Error Propagation

- For error propagation we assume that input data is normally distributed.
- Scale the relative covariance matrices by the *a priori* reference variance if results are consistent with assumptions, or by the *a posteriori* reference variance if results are not consistent with assumptions.
- Random variation in the input observations is transformed by the model into corresponding random variation in the resulting unknown parameters.
- This dispersion or uncertainty in the computed unknown parameters can be scaled to a given probability level, and presented in 1D (confidence interval) or 2D (confidence ellipse).
- Higher dimensionality is difficult to visualize.
- Confidence ellipses can be absolute, with reference to a point, or relative, with reference to the coordinate differences between two points.

Error Propagation

$$y = a_1 x_1 + a_2 x_2 + a_3 x_3$$

$$\sigma_y^2 = a_1^2 \sigma_1^2 + a_2^2 \sigma_2^2 + a_3^2 \sigma_3^2$$

If x's are uncorrelated

$$\Sigma_x = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{bmatrix}$$

To allow for correlation between the x's

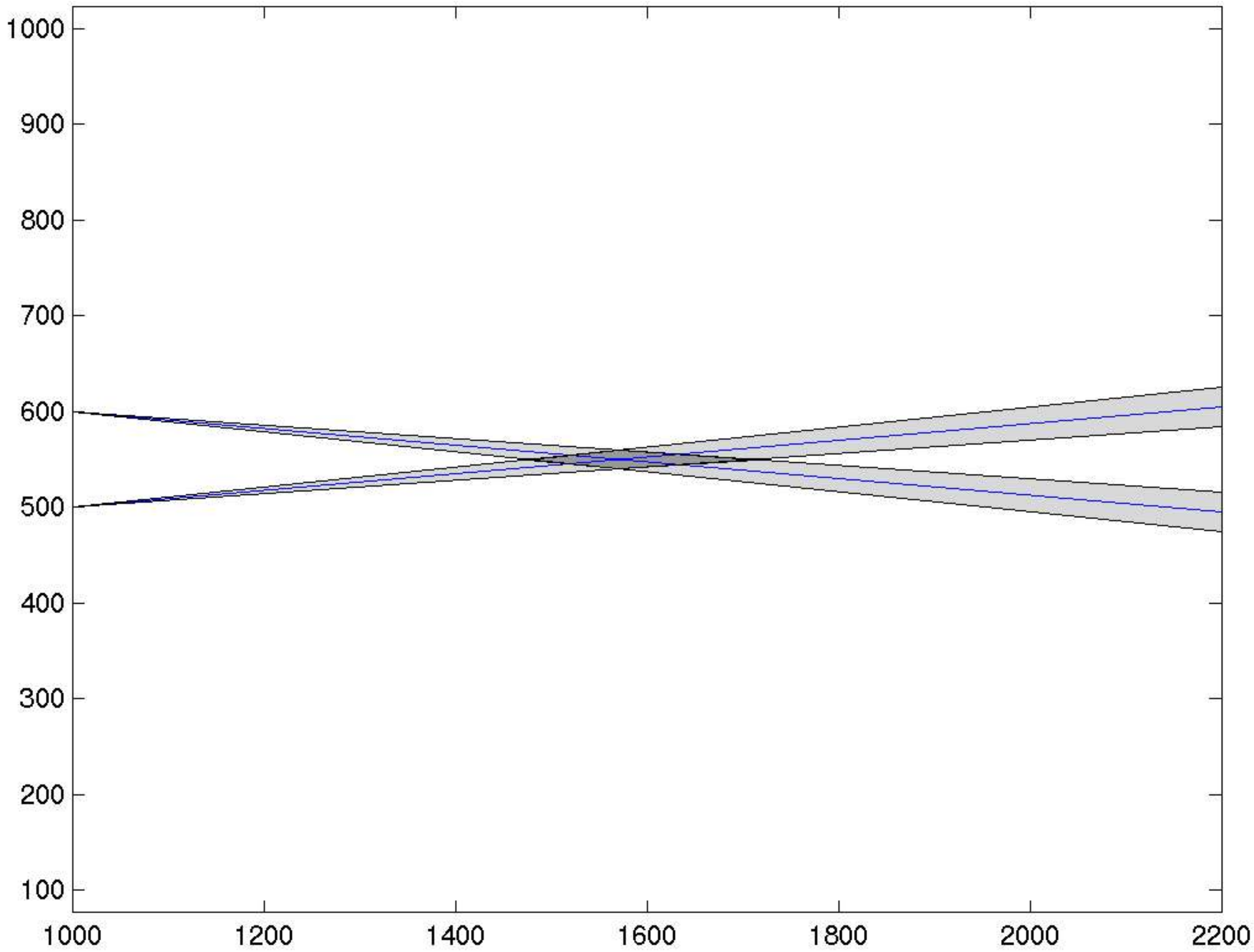
$$y = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \mathbf{Ax}$$

Express the function in matrix form

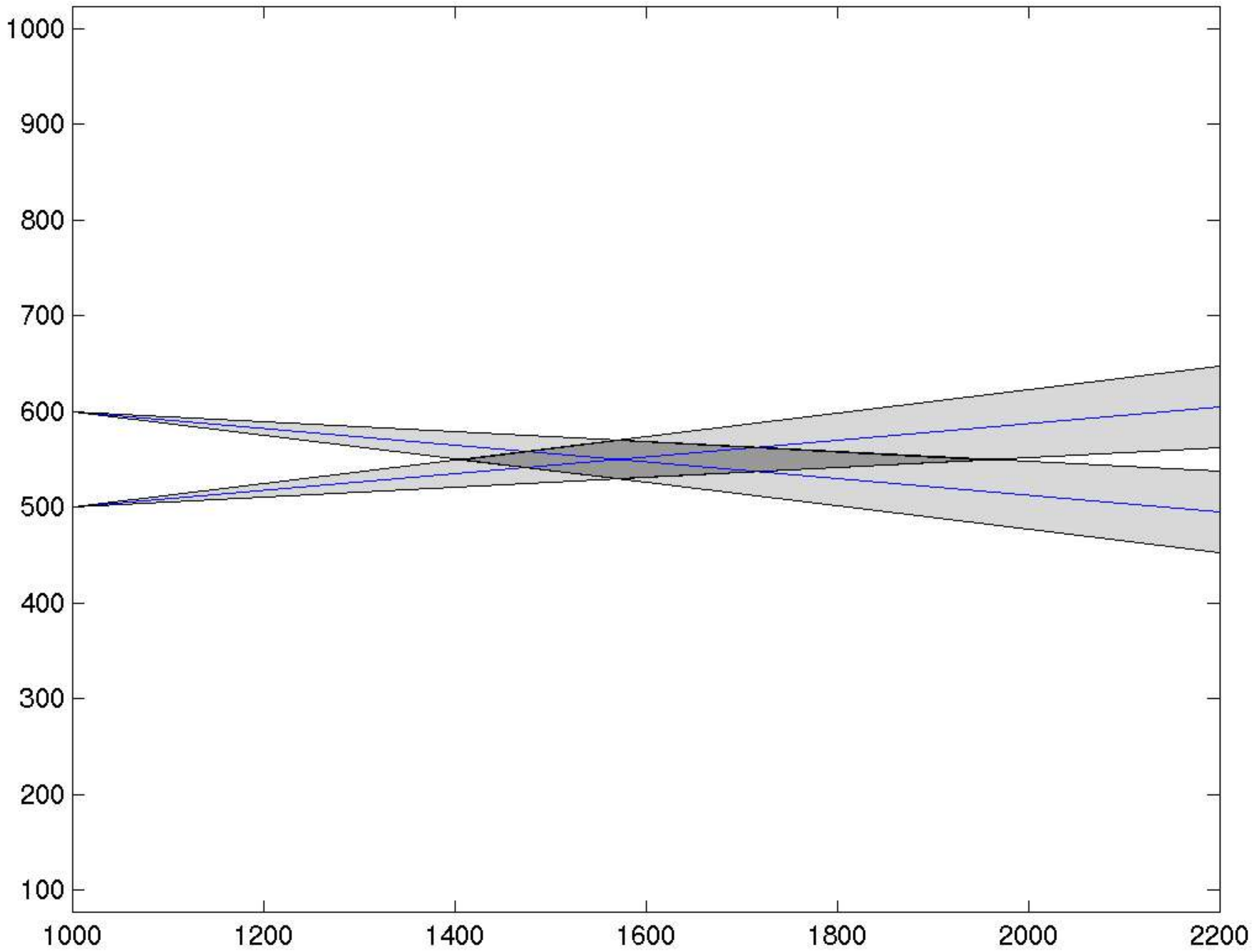
$$\sigma_y^2 = \mathbf{A \Sigma_x A^T}$$

