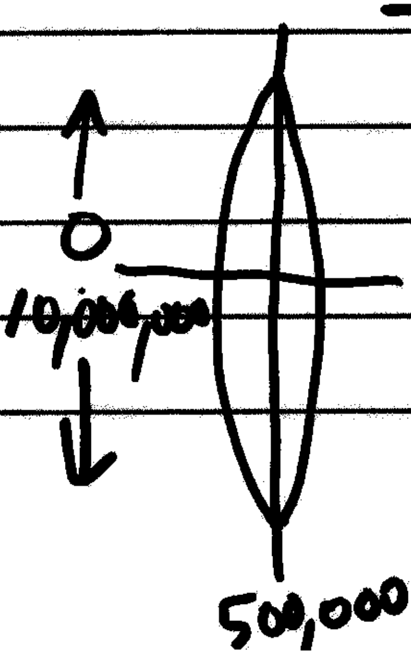
Lambert Conic

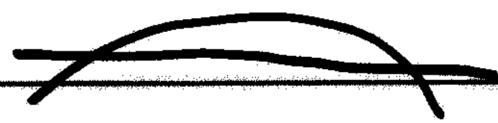
UTM: universal Transverse Mercator



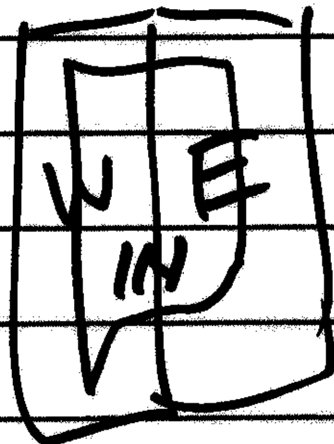
60 Zones
each 6°

(X, Y, H)

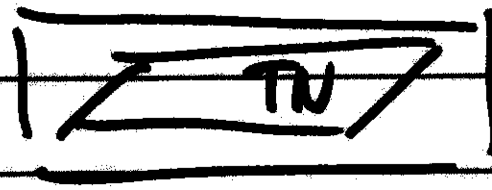




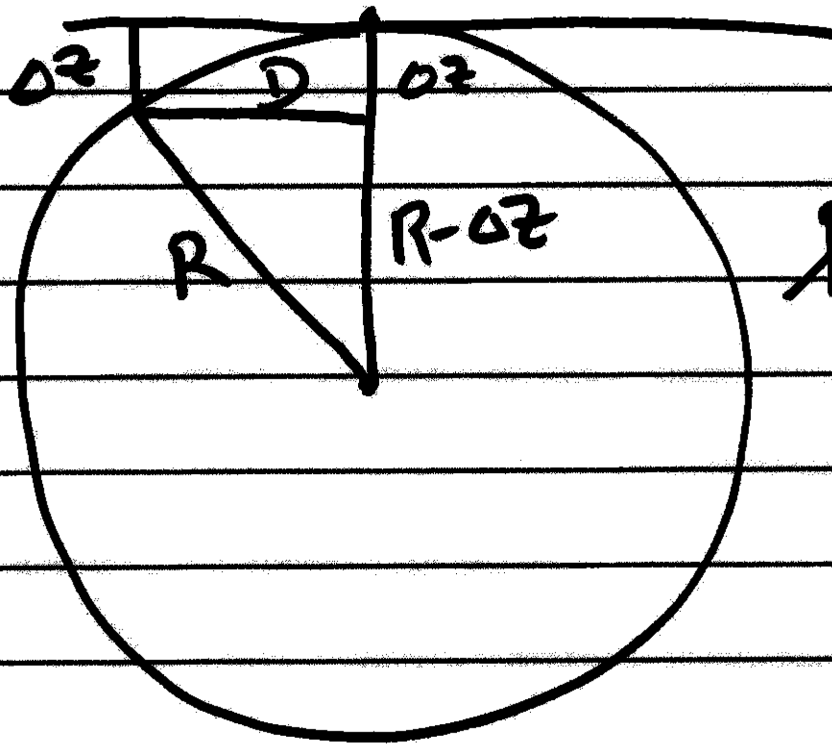
SPC : state plane coordinates



2 T.M.
zones



1 Lambert
Conic Zone



$$R^2 = D^2 + (R-\Delta z)^2$$

$$\cancel{R^2} = D^2 + \cancel{R^2} + \Delta z^2 - 2R\Delta z$$

$$\underbrace{D^2 + \Delta z^2}_{\cancel{\Delta z^2}} = 2R\,\Delta z$$

$$D^2 = 2R\Delta z$$

$$\boxed{\Delta z = \frac{D^2}{2R}}$$

D(m)	Δz
10	0
100	.0008
500	.02
1000	.08
5000	1.9
10000	7.8