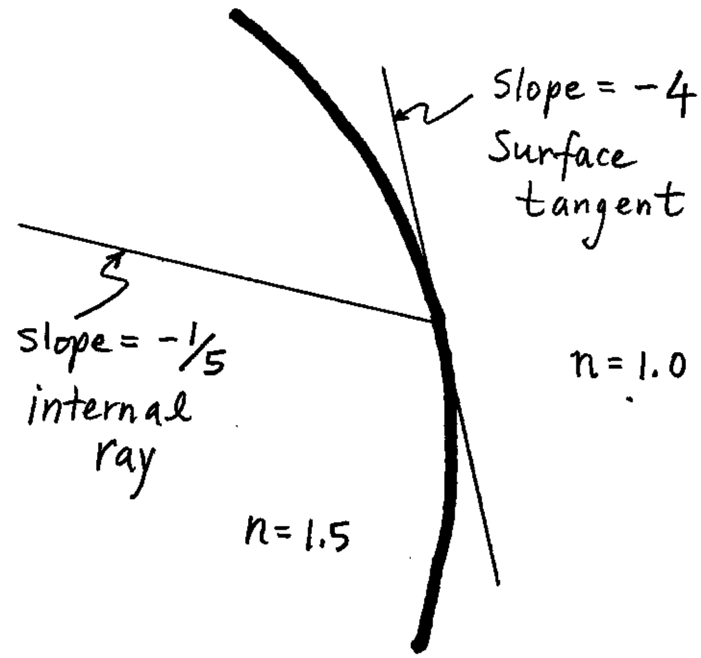
GRAD 590D EXAM I

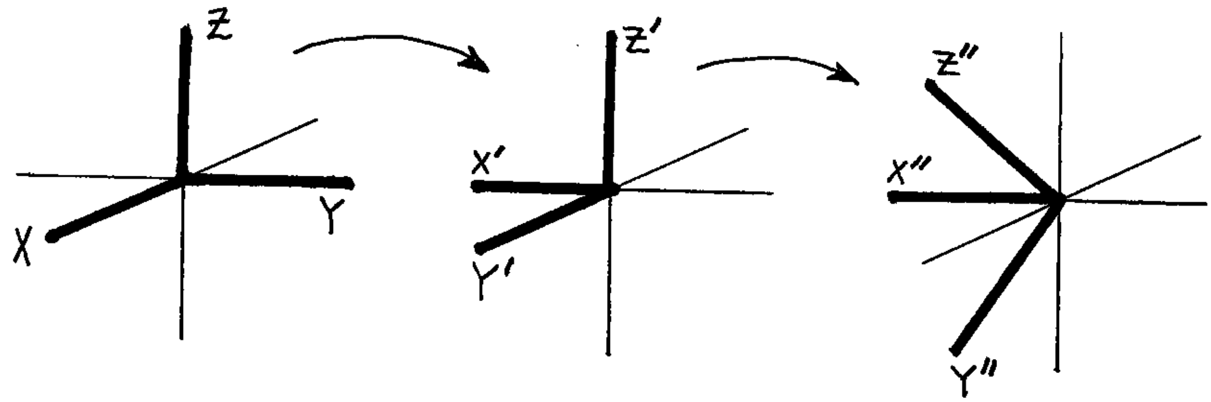
16 Oct. 2007

Name ______________________

1. In the sketch a ray internal to the lens approaches from the left. Slopes are given for the ray and the tangent surface of the lens. Find θ_1 and θ_2 of the incoming and exiting ray with respect to the surface normal.



2.



Construct the rotation matrix M, numerically, via 2 sequential rotations, so that

$$\begin{pmatrix} X'' \\ Y'' \\ Z'' \end{pmatrix} = M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Note that each elementary rotation has magnitude of either 90° or 45°.

