Advanced Geospatial Estimation
Exam 1, 1 April 2009
(1 page of notes allowed)

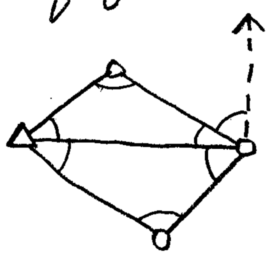
Name ______________________

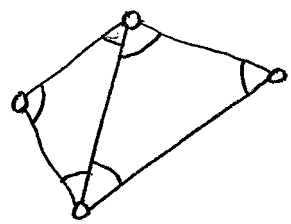
1. The following symbols have the indicated meaning:

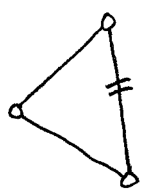
$\measuredangle$ = angle observation, $\nparallel$ = distance observation,

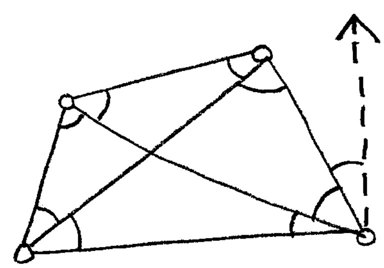
$\uparrow$ = north direction, Δ = ground control point

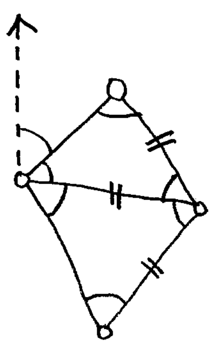
What is the number of minimal constraints needed for each figure? (All 2D, except (f) is 3D)

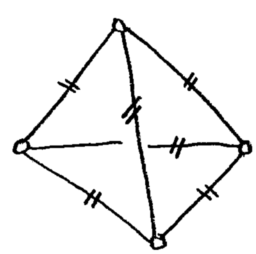
(a)


(b)


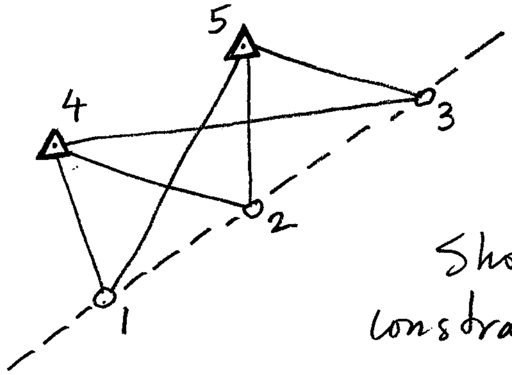
(c)


(d)


(e)


(f) (3D)


2.



In the 2D network shown we wish to constrain points 1, 2, and 3 to lie along a line. Show how you can write the needed constraint equations in two ways: (a) with added parameters, and (b) without added parameters. In each case show the linearized constraint matrix either as C or D_1 & D_2

3. Solve the following LS problem: Fit a <u>line</u> to the data:

X	Y	σ_y
1	1	0.5
2	3	0.5
3	2	0.5

X: constant
Y: observation

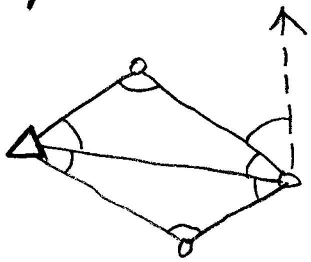
Enforce the <u>prior</u> knowledge that

$$m \text{ (slope)} = 0.4 \quad \text{with } \sigma = 0.4$$
$$b \text{ (intercept)} = 1.5 \quad \text{with } \sigma = 0.4$$

Adv. Geospa. Est. EXAM 1 Solution
April 2009

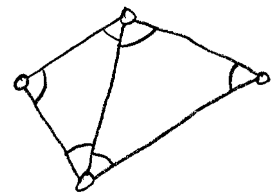
1. Number of minimal constraints:

(a)



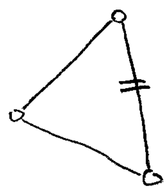
1: scale

(b)