Rigorous error propagation including correlations and covariances

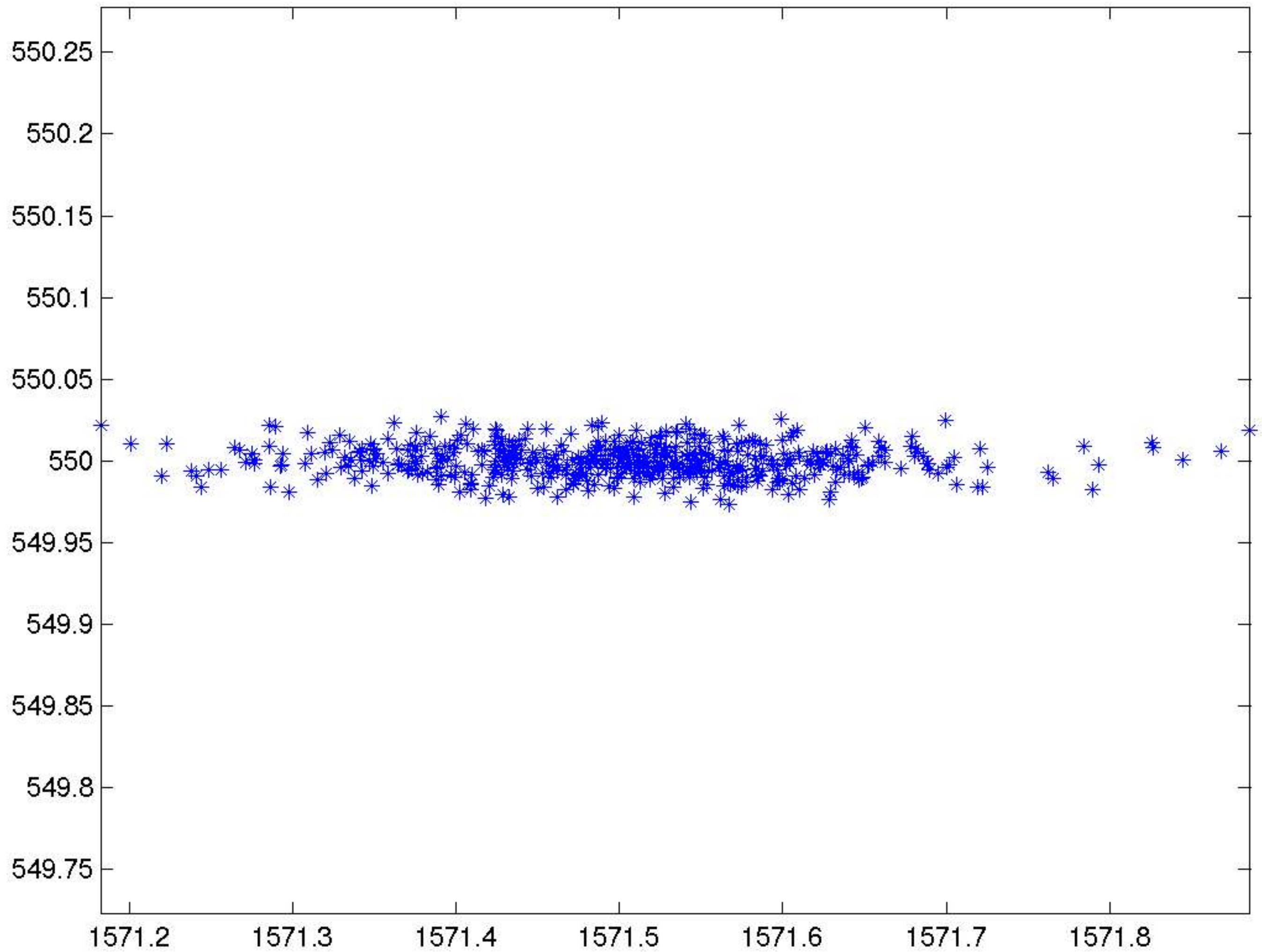
Propagate Angular Uncertainty into an Uncertainty Region around the Intersection Point



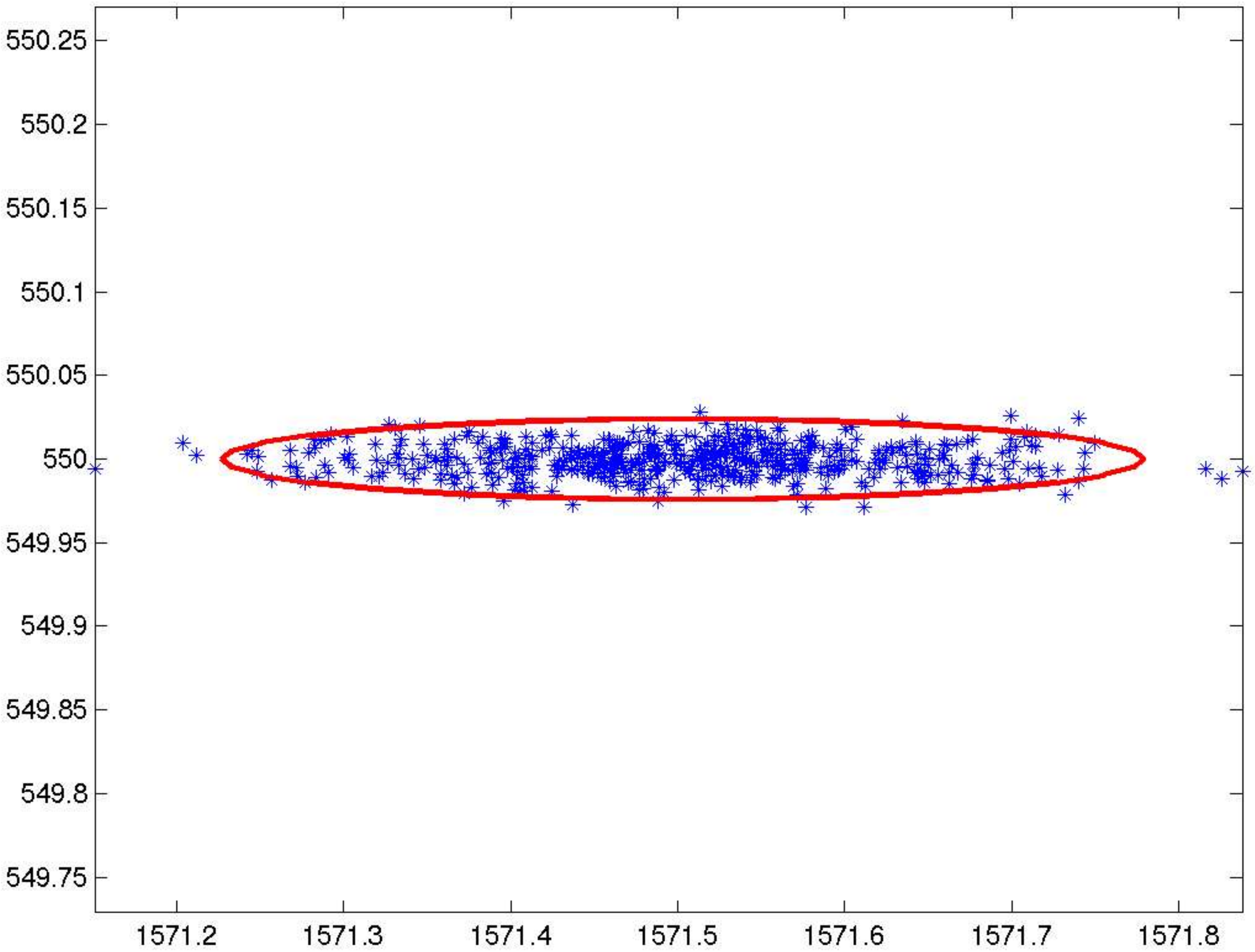
Increase the Angular Uncertainty



Monte Carlo Simulation Yields an Equivalent Picture of the Uncertainty Region as a Scatter Diagram