3. A 4-parameter transformation of the form,

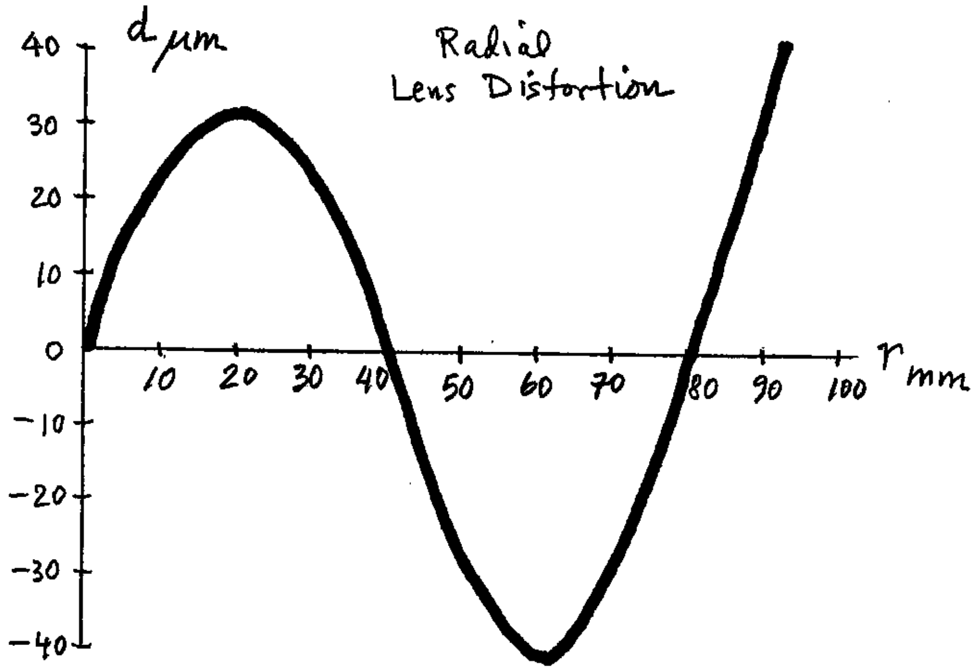
$$\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

is expressed numerically as,

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1.000 & 1.732 \\ -1.732 & -1.000 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} 1.000 \\ 2.000 \end{pmatrix}$$

what are λ and θ ?

4.



An image point $(x, y) = (50.000, 33.166)$ mm is expressed with respect to the calibrated principal point. Compute the correction for radial lens distortion and apply the correction to the point coordinates.

5. For the given interior and exterior orientation data, and for the given image point, find the intersection of the ray with the plane, $Y_A = 50\,m$.

$$\begin{bmatrix} x - x_o \\ y - y_o \\ -f \end{bmatrix} = \lambda M \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix}$$

$x_o = 0$
$y_o = 0$
$f = 100$ mm
$x = 10$ mm
$y = 20$ mm
$X_L = 100$ m
$Y_L = 100$ m
$Z_L = 100$ m

$$M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

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useful facts:

$$M_\omega = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix}$$ about X

$$M_\varphi = \begin{bmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{bmatrix}$$ about Y

$$M_\kappa = \begin{bmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ about Z

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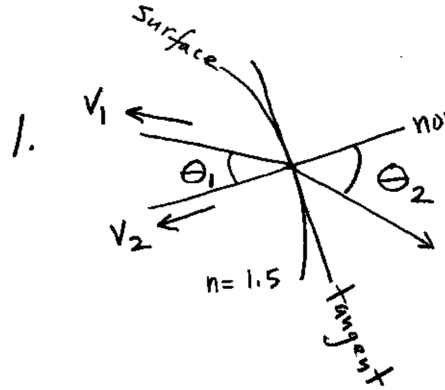
Solution 28-Oct-07

1.



$\vec{V}_1 : (-5, 1)$ if tangent slope is -4, then
$\vec{V}_2 : (-4, -1)$ normal slope is $1/4$

unit vectors : $\vec{\mu}_1 : (-.9806, .1961)$
$\vec{\mu}_2 : (-.9701, -.2425)$

$$\cos\theta_1 = \vec{\mu}_1 \bullet \vec{\mu}_2 = (-.9806)(-.9701) + (.1961)(-.2425)$$
$$= .9037, \quad \underline{\theta_1 = 25.35^\circ}$$

Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$(1.5)(.42811) = (1.0) \sin\theta_2 = .6422$, $\underline{\theta_2 = 39.95^\circ}$

recall $\vec{A} \bullet \vec{B} = |A||B| \cos\theta$, $\dfrac{\vec{A}}{|\vec{A}|} \bullet \dfrac{\vec{B}}{|\vec{B}|} = \cos\theta$