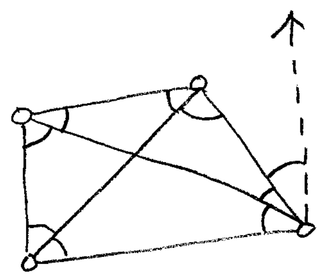
4: T_x, T_y, scale, Orientation

(c)



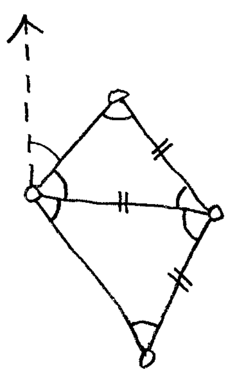
Not counted: figure not complete

(d)



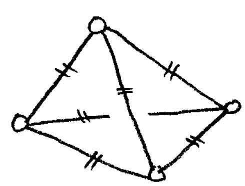
3: T_x, T_y, Scale

(e)



2: T_x, T_y

(f) (3D – tetrahedron)



6: $T_x, T_y, T_z, \underbrace{\omega, \varphi, k}_{\text{orientation}}$

2. Constraints with added parameters : $\underline{m}, \underline{b}$

(a)

$$Y_1 = mX_1 + b \qquad F_{c_1} = Y_1 - mX_1 - b = 0$$
$$Y_2 = mX_2 + b \qquad F_{c_2} = Y_2 - mX_2 - b = 0$$
$$Y_3 = mX_3 + b \qquad F_{c_3} = Y_3 - mX_3 - b = 0$$

linearize :

$$\begin{array}{cccccccc} \underline{x_1} & \underline{y_1} & \underline{x_2} & \underline{y_2} & \underline{x_3} & \underline{y_3} & \underline{m} & \underline{b} \\ \end{array}$$

$$\left[\begin{array}{cccccc:cc} -m & 1 & 0 & 0 & 0 & 0 & -x_1 & -1 \\ 0 & 0 & -m & 1 & 0 & 0 & -x_2 & -1 \\ 0 & 0 & 0 & 0 & -m & 1 & -x_3 & -1 \end{array}\right] \left[\begin{array}{c} \Delta x_1 \\ \Delta y_1 \\ \Delta x_2 \\ \Delta y_2 \\ \Delta x_3 \\ \Delta y_3 \\ \hdashline \Delta m \\ \Delta b \end{array}\right] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

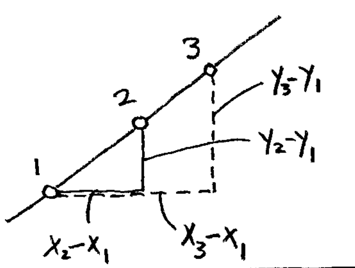
$$C \cdot \Delta = g$$

— OR —

$$\begin{bmatrix} -m & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -m & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -m & 1 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta y_1 \\ \Delta x_2 \\ \Delta y_2 \\ \Delta x_3 \\ \Delta y_3 \end{bmatrix} + \begin{bmatrix} -x_1 & -1 \\ -x_2 & -1 \\ -x_3 & -1 \end{bmatrix} \begin{bmatrix} \Delta m \\ \Delta b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$D_1 \cdot \Delta + D_2 \, \Delta' = h$$

(b) without A.P.'s



$$\frac{Y_3 - Y_1}{X_3 - X_1} = \frac{Y_2 - Y_1}{X_2 - X_1} \quad ; \quad (X_2 - X_1)(Y_3 - Y_1) = (X_3 - X_1)(Y_2 - Y_1)$$

$$X_2Y_3 + X_1Y_1 - X_2Y_1 - X_1Y_3 = X_3Y_2 + X_1Y_1 - X_3Y_1 - X_1Y_2$$

$$F_c = X_2Y_3 - X_2Y_1 - X_1Y_3 - X_3Y_2 + X_3Y_1 + X_1Y_2 = 0$$

linearize :

$$\begin{array}{cccccc} \underline{X_1} & \underline{Y_1} & \underline{X_2} & \underline{Y_2} & \underline{X_3} & \underline{Y_3} \end{array}$$

$$\begin{bmatrix} Y_2 - Y_3 & X_3 - X_2 & Y_3 - Y_1 & X_1 - X_3 & Y_1 - Y_2 & X_2 - X_1 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta y_1 \\ \Delta x_2 \\ \Delta y_2 \\ \Delta x_3 \\ \Delta y_3 \end{bmatrix} = 0$$

$$C \cdot \Delta = g$$

OR

$$\tan^{-1}\left(\frac{X_2 - X_1}{Y_2 - Y_1}\right) = \tan^{-1}\left(\frac{X_3 - X_2}{Y_3 - Y_2}\right)$$

linearize!