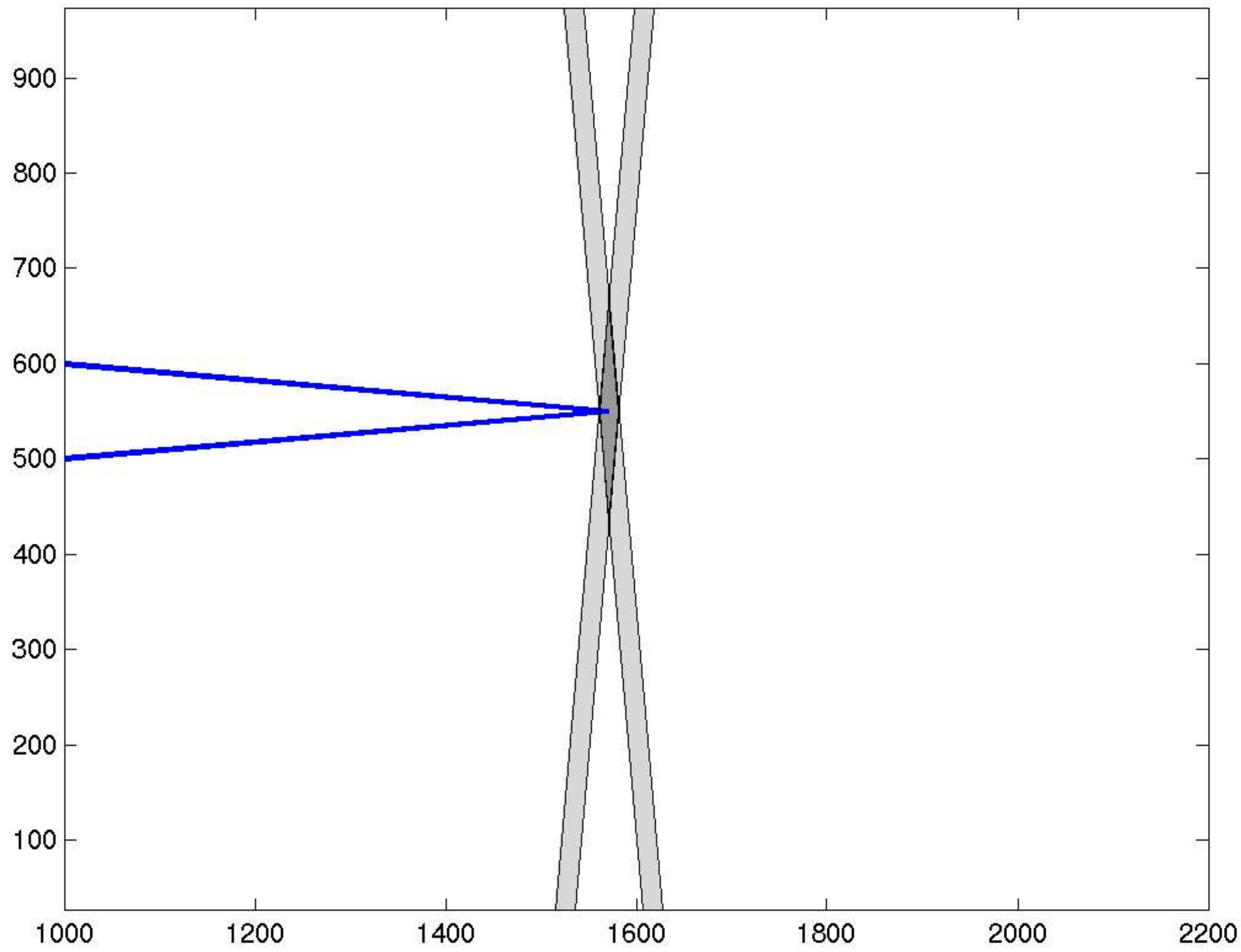


From the Least Squares Equations We Can Compute a Confidence Region (Error Ellipse)

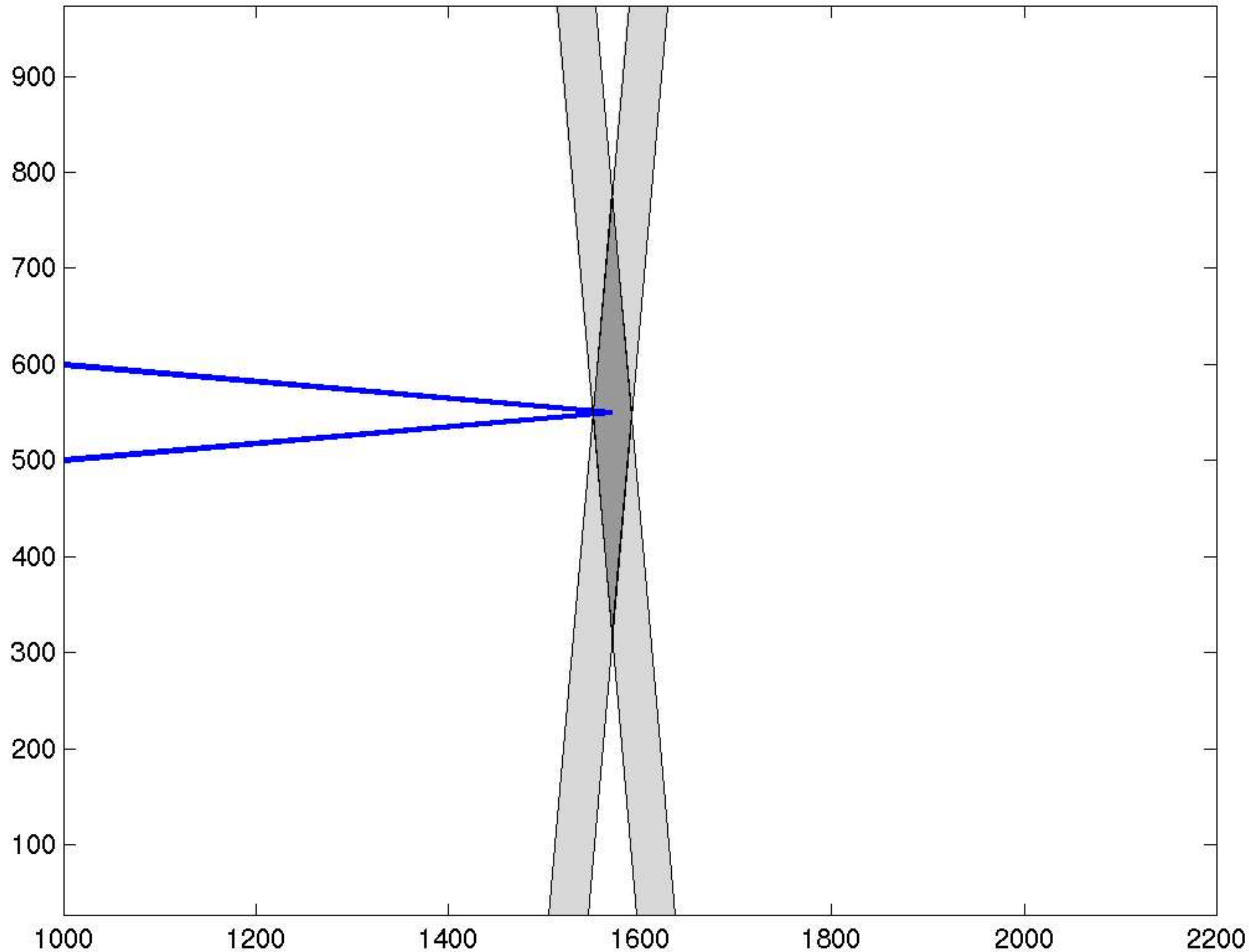


Notice that if we assume an uncertainty this can be plotted *without making any actual observations.* Thus we have a design mode where we can predict confidence ellipses based on assumed observation error and network geometry.

