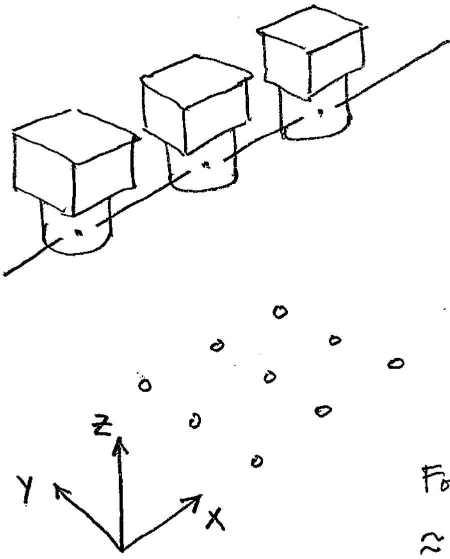
Adj. Geospa. Obs. Homework 6

assigned Friday 18 Nov, 2016 : due Monday 28 Nov.

1. 3 frame cameras view an array of 9 control points. All cameras have $(x_0, y_0, f) = (0, 0, 30.000)$ mm. Solve the constrained LS problem for the exterior orientation parameters $(\omega, \varphi, \kappa, X_L, Y_L, Z_L)_{1,2,3}$, where the perspective centers lie along a line, and also $\omega_1 = \omega_2 = \omega_3$, $\varphi_1 = \varphi_2 = \varphi_3$, and $\kappa_1 = \kappa_2 = \kappa_3$.



The given image coordinates have $\sigma = .025$ mm.

For initial approximations: $X_{L_1} \approx 2.5$ m, ΔX between cameras ≈ 0.5 m, $Y_{L_{1,2,3}} \approx 12.0$ m, $Z_{L_{1,2,3}} \approx 6.0$ m, $\omega \approx \varphi \approx \kappa \approx 0$

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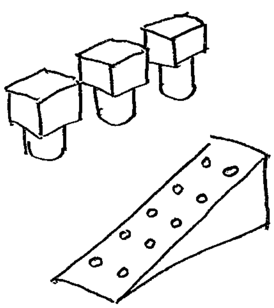
2. Using your results from problem 1, Fix the exterior orientation parameters of all 3 cameras, and solve for the 10 unknown object points which are constrained to lie in a plane. $\boxed{\sigma = 0.03}$



(a) use the linear intersection equations to get initial approximations for the object points

(b) you may first solve the unconstrained LS problem in order to get plane parameter approximations.

extra credit: show the constraint equations, in case you are NOT able to use "added parameters".

both 1 & 2 are indirect observation problems

Equations & partial derivatives for collinearity

$$\begin{bmatrix} x - x_0 \\ y - y_0 \\ -f \end{bmatrix} = \lambda M \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix}$$

$$\begin{bmatrix} x - x_0 \\ y - y_0 \\ -f \end{bmatrix} = \lambda \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

$$\frac{x - x_0}{-f} = \frac{U}{W}, \quad \frac{y - y_0}{-f} = \frac{V}{W}$$

$$x = x_0 - f\, U/W$$
$$y = y_0 - f\, V/W$$

$$\boxed{\begin{aligned} F_x &= x - x_0 + f\, U/W = 0 \\ F_y &= y - y_0 + f\, V/W = 0 \end{aligned}}$$

Generic Partial Derivatives

$$\frac{\partial F_x}{\partial p} = f \cdot \frac{W \frac{\partial U}{\partial p} - U \frac{\partial W}{\partial p}}{W^2}$$

$$\frac{\partial F_y}{\partial p} = f \cdot \frac{W \frac{\partial V}{\partial p} - V \frac{\partial W}{\partial p}}{W^2}$$

$$\boxed{\begin{aligned} \frac{\partial F_x}{\partial p} &= \frac{f}{W}\left[\frac{\partial U}{\partial p} - \frac{U}{W}\frac{\partial W}{\partial p}\right] \\ \frac{\partial F_y}{\partial p} &= \frac{f}{W}\left[\frac{\partial V}{\partial p} - \frac{V}{W}\frac{\partial W}{\partial p}\right] \end{aligned}}$$

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = M \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix}$$

$$\frac{\partial}{\partial \omega} \begin{bmatrix} U \\ V \\ W \end{bmatrix} = M_\kappa M_\phi \frac{\partial M_\omega}{\partial \omega} \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix}$$

$$\frac{\partial}{\partial \phi} \begin{bmatrix} U \\ V \\ W \end{bmatrix} = M_\kappa \frac{\partial M_\phi}{\partial \phi} M_\omega \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix}$$

$$\frac{\partial}{\partial \kappa} \begin{bmatrix} U \\ V \\ W \end{bmatrix} = \frac{\partial M_\kappa}{\partial \kappa} M_\phi M_\omega \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix}$$

$$\frac{\partial}{\partial X} \begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} m_{11} \\ m_{21} \\ m_{31} \end{bmatrix}, \quad \frac{\partial}{\partial X_L} \begin{bmatrix} U \\ V \\ W \end{bmatrix} = -\begin{bmatrix} m_{11} \\ m_{21} \\ m_{31} \end{bmatrix}$$

$$\frac{\partial}{\partial Y} \begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} m_{12} \\ m_{22} \\ m_{32} \end{bmatrix}, \quad \frac{\partial}{\partial Y_L} \begin{bmatrix} U \\ V \\ W \end{bmatrix} = -\begin{bmatrix} m_{12} \\ m_{22} \\ m_{32} \end{bmatrix}$$

