Adj. of Geospatial Obs. CE597 **EXAM 1** 6 Nov, 2019

1. LS solution for Indirect observations : $\Delta = (B^T W B)^{-1} B^T W f$

$$W = \begin{bmatrix} \sigma_0^2/\sigma_1^2 & & 0 \\ & \sigma_0^2/\sigma_2^2 & \\ 0 & & \ddots \; \sigma_0^2/\sigma_n^2 \end{bmatrix}$$

, Show that the estimation of Δ does NOT depend on value of σ_0^2.

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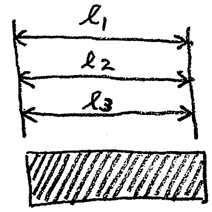
2. You make 3 measurements of a length. You want to adjust by LS.



(a) show condition equations for Indirect Observations, in the form of $v + B\Delta = f$

(b) show condition equations for Observations Only, in the form of $Av = f$

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3. A survey network problem has distance and direction observations, with uncertainties ; $\sigma_{dist} = 0.00002$ km , $\sigma_{dir} = 2'$ (arc minutes)

Show valid weights for the LS adjustment, where km are the length units.

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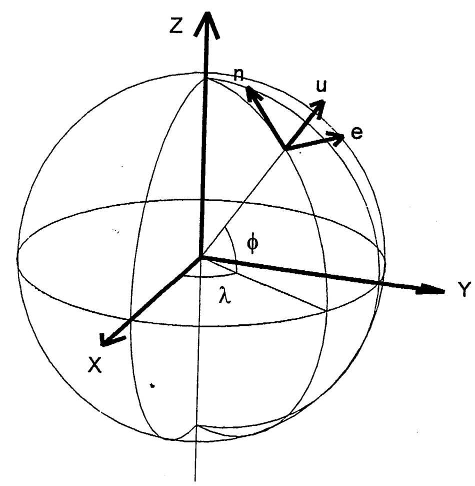
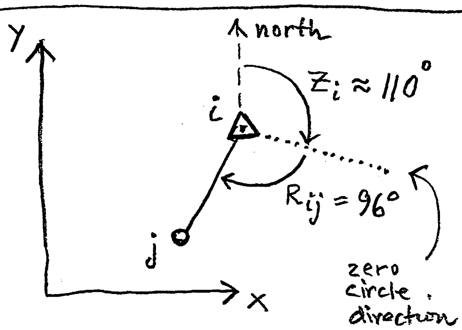
4. The rotation sequence we used to transform $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow \begin{bmatrix} e \\ n \\ u \end{bmatrix}$ (make parallel) was 1. $M_Z(\lambda + 90°)$ 2. $M_X(90° - \phi)$

Find another rotation sequence to make the same transformation.



5.



$(X_i, Y_i) = (100, 100)$

$(X_j, Y_j) \approx (80, 60)$

Z_i : orientation unknown

R_{ij} : direction observation

This is a 2D problem.

(a) Write the direction condition equation for Indirect Observations.

(b) Evaluate numerically the right hand side term, f, if the equation is linearized.

Show your work.

Exam 1 solution

$$W = \sigma_0^2 \begin{bmatrix} \frac{1}{\sigma_1^2} & & & \\ & \frac{1}{\sigma_2^2} & & \\ & & \ddots & \\ & & & \frac{1}{\sigma_n^2} \end{bmatrix} = \sigma_0^2 \Sigma^{-1}$$

$$\Delta = \left(B^T \sigma_0^2 \Sigma^{-1} B\right)^{-1} B^T \sigma_0^2 \Sigma^{-1} f$$

$$\Delta = \cancel{\frac{1}{\sigma_0^2}} \left(B^T \Sigma^{-1} B\right)^{-1} B^T \cancel{\sigma_0^2} \Sigma^{-1} f = \left(B^T \Sigma^{-1} B\right)^{-1} B^T \Sigma^{-1} f$$

$\boxed{n=3,\ n_0=1,\ r=2}$

2.

$$\begin{aligned} l_1+v_1 &= x \\ l_2+v_2 &= x \\ l_3+v_3 &= x \end{aligned} \qquad \begin{aligned} v_1 - x &= -l_1 \\ v_2 - x &= -l_2 \\ v_3 - x &= -l_3 \end{aligned} \qquad \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} x = \begin{bmatrix} -l_1 \\ -l_2 \\ -l_3 \end{bmatrix}$$

$$\begin{aligned} \hat{l}_1 &= \hat{l}_2 \\ \hat{l}_1 &= \hat{l}_3 \end{aligned} \qquad \begin{aligned} l_1+v_1 &= l_2+v_2 \\ l_1+v_1 &= l_3+v_3 \end{aligned} \qquad \begin{aligned} v_1-v_2 &= -(l_1-l_2) \\ v_1-v_3 &= -(l_1-l_3) \end{aligned} \qquad \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = - \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$$

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3. $\sigma_{dir} = 2' = \frac{2}{60}\text{ deg} = \frac{2}{60} \cdot \frac{\pi}{180}\text{ rad} = .0005818\text{ rad}$ } let $\sigma_0 = .0005818$

$\sigma_{dist} = .00002\text{ km}$

$$W_{dir} = \frac{(.0005818)^2}{(.0005818)^2} = 1, \quad W_{dist} = \frac{(.0005818)^2}{(.00002)^2} = 846$$

4.

1. $M_z(\lambda)$	1. $M_z(-(90-\lambda))$	1. $M_z(-((90-\lambda)+90))$
2. $M_y(90^\circ-\phi)$	2. $M_x(-(90-\phi))$	2. $M_y(-(90-\phi))$
3. $M_z(90^\circ)$	3. $M_z(180^\circ)$	3. $M_z(-90)$

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5. $Z_i + R_{ij} = Az_{ij}$, $F_{dir} = R_{ij} + z_i - Az_{ij} = 0$, (a) $\boxed{F_{dir} = R_{ij} + z_i - \operatorname{atan}\left(\frac{x_j - x_i}{y_j - y_i}\right) = 0}$

(b) $f = -F$: $-\left(R_{ij} + z_i - \operatorname{atan}\left(\frac{80-100}{60-100}\right)\right)$, $\operatorname{atan}\left(\frac{-20}{-40}\right) = 26.56 + 180 = 206.56$

adjust for quadrant

$$f = -(96^\circ + 110^\circ - 206.56^\circ) \cdot \frac{\pi}{180} = -(-.56^\circ) \cdot \frac{\pi}{180} = .00977\text{ R}$$