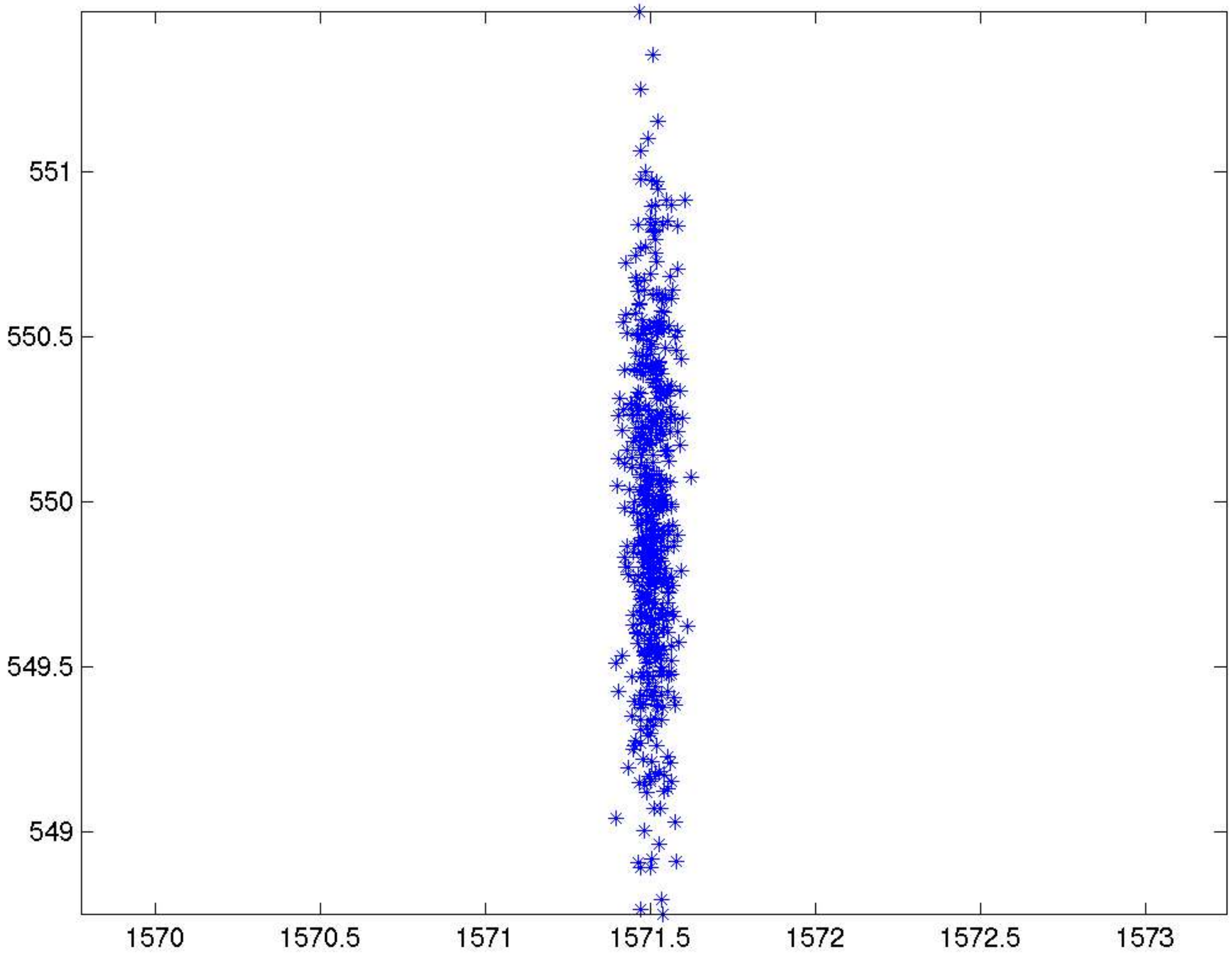
Same Idea – Let's Look at Uncertainty in Distance and Its Effect on the Uncertainty in the Position



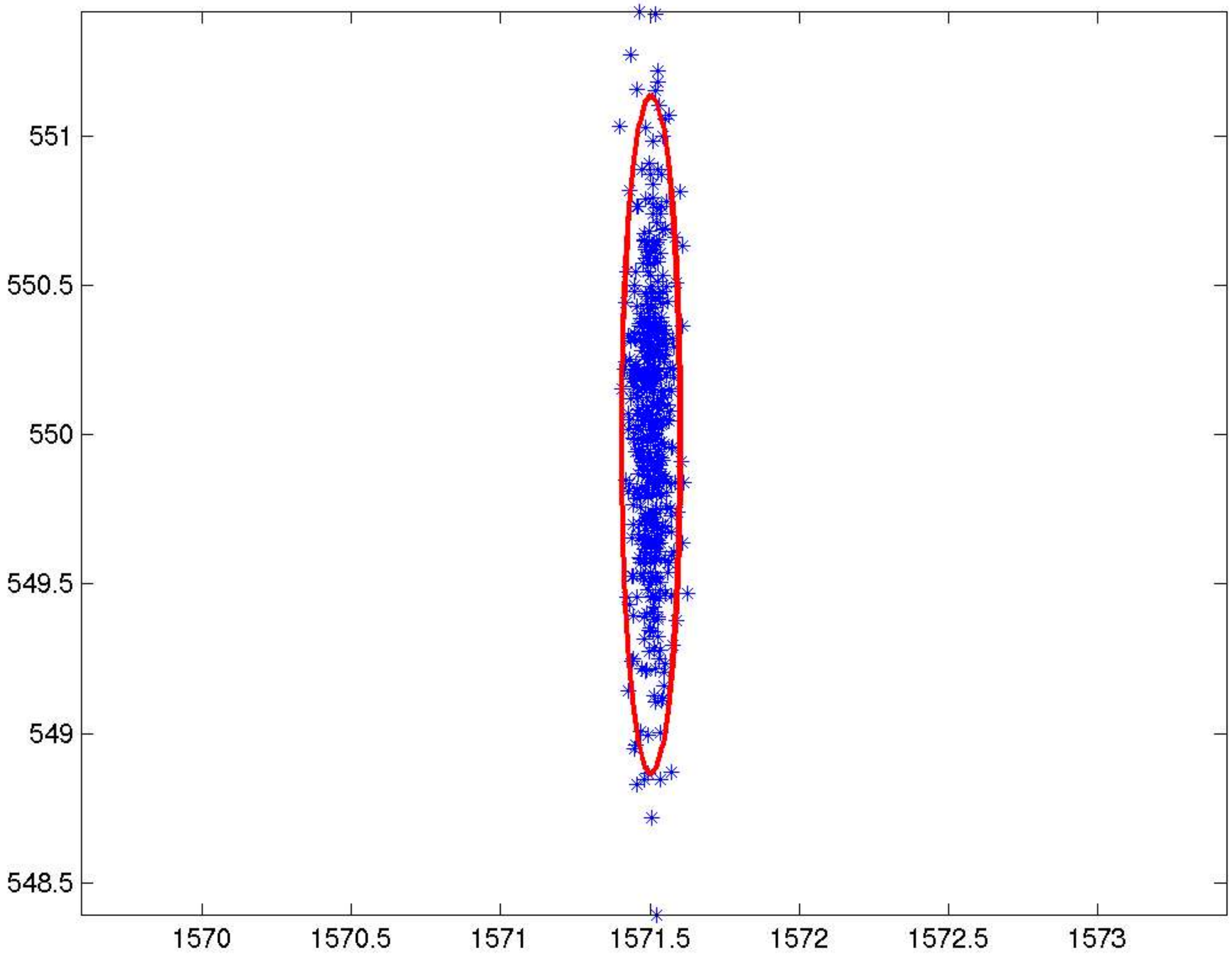
Increase the Uncertainty in the Distance



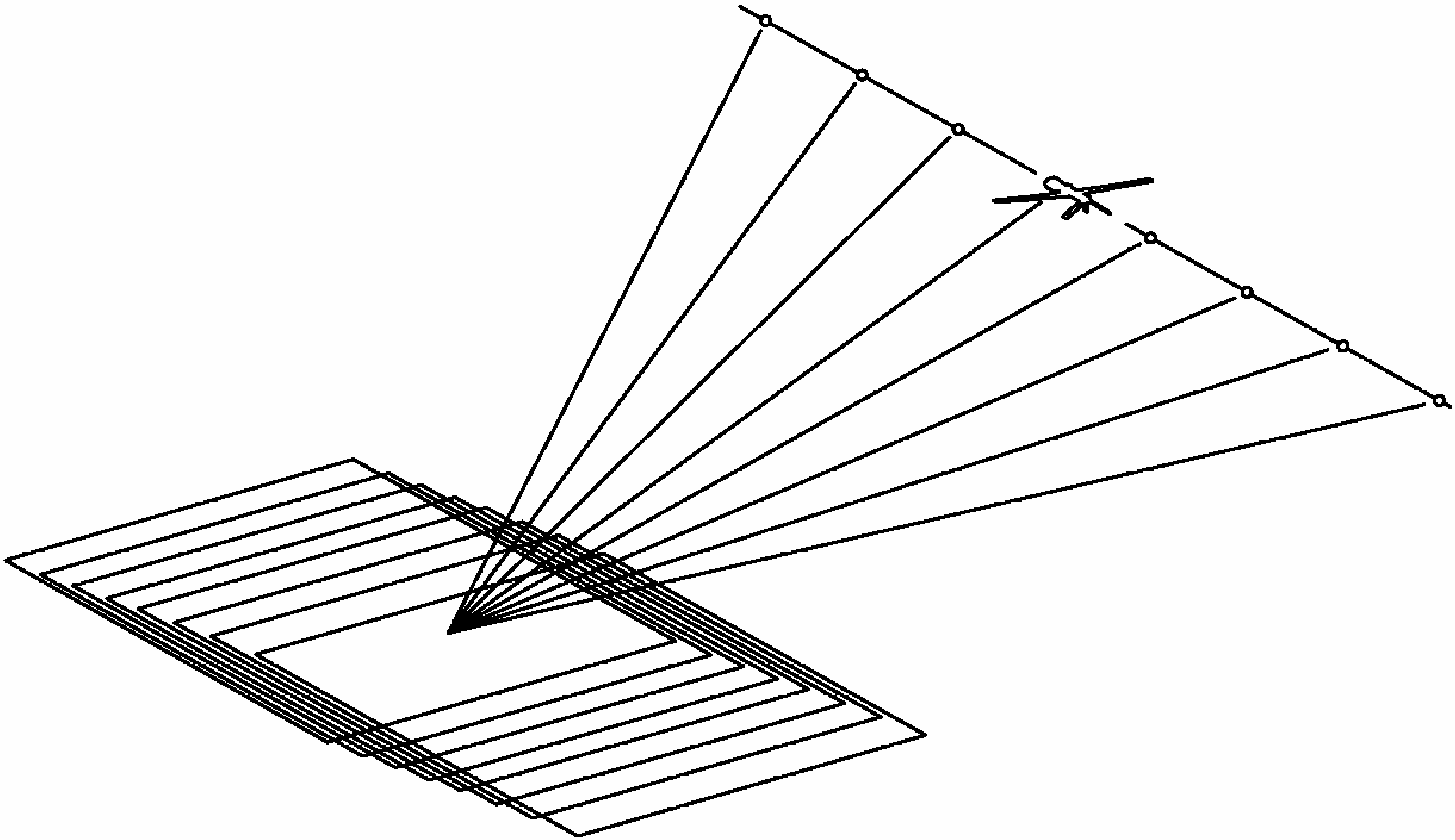
Monte Carlo Simulation on the Same Geometric Model yields a Scatter Diagram