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2. By the right hand rule, first rotation is $\theta_z = -90^\circ$, second rotation is $\theta_x = -45^\circ$

$$M = M_x(-45^\circ)\, M_z(-90^\circ) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-45) & \sin(-45) \\ 0 & -\sin(-45) & \cos(-45) \end{bmatrix} \begin{bmatrix} \cos(-90) & \sin(-90) & 0 \\ -\sin(-90) & \cos(-90) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & .707 & -.707 \\ 0 & .707 & .707 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ .707 & 0 & -.707 \\ .707 & 0 & .707 \end{bmatrix}$$

note: the matrix for the rotation which is applied <u>first</u>, goes on the <u>right</u> in the matrix product

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3. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$, $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} = \lambda \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$a^2 + b^2 = \lambda^2 (\underbrace{\cos^2\theta + \sin^2\theta}_{=1}) = \lambda^2$, there are 2 solutions : $\lambda = +/- \sqrt{a^2+b^2}$

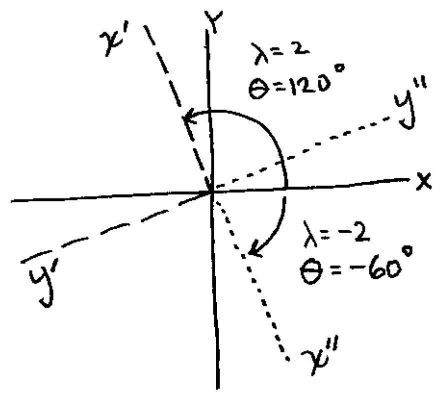
$\lambda = +2$ or -2

$\lambda = 2 \Rightarrow \cos\theta = -.5, \sin\theta = .866, \theta = 120^\circ$

$\lambda = -2 \Rightarrow \cos\theta = .5, \sin\theta = -.866, \theta = -60^\circ$

$\theta = \tan^{-1}\left(\dfrac{\sin\theta}{\cos\theta}\right)$ use signs of numerator and denominator to put in correct Quadrant

$\frac{+}{-}$	$\frac{+}{+}$
$\frac{-}{-}$	$\frac{-}{+}$



4. for $(X, y) = (50, 33.166)$, $r = \sqrt{50^2 + 33.166^2} = 60$

interpolate from graph: $d = -40\mu m = -0.040$ mm

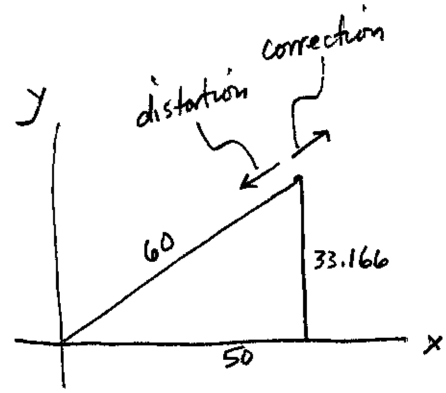
$\Rightarrow$ correction is $+0.040$

$\frac{dx}{x} = \frac{dr}{r}$, $dx = \frac{.040}{60} \cdot 50 = .033$

$dy = \frac{.040}{60} \cdot 33.166 = .022$

$X_{corr.} = 50.000 + .033 = \underline{50.033}$

$y_{corr.} = 33.166 + .022 = \underline{33.188}$



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5. Intersect with $Y = 50$. write collinearity equations with constants substituted for variables:

$$\begin{pmatrix} 10 \\ 20 \\ -100 \end{pmatrix} = \lambda \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} X-100 \\ 50-100 \\ Z-100 \end{pmatrix} \quad \text{solve for } X \,\&\, Z$$

$$\frac{1}{\lambda} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 10 \\ 20 \\ -100 \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} -100 \\ 10 \\ 20 \end{pmatrix} = \begin{pmatrix} X-100 \\ -50 \\ Z-100 \end{pmatrix}$$

$$\frac{-100}{10} = \frac{X-100}{-50}, \quad (-50)(-10) = X - 100, \quad X = 100 + 500 = \underline{600}$$

$$\frac{20}{10} = \frac{Z-100}{-50}, \quad (-50)(2) = Z - 100, \quad Z = 100 - 100 = \underline{0}$$

Note: this mathematical solution cannot actually be realized since it corresponds to the ray exiting through the rear of the camera.