

1.

$$n = 15$$

$$n_0 = 10$$

$$r = 5$$

$n = 10$ parameters
elev. at unknown points

$x_1, x_2, x_3, x_4, x_6, x_7, x_8,$

x_{10}, x_{11}, x_{12}

$$\sigma_{14} = .02\sqrt{10} = .063$$

$$\sigma_0 = \hat{\sigma}_{14}$$

$$W_i = \frac{\sigma_0^2}{\sigma_i^2}$$

$$l_1 + v_1 = x_2 - x_1$$

$$l_2 + v_2 = x_{10} - x_2$$

$$l_3 + v_3 = x_{11} - x_{10}$$

$$l_4 + v_4 = x_{11} - x_{12}$$

$$l_5 + v_5 = x_{12} - x_7$$

$$l_6 + v_6 = x_7 - 195.90$$

$$l_7 + v_7 = 195.90 - x_6$$

$$l_8 + v_8 = x_1 - x_6$$

$$l_9 + v_9 = 195.90 - x_4$$

$$l_{10} + v_{10} = x_8 - x_4$$

$$l_{11} + v_{11} = x_7 - x_8$$

$$l_{12} + v_{12} = x_4 - x_3$$

$$l_{13} + v_{13} = x_3 - x_2$$

$$l_{14} + v_{14} = 194.80 - x_{10}$$

$$l_{15} + v_{15} = x_8 - 194.80$$

$$v_1 - x_2 + x_1 = -l_1$$

$$v_2 - x_{10} + x_2 = -l_2$$

$$v_3 - x_{11} + x_{10} = -l_3$$

$$v_4 - x_{11} + x_{12} = -l_4$$

$$v_5 - x_{12} + x_7 = -l_5$$

$$v_6 - x_7 = -l_6 - 195.90$$

$$v_7 + x_6 = 195.90 - l_7$$

$$v_8 - x_1 + x_6 = -l_8$$

$$v_9 + x_4 = 195.90 - l_9$$

$$v_{10} - x_8 + x_4 = -l_{10}$$

$$v_{11} - x_7 + x_8 = -l_{11}$$

$$v_{12} - x_4 + x_3 = -l_{12}$$

$$v_{13} - x_3 + x_2 = -l_{13}$$

$$v_{14} + x_{10} = 194.80 - l_{14}$$

$$v_{15} - x_8 = -194.80 - l_{15}$$

$$B = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_6 & x_7 & x_8 & x_{10} & x_{11} & x_{12} \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

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hw2_1_sol
sig0 =
0.0632

      x1      x2      x3      x4      x6      x7      x8      x10      x11      x12
delt =
189.2560 190.0151 192.3574 194.2095 188.6607 199.3724 196.0690 192.1488 201.2646 200.6615

      #      l      ns      sig      wt      d      f      v      lhat
ans =
1.0000 0.7610 3.0000 0.0346 3.3333      0 -0.7610 -0.0018 0.7592
2.0000 2.1440 5.0000 0.0447 2.0000      0 -2.1440 -0.0103 2.1337
3.0000 9.1380 8.0000 0.0566 1.2500      0 -9.1380 -0.0222 9.1158
4.0000 0.5920 4.0000 0.0400 2.5000      0 -0.5920 0.0111 0.6031
5.0000 1.2640 9.0000 0.0600 1.1111      0 -1.2640 0.0250 1.2890
6.0000 3.4720 6.0000 0.0490 1.6667 -195.9000 -199.3720 0.0004 3.4724
7.0000 7.2350 7.0000 0.0529 1.4286 195.9000 188.6650 0.0043 7.2393
8.0000 0.5990 6.0000 0.0490 1.6667      0 -0.5990 -0.0037 0.5953
9.0000 1.7100 6.0000 0.0490 1.6667 195.9000 194.1900 -0.0195 1.6905
10.0000 1.8360 5.0000 0.0447 2.0000      0 -1.8360 0.0235 1.8595
11.0000 3.2980 2.0000 0.0283 5.0000      0 -3.2980 0.0054 3.3034
12.0000 1.8420 7.0000 0.0529 1.4286      0 -1.8420 0.0101 1.8521
13.0000 2.3350 5.0000 0.0447 2.0000      0 -2.3350 0.0072 2.3422
14.0000 2.6440 10.0000 0.0632 1.0000 194.8000 192.1560 0.0072 2.6512
15.0000 1.2750 3.0000 0.0346 3.3333 -194.8000 -196.0750 -0.0060 1.2690
diary off

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2. $\hat{x} = aX + bY + c$
 $\hat{y} = -bX + aY + d$
 $\hat{x} - aX - bY - c = 0$
 $\hat{y} + bX - aY - d = 0$