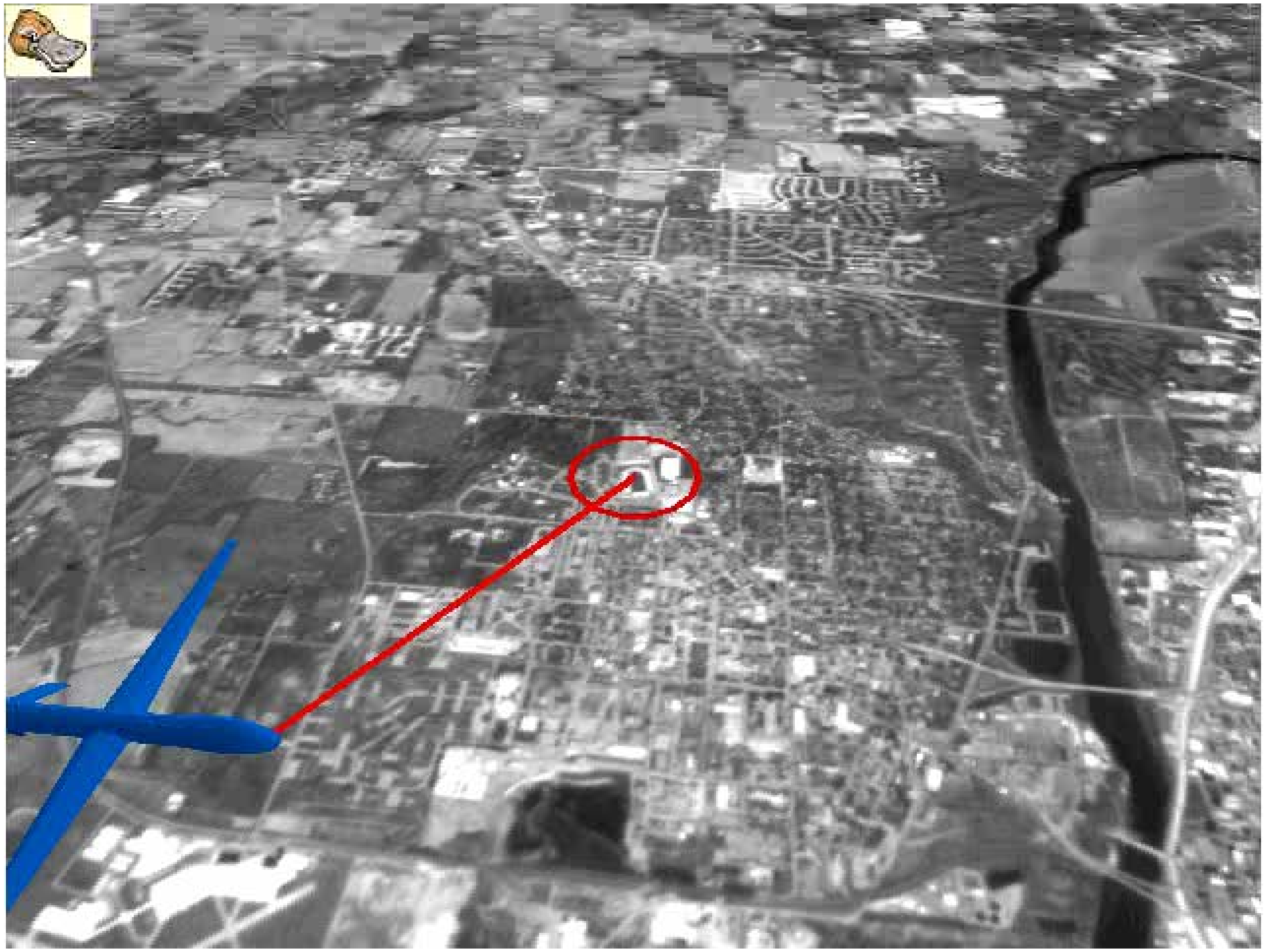
Compute the Corresponding Confidence Ellipse from the Mathematical Model



Multiple Ray Intersections to Define Target Location



Graphic Animation



Covariance Matrices and Confidence Regions

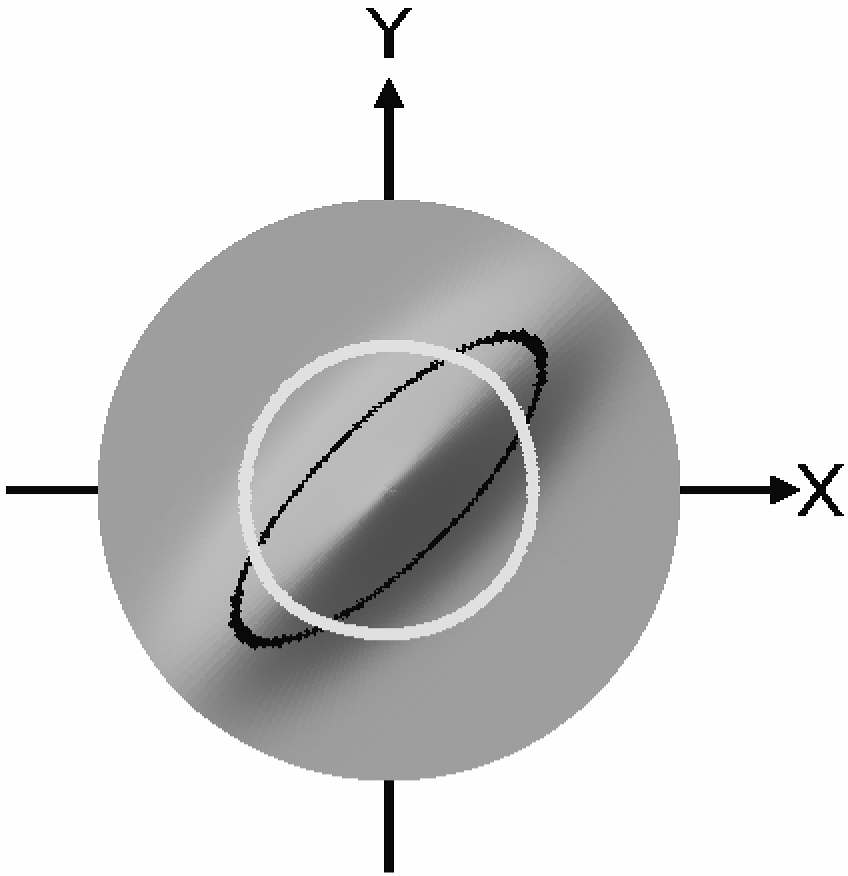
Covariance Matrix, Uncorrelated

$$\mathbf{\Sigma} = \begin{bmatrix} {\sigma_x}^2 & 0 \\ 0 & {\sigma_y}^2 \end{bmatrix}$$

Covariance Matrix, Correlated

$$\mathbf{\Sigma} = \begin{bmatrix} {\sigma_x}^2 & \sigma_{xy} \\ \sigma_{xy} & {\sigma_y}^2 \end{bmatrix}$$

Error Region Tutorial



$$\Sigma_{xx} = \sigma_0^2 Q_{xx}$$

Geometry

$$\mathbf{Q_{xx}} = \left(\mathbf{B^T W B}\right)^{-1}$$

Uncertainty of Observations

σ_0^2 prior (if consistent with results)

$\hat{\sigma}_0^2$ post (if prior not consistent with results)