3. unified LS, indirect observations, LINEAR!

$y = mx + b$

$y + v_y = mx + b$

$$\boxed{v_y - mx - b = -y}$$

conventional	unified
$n = 3$	$n = 3 + 2 = 5$
$n_0 = 2$	$n_0 = 2$
$r = 1$	$r = 3$
$u = 2$	$u = 2$
$c = r + u = 3$	$c = r + u = 5 \begin{cases} 3 \text{ conventional} \\ 2 \text{ unified} \end{cases}$

$$B = \begin{bmatrix} -1 & -1 \\ -2 & -1 \\ -3 & -1 \end{bmatrix} \quad f = \begin{bmatrix} -1 \\ -3 \\ -2 \end{bmatrix} \quad \sigma_0 = 0.5, \quad W = I_3$$

$$W_{xx} = \begin{bmatrix} \frac{(0.5)^2}{(0.4)^2} & 0 \\ 0 & \frac{(0.5)^2}{(0.4)^2} \end{bmatrix} = \begin{bmatrix} 1.56 & 0 \\ 0 & 1.56 \end{bmatrix}$$

use "total parameter" case:

$$\Delta = (N + W_{xx})^{-1}(t + W_{xx}x)$$

$$N = \begin{bmatrix} 14 & 6 \\ 6 & 3 \end{bmatrix} \quad W_{xx} = \begin{bmatrix} 1.56 & 0 \\ 0 & 1.56 \end{bmatrix} \quad t = \begin{bmatrix} 13 \\ 6 \end{bmatrix} \quad x = \begin{bmatrix} .4 \\ 1.5 \end{bmatrix}$$

$$N + W_{xx} = \begin{bmatrix} 15.56 & 6 \\ 6 & 4.56 \end{bmatrix}, \text{ inverse: } C = \begin{bmatrix} 4.56 & -6 \\ -6 & 15.56 \end{bmatrix}$$

$C^T = C, \quad |N + W_{xx}| = 70.9536 - 36 = 34.9536$

$$(N + W_{xx})^{-1} = \frac{C^T}{|\cdot|} = \frac{\begin{bmatrix} 4.56 & -6 \\ -6 & 15.56 \end{bmatrix}}{34.9536} = \begin{bmatrix} .1305 & -.1717 \\ -.1717 & .4452 \end{bmatrix}$$

$$t + W_{xx}x = \begin{bmatrix} 13 \\ 6 \end{bmatrix} + \begin{bmatrix} 1.56 & 0 \\ 0 & 1.56 \end{bmatrix}\begin{bmatrix} .4 \\ 1.5 \end{bmatrix} = \begin{pmatrix} 13 \\ 6 \end{pmatrix} + \begin{pmatrix} .624 \\ 2.34 \end{pmatrix} = \begin{pmatrix} 13.624 \\ 8.34 \end{pmatrix}$$

$$\Delta = \begin{bmatrix} .1305 & -.1717 \\ -.1717 & .4452 \end{bmatrix}\begin{bmatrix} 13.624 \\ 8.34 \end{bmatrix} = \begin{bmatrix} 0.3460 \\ 1.3737 \end{bmatrix} = \begin{bmatrix} \hat{m} \\ \hat{b} \end{bmatrix}$$

manual inverse

$$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{C^T}{|A|}$$

$$c_{ij} = (-1)^{i+j} m_{ij}$$

Linear ULS:

Δ: total parameter

$t = B^T W_e f$

$\Delta = (N + W_{xx})^{-1}(t + W_{xx}x)$

$V_x = \Delta - x$

Δ: correction

$\bar{f} = d - Al - Bx$

$\dot{t} = B^T W_e \bar{f}$

$\Delta = (N + W_{xx})^{-1}\dot{t}$