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} + \begin{bmatrix} -X & -Y & -1 & 0 \\ -Y & X & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix}$$

$$V + B \Delta = f$$

#points = 8, $n = 2 \times \#pts = 16$

$n_0 = 4, r = 12$

$\sigma_x = 0.1, \sigma_y = 0.1 \Rightarrow W = I$

$$\begin{bmatrix} v_{x_1} \\ v_{y_1} \\ v_{x_2} \\ v_{y_2} \\ \vdots \\ v_{x_8} \\ v_{y_8} \end{bmatrix} + \begin{bmatrix} -X_1 & -Y_1 & -1 & 0 \\ -Y_1 & X_1 & 0 & -1 \\ -X_2 & -Y_2 & -1 & 0 \\ -Y_2 & X_2 & 0 & -1 \\ \vdots & \vdots & \vdots & \vdots \\ -X_8 & -Y_8 & -1 & 0 \\ -Y_8 & X_8 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -x_1 \\ -y_1 \\ -x_2 \\ -y_2 \\ \vdots \\ -x_8 \\ -y_8 \end{bmatrix}$$

$V + B \Delta = f$

$$B = \begin{bmatrix} -1 & -3 & -1 & 0 \\ -3 & 1 & 0 & -1 \\ -4 & -4 & -1 & 0 \\ -4 & 4 & 0 & -1 \\ -3 & -2 & -1 & 0 \\ -2 & 3 & 0 & -1 \\ -3 & -3 & -1 & 0 \\ -3 & 3 & 0 & -1 \\ -3 & -5 & -1 & 0 \\ -5 & 3 & 0 & -1 \\ -4 & -4 & -1 & 0 \\ -4 & 4 & 0 & -1 \\ -5 & -2 & -1 & 0 \\ -2 & 5 & 0 & -1 \\ -5 & -3 & -1 & 0 \\ -3 & 5 & 0 & -1 \end{bmatrix} f = \begin{bmatrix} -3.83 \\ -0.83 \\ -7.54 \\ -0.46 \\ -5.58 \\ +1.08 \\ -5.98 \\ +0.37 \\ -7.04 \\ -2.20 \\ -7.50 \\ -0.57 \\ -7.70 \\ +2.23 \\ -8.17 \\ +0.98 \end{bmatrix}$$

$a = k \cos \theta$
 $b = k \sin \theta$
 $a^2 + b^2 = k^2 \cos^2 \theta + k^2 \sin^2 \theta$
 $= k^2 (\cos^2 \theta + \sin^2 \theta)$
 $= k^2 \cdot 1$

$\Rightarrow k = \sqrt{a^2 + b^2}, 2 \text{ values } +/-, k = \pm 1.1913$

$\oplus \cos \theta = a/k, \sin \theta = b/k$
 $\text{atan2}(\sin \theta, \cos \theta) \rightarrow 23.5581$

$\ominus \cos \theta = a/k, \sin \theta = b/k$
 $\text{atan2}(\sin \theta, \cos \theta) \rightarrow 23.5581 + 180^\circ$

Reasonable strategy: allow only POSITIVE scale factor

hw2_2_sol

np = 8
 n = 16
 n0 = 4
 u = 4
 B =

| | | | |
|----|----|----|----|
| -1 | -3 | -1 | 0 |
| -3 | 1 | 0 | -1 |
| -4 | -4 | -1 | 0 |
| -4 | 4 | 0 | -1 |
| -3 | -2 | -1 | 0 |
| -2 | 3 | 0 | -1 |
| -3 | -3 | -1 | 0 |
| -3 | 3 | 0 | -1 |
| -3 | -5 | -1 | 0 |
| -5 | 3 | 0 | -1 |
| -4 | -4 | -1 | 0 |
| -4 | 4 | 0 | -1 |
| -5 | -2 | -1 | 0 |
| -2 | 5 | 0 | -1 |
| -5 | -3 | -1 | 0 |
| -3 | 5 | 0 | -1 |

f_t =

| | | | | | | | |
|----------------------|---------|---------|---------|---------|--------|---------|--------|
| Columns 1 through 8 | | | | | | | |
| -3.8300 | -0.8300 | -7.5400 | -0.4600 | -5.5800 | 1.0800 | -5.9800 | 0.3700 |
| Columns 9 through 16 | | | | | | | |
| -7.0400 | -2.2000 | -7.5000 | -0.5100 | -7.7000 | 2.2300 | -8.1700 | 0.9800 |

del_t =

| | | | | |
|--------|--------|--------|---------|--------------|
| 1.0921 | 0.4762 | 1.2978 | -1.9651 | = a, b, c, d |
|--------|--------|--------|---------|--------------|

v_t =

| | | | | | | | |
|----------------------|---------|--------|---------|---------|---------|--------|---------|
| Columns 1 through 8 | | | | | | | |
| -0.0117 | 0.0049 | 0.0306 | 0.0385 | -0.0537 | -0.1295 | 0.0224 | 0.2526 |
| Columns 9 through 16 | | | | | | | |
| -0.0853 | -0.1333 | 0.0706 | -0.0115 | 0.0104 | 0.0682 | 0.0165 | -0.0897 |

diary off

||v|| ok

$\sigma_x = \sigma_y = 0.1$