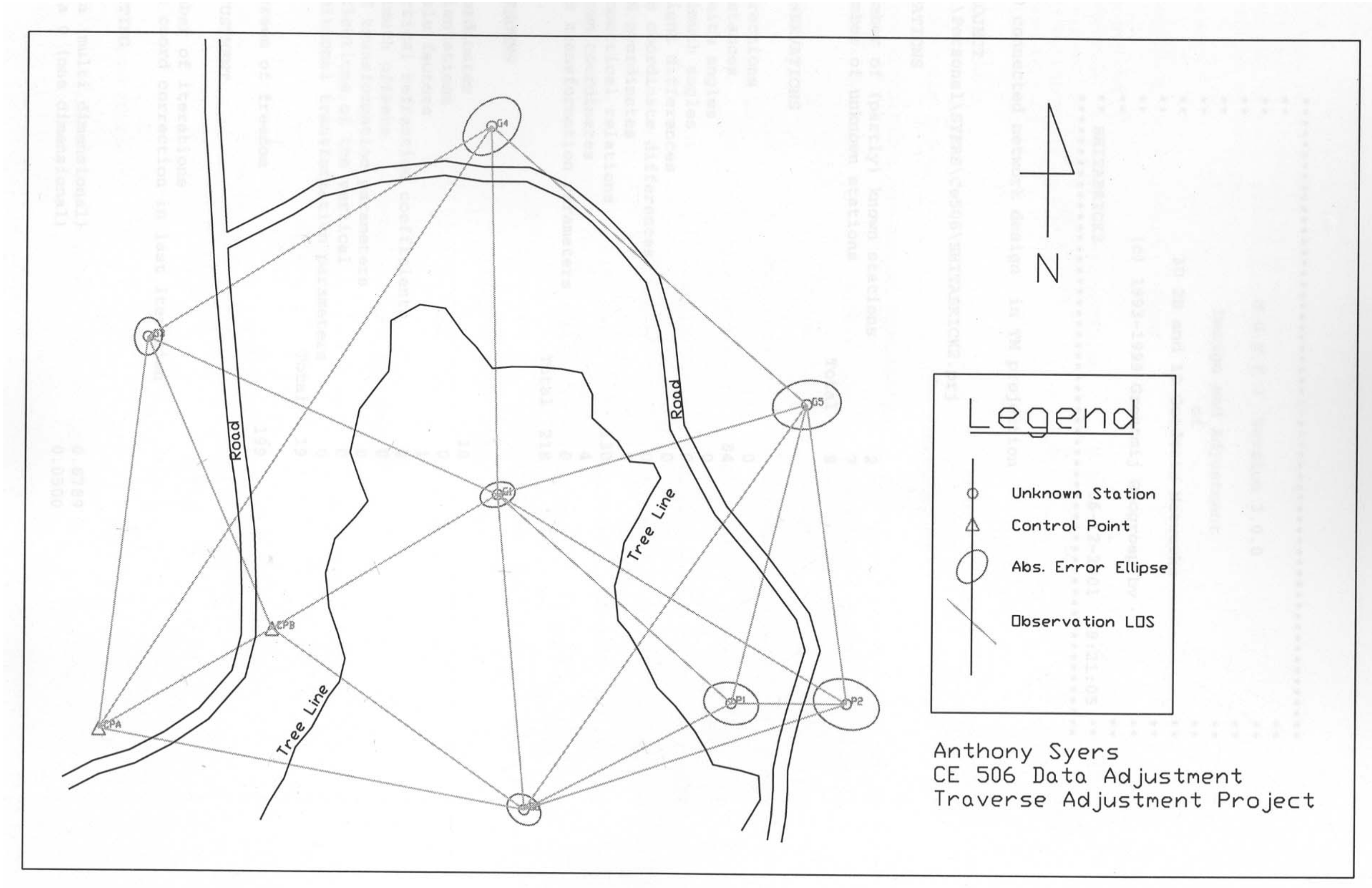
$$w_i = \sigma_0^2 / \sigma_i^2$$

Pre-analysis

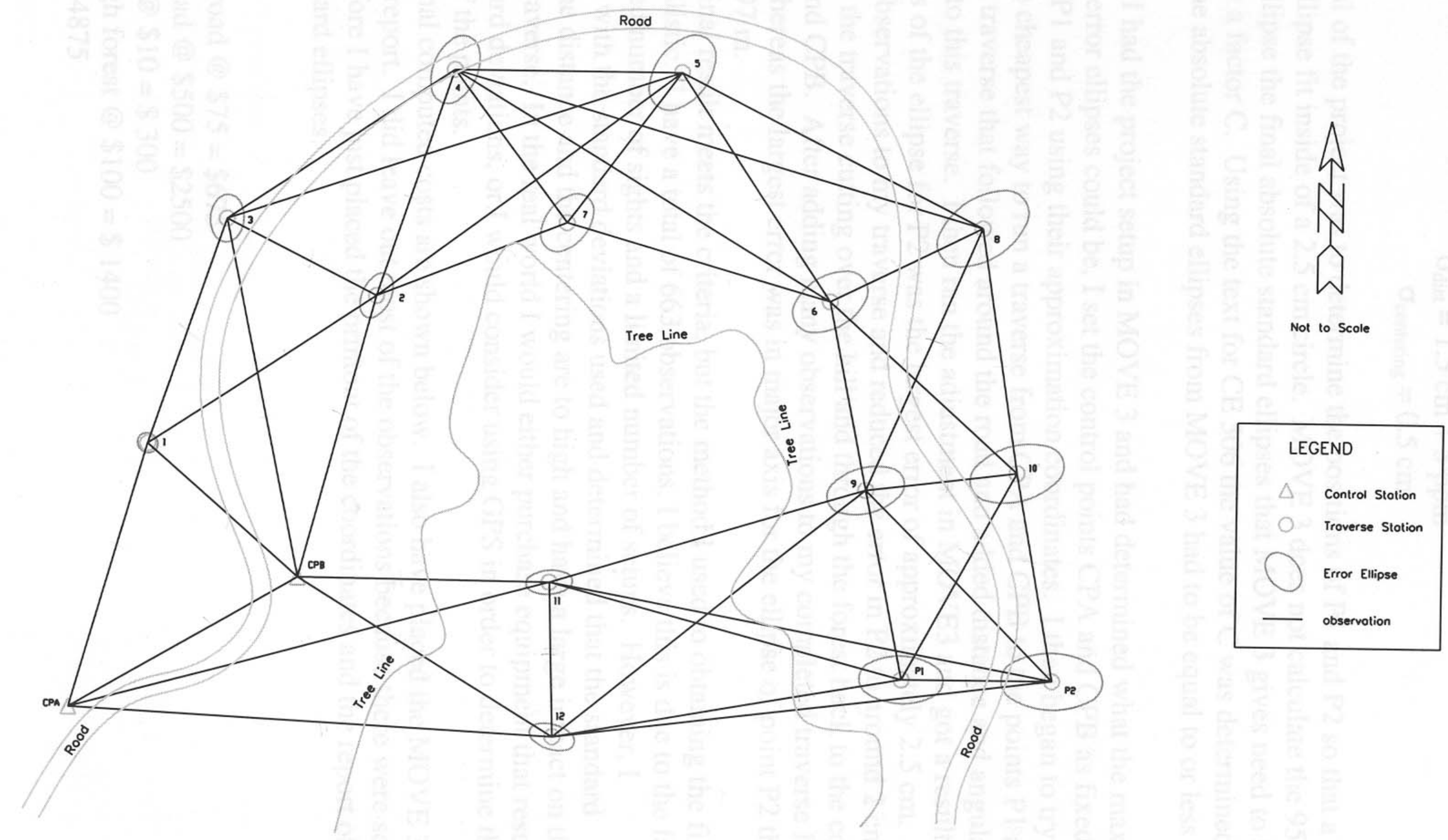
Prior to any deployment of equipment or instruments in the field, you can propose a geometry (network) layout, an observation scheme, and an uncertainty associated with each observation. Thus you have all of the information needed on the previous page to compute the error propagation associated with the proposed measurement and adjustment task. Thus you can pre-compute error (confidence) ellipses or circles. This is known as pre-analysis. Some surveying examples follow.

