

## Homework 5 GPS Navigation Solution

Solve the least squares problem to find the receiver position given the observed pseudoranges and the tabulated ephemeris data. For this indirect observation problem, the condition equation is,

$$PR_i = \left[ (X - X_{S_i})^2 + (Y - Y_{S_i})^2 + (Z - Z_{S_i})^2 \right]^{1/2} + c \cdot DT$$

OR

$$F_i = PR_i - \left[ (X - X_{S_i})^2 + (Y - Y_{S_i})^2 + (Z - Z_{S_i})^2 \right]^{1/2} - c \cdot DT = 0$$

For our data the pseudo range observation, C1, is corrupted by 3 systematic errors : (1) receiver clock bias, common to all observations at the same epoch (time), (2) satellite clock bias, slowly varying function of time, per satellite, and (3) troposphere delay, dependent on environmental (weather) conditions and on the elevation angle to the satellite (at zenith minimum travel, at low elevation angle maximum travel & delay through the troposphere). Assume  $\sigma_{PR} = 5\text{ m}$ . Make a 2-sided global test on reference variance at  $\alpha = .05$ . Make a 90% confidence interval for each of  $\mu_x, \mu_y, \mu_z$ . Find all data in hw5-data2.m4t. Assigned Friday 19 October, 2018, due ----- See accompanying template & needed functions.

I provide tropom (via Kai Borre) and interp-lagr.m,  
You provide xyz2geo.m and xyz2enu.m.

## TEMPLATE

initialize geometric range<sub>i</sub> = pseudorange<sub>i</sub> , per obs

$$\Delta t_i = \text{geom. range}_i / c \quad (\text{units!})$$

initialize unknowns  $X, Y, Z, DT$  for receiver  $km, \mu\text{sec}$

Same  $X_{\text{ref}}, Y_{\text{ref}}, Z_{\text{ref}}$

Initialize C, enviro. constants

load ephemeris data  $sX, sY, sZ, scEk$  10 epochs @ 15 min., 32 sats.

create indep. variable  $t = [-60 \ -45 \ -30 \ -15 \ 0 \ 15 \ 30 \ 45 \ 60 \ 75]$

load observation data C1, sats, note time epoch of obs.

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for iter = 1:10          (convergence check later)
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$$B = \text{zeros}(nsat, 4)$$

$$f = \text{zeros}(n_{\text{sat}}, 1)$$

for  $i = 1 : nsat$

$$t_x = t_0 - \Delta t_i \quad (t_x: \text{transmission time})$$

$$x = sX(\cdot, \text{sats}(i))$$

$X_{\text{ef}} = \text{interp\_lagr}(10, t, x, t_x)$

$$y = \text{sy}(\text{sats}(i))$$

$$Y_{\text{ef}} = \text{interp-lagr}(10, t, y, t_x)$$

$$z = \text{set}(\dots, \text{sets}(i))$$

$$Z_{\text{ef}} = \text{interp-lagr}(10, t, z, t_x)$$

$$clk = sclk \left( \vdots, \text{sats}(i) \right)$$

`Clk = interp_lagr(10, t, clk, tx)`

$$\Theta = (\Delta t / \text{length of sidereal day}) * 2\pi$$

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$$\begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix}_i = M_z(\theta) \begin{bmatrix} X_{ecf} \\ Y_{ecf} \\ Z_{ecf} \end{bmatrix} \quad \text{rotate into receive epoch}$$

correct PR for 2 sys. errors

+  $C * \text{Clk}$  sat clock, units

- troposphere delay term (length units)

for tropo correction, need local elevation angle to sat

$XYZ_{\text{sat}} \rightarrow ENU_{\text{sat}} \rightarrow \text{elevation angle}$

evaluate const. eqn. (i)  $\notin$  partials

$$F = \dots$$

$$\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}, \frac{\partial F}{\partial D_T} = \dots$$

$$B(i, :) = [\dots]$$

$$f(i) = (\cdot)$$

end (for  $i = 1 : n_{\text{sat}}$ )

args for tropo.m  
 all height args: 0.236 km  
 pressure: 1018.9 mb  
 temp: 277.6 deg. K  
 humidity: 82.2%  
 you compute elev. angle  
 and sine(elev\_ang)

form and solve NE

make parameter corrections

$$\text{geometric range}_i = \left[ (X - X_{s_i})^2 + (Y - Y_{s_i})^2 + (Z - Z_{s_i})^2 \right]^{1/2}$$

$$\Delta t_i = \text{geom. range}_i / C$$

end (for iter = 1 : 10)

compute  $V$ .

compute ENU of est. receiver pos. with respect to  $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\text{Ref}}$   
 post adjustment statistics