Important caveat: conventional error propagation only considers *random* errors, not *biases or systematic errors.* If you suspect, or know that these are present, then you must inflate EP results by that knowledge ("consider" covariance concept, Tapley or Montenbruck on orbit estimation)

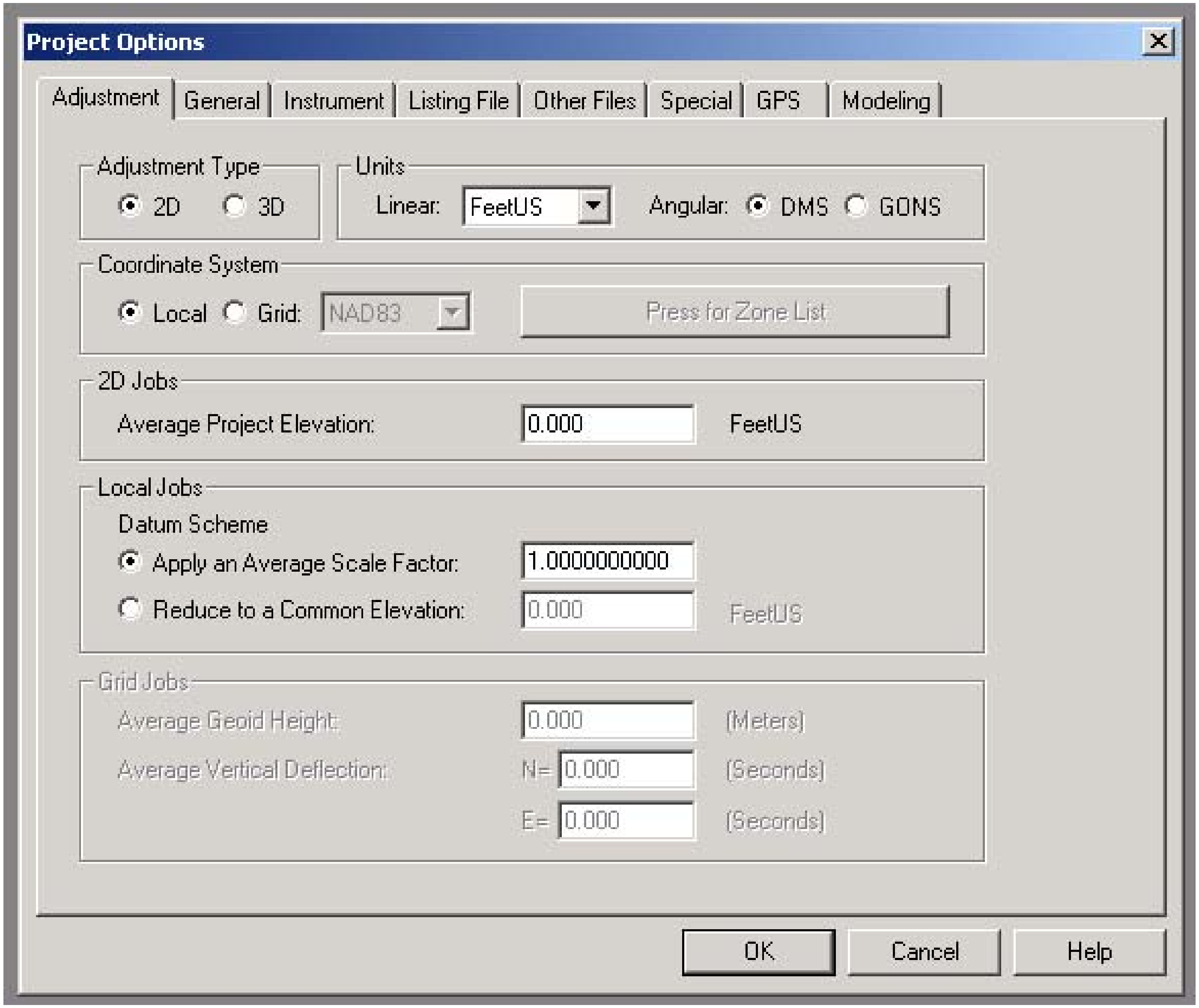
Student Design Example Using Starnet or Move3



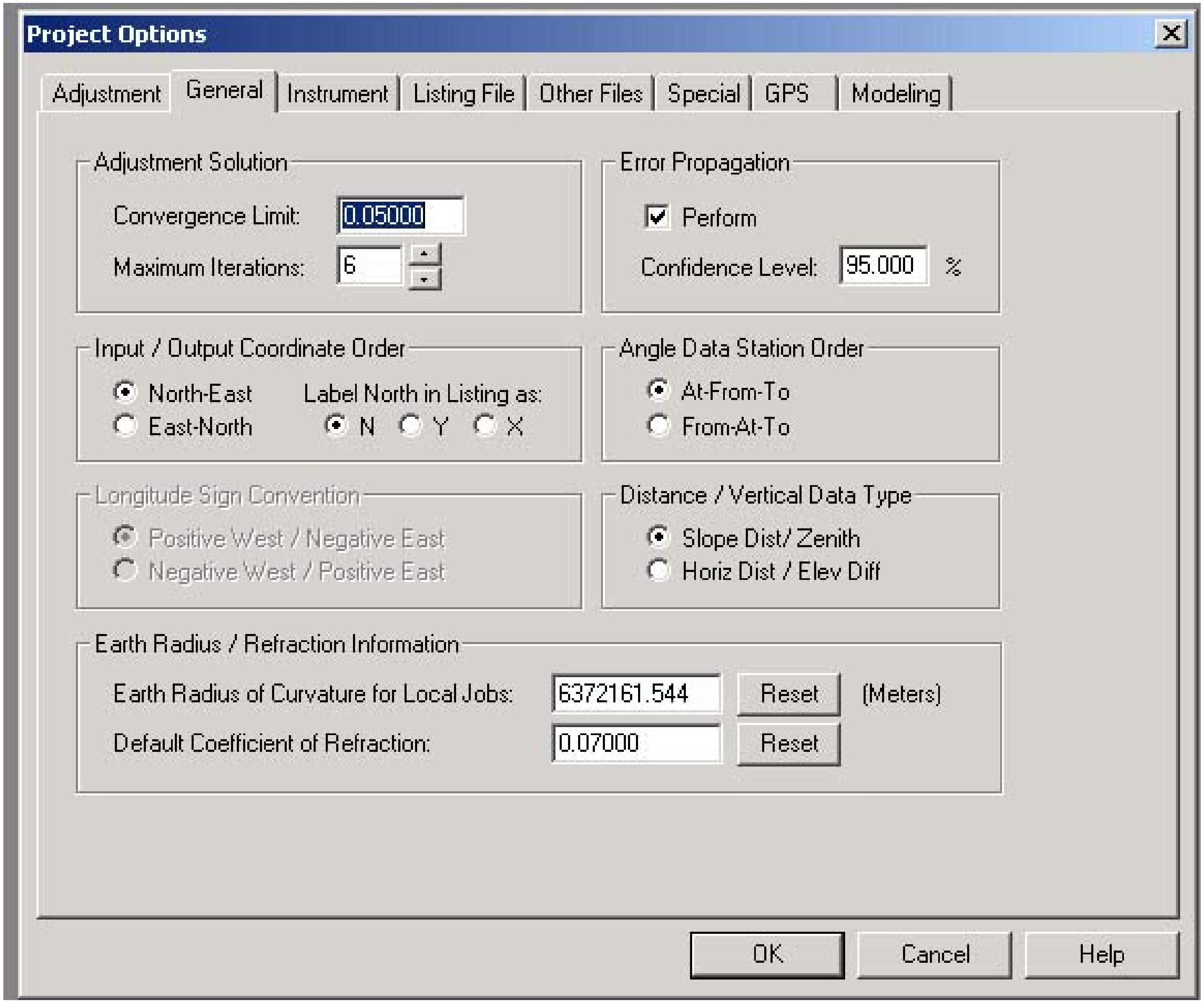
Another Student Design from Data Adjustment I



Starnet Adjustment Option Screen



Starnet Option General Screen

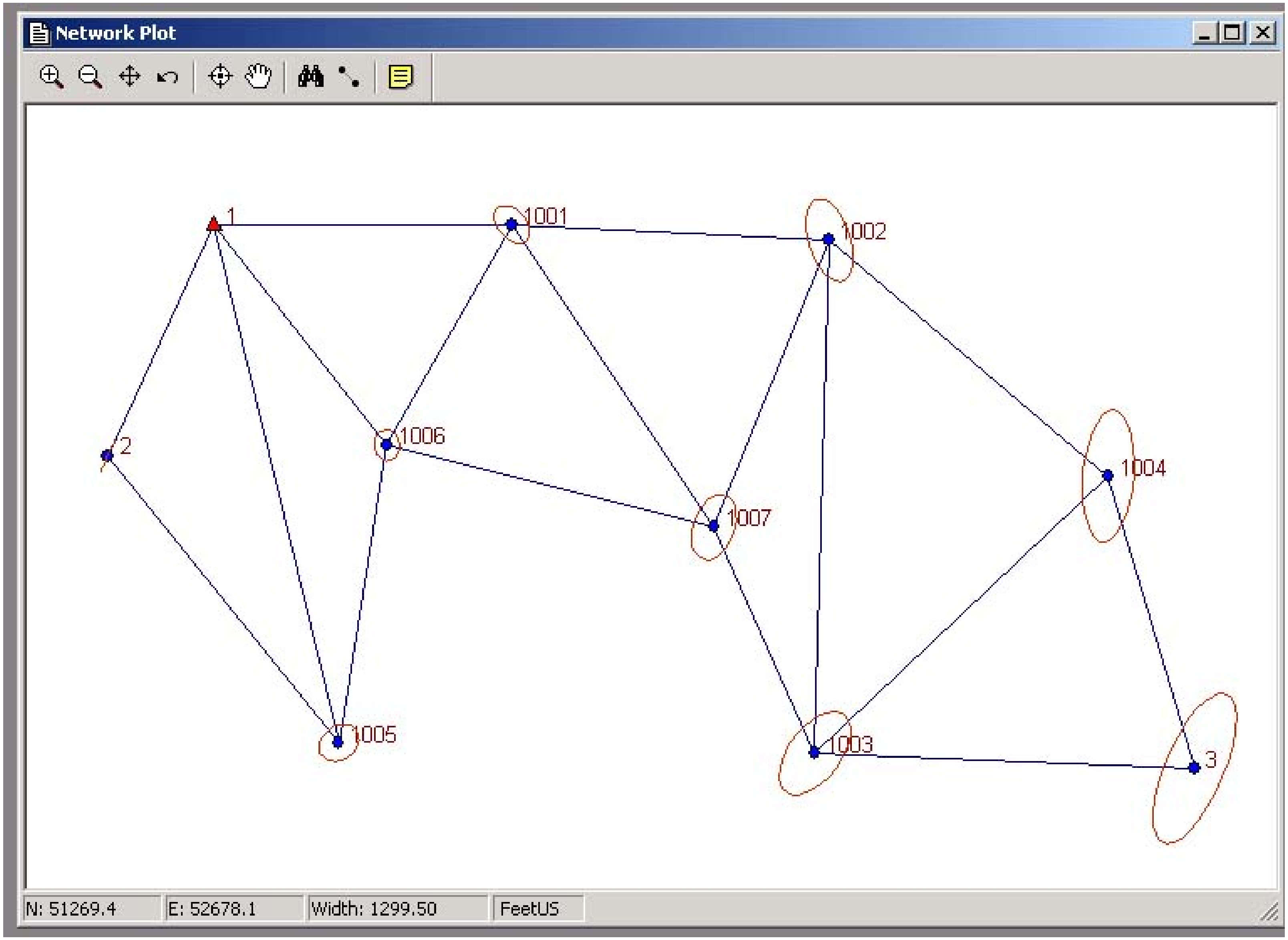


Preanalysis.dat

```
# Preanalysis
#
C    1        51160    52640  ! !
C    2        50935    52530
C    3        50630    53660
C    1001     51160    52950
C    1002     51145    53280
C    1003     50645    53265
C    1004     50915    53570
C    1005     50655    52770
C    1006     50945    52820
C    1007     50865    53160
#
B    1-2        ?   !
#B   1003-3     ?   5.0   # Uncomment this line to add another bearing
#
TB   2
T    1
T    1001
T    1002
T    1004
T    3
T    1003
T    1007
T    1006
T    1005
T    2
TE   1
#
M    1-2-1005
M    1-2-1006
M    1001-1-1006
M    1001-1-1007
M    1002-1001-1007
M    1002-1001-1003
M    1004-1002-1003
A    1003-1007-1002
A    1003-1007-1004
A    1007-1006-1001
A    1007-1006-1002
A    1006-1005-1
A    1006-1005-1001
A    1005-2-1
```

Starnet input data file format

Starnet Preanalysis Plot Screen



Preanalysis.lst

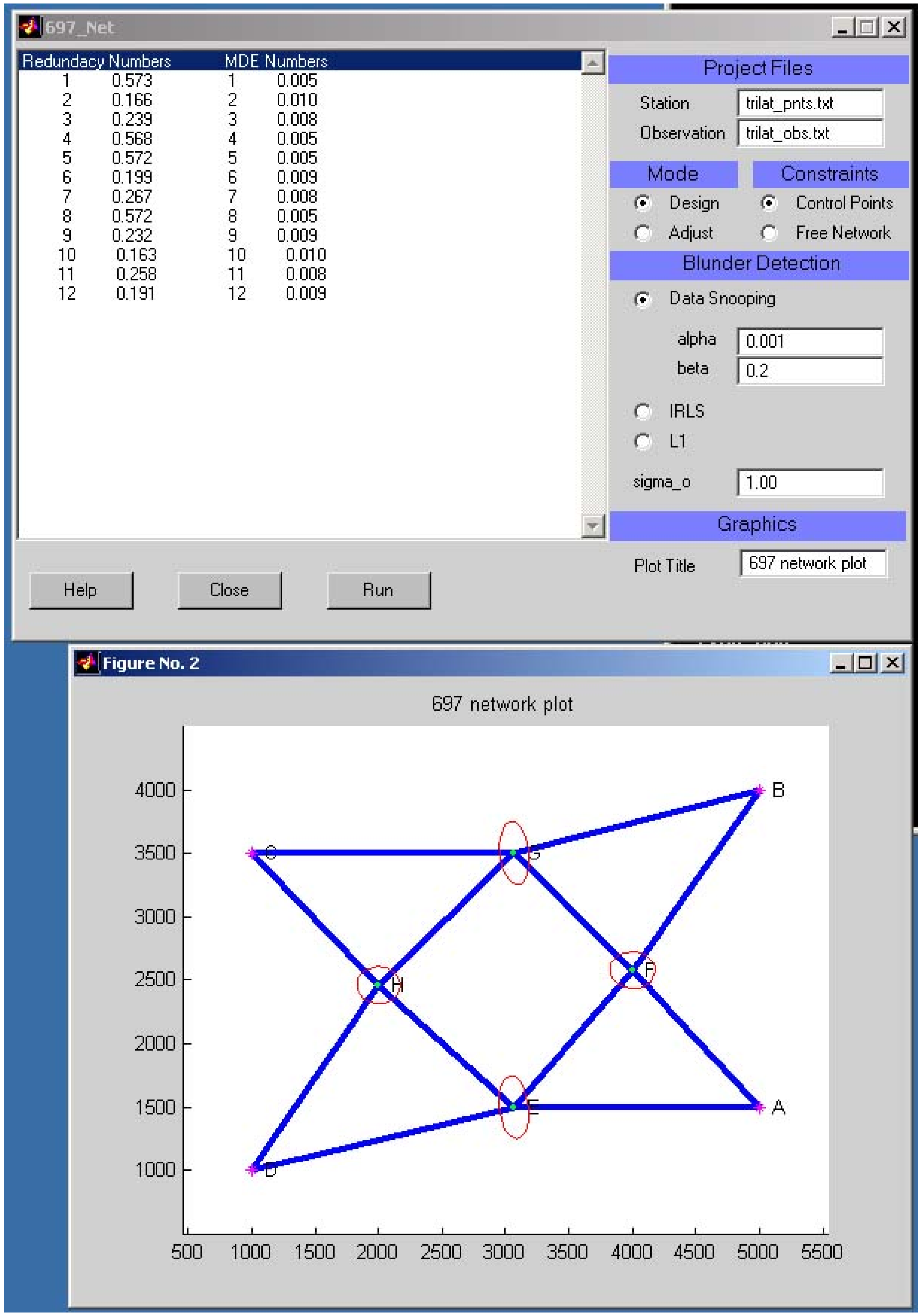
Station Coordinate Error Ellipses (FeetUS)
Confidence Region = 95%

Station	Semi-Major Axis	Semi-Minor Axis	Azimuth of Major Axis
1	0.00000	0.00000	0-00
2	0.01892	0.00000	26-03
3	0.08916	0.03341	24-46
1001	0.02462	0.01585	135-15
1002	0.04688	0.02421	160-48
1003	0.05416	0.02938	40-15
1004	0.07220	0.02947	5-03
1005	0.02431	0.01853	62-18
1006	0.01675	0.01473	176-24
1007	0.03688	0.02354	22-32

Relative Error Ellipses (FeetUS)
Confidence Region = 95%

Stations From	To	Semi-Major Axis	Semi-Minor Axis	Azimuth of Major Axis
1	2	0.01892	0.00000	26-03
1	1001	0.02462	0.01585	135-15
1	1005	0.02431	0.01853	62-18
1	1006	0.01675	0.01473	176-24
1001	1002	0.02989	0.01912	162-42
1001	1006	0.01961	0.01453	126-50
1001	1007	0.02871	0.01646	56-21
1002	1003	0.04565	0.01740	95-09
1002	1004	0.03725	0.02171	27-18
1002	1007	0.02871	0.01491	120-31
1003	3	0.04660	0.02131	8-34
1003	1004	0.04288	0.01682	142-56
1003	1007	0.02603	0.02039	80-46
1004	3	0.03493	0.02296	62-45
1005	2	0.02324	0.01698	74-38
1005	1006	0.02120	0.01780	14-00
1006	1007	0.02952	0.01913	32-55

Starnet output listing for the preanalysis project. Dimensions of absolute and relative error ellipses are given



Adjustment program created by students in Geomatics program. Planned capabilities include blunder detection by L1, IRLS, Data Snooping, also free network adjustment, preanalysis and adjustment modes. Matlab GUI tools and numerical and symbolic processing capabilites provide a very rich environment