$$\frac{\partial}{\partial Z} \begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} m_{13} \\ m_{23} \\ m_{33} \end{bmatrix}, \quad \frac{\partial}{\partial Z_L} \begin{bmatrix} U \\ V \\ W \end{bmatrix} = -\begin{bmatrix} m_{13} \\ m_{23} \\ m_{33} \end{bmatrix}$$

$$\frac{\partial M_\omega}{\partial \omega} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin\omega & \cos\omega \\ 0 & -\cos\omega & -\sin\omega \end{bmatrix}$$

$$\frac{\partial M_\phi}{\partial \phi} = \begin{bmatrix} -\sin\phi & 0 & -\cos\phi \\ 0 & 0 & 0 \\ \cos\phi & 0 & -\sin\phi \end{bmatrix}$$

$$\frac{\partial M_\kappa}{\partial \kappa} = \begin{bmatrix} -\sin\kappa & \cos\kappa & 0 \\ -\cos\kappa & -\sin\kappa & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Linear Intersection Equations for Initial Approximations

$$\frac{1}{\lambda} M^T \begin{bmatrix} x - x_0 \\ y - y_0 \\ -f \end{bmatrix} = \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix}$$

$$\frac{1}{\lambda} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix}$$

$$u/w = X - X_L / Z - Z_L = C_1$$

$$v/w = Y - Y_L / Z - Z_L = C_2$$

$$C_1 Z - C_1 Z_L = X - X_L$$
$$C_2 Z - C_2 Z_L = Y - Y_L$$

$$X - C_1 Z = X_L - C_1 Z_L$$
$$Y - C_2 Z = Y_L - C_2 Y_L$$

$$\begin{bmatrix} 1 & 0 & -C_1 \\ 0 & 1 & -C_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_L - C_1 Z_L \\ Y_L - C_2 Z_L \end{bmatrix}$$

for 1 image

for 3 images

$$\begin{bmatrix} 1 & 0 & -C_1 \\ 0 & 1 & -C_2 \\ \hline 1 & 0 & -C_1 \\ 0 & 1 & -C_2 \\ \hline 1 & 0 & -C_1 \\ 0 & 1 & -C_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_L - C_1 Z_L \\ Y_L - C_2 Z_L \\ X_L - C_1 Z_L \\ Y_L - C_2 Z_L \\ X_L - C_1 Z_L \\ Y_L - C_2 Z_L \end{bmatrix}$$

C_1, C_2, X_L, Y_L, Z_L per image

$$Bx \approx f, \quad x = (B^T B)^{-1} B^T f$$

Suggested function syntax for HW6: call function for each point observed in each image

$$[\quad * \quad] = \text{collin}(x_0, y_0, f, x, y, \omega, \varphi, \kappa, X_L, Y_L, Z_L, X, Y, Z)$$

$$* \quad \begin{bmatrix} \frac{\partial F_x}{\partial \omega} & \frac{\partial F_x}{\partial \varphi} & \frac{\partial F_x}{\partial \kappa} & \frac{\partial F_x}{\partial X_L} & \frac{\partial F_x}{\partial Y_L} & \frac{\partial F_x}{\partial Z_L} & \frac{\partial F_x}{\partial X} & \frac{\partial F_x}{\partial Y} & \frac{\partial F_x}{\partial Z} & F_x \\ \frac{\partial F_y}{\partial \omega} & \frac{\partial F_y}{\partial \varphi} & \frac{\partial F_y}{\partial \kappa} & \frac{\partial F_y}{\partial X_L} & \frac{\partial F_y}{\partial Y_L} & \frac{\partial F_y}{\partial Z_L} & \frac{\partial F_y}{\partial X} & \frac{\partial F_y}{\partial Y} & \frac{\partial F_y}{\partial Z} & F_y \end{bmatrix}$$

$$\underbrace{\quad\omega \ldots Z_L\quad}_{\text{partials for resection problem}} \quad \underbrace{\quad X, Y, Z\quad}_{\text{partials for intersection problem}} \quad \underbrace{\quad F\quad}_{\text{value of the condition equation}}$$

#	x1	y1	x2	y2	x3	y3
1	-6.818	8.696	-9.133	8.873	-11.476	9.018
2	2.131	8.351	-0.255	8.543	-2.619	8.655
3	11.061	7.984	8.621	8.155	6.202	8.264
4	-7.156	-0.300	-9.477	-0.110	-11.851	0.112
5	1.836	-0.575	-0.482	-0.392	-2.923	-0.136
6	10.776	-0.898	8.363	-0.675	5.920	-0.529
7	-7.516	-9.361	-9.899	-9.114	-12.226	-8.833
8	1.549	-9.613	-0.844	-9.345	-3.222	-9.137
9	10.530	-9.900	8.074	-9.626	5.612	-9.378

#	X	Y	Z
1	1.500	13.500	1.000
2	3.000	13.500	1.000
3	4.500	13.500	1.000
4	1.500	12.000	1.000
5	3.000	12.000	1.000
6	4.500	12.000	1.000
7	1.500	10.500	1.000
8	3.000	10.500	1.000
9	4.500	10.500	1.000

hw6_2_dat

#	x1	y1	x2	y2	x3	y3
1	-7.782	-3.533	-10.407	-3.326	-12.907	-3.058
2	-3.030	-3.792	-5.658	-3.559	-8.302	-3.273
3	2.114	-4.014	-0.678	-3.819	-3.391	-3.600
4	7.540	-4.374	4.649	-4.107	1.801	-3.857
5	13.356	-4.715	10.341	-4.424	7.388	-4.136
6	-7.640	3.054	-10.199	3.241	-12.704	3.415
7	-2.823	2.926	-5.474	3.169	-8.127	3.336
8	2.354	2.890	-0.424	3.123	-3.126	3.270
9	7.871	2.860	4.933	2.994	2.044	3.263
10	13.738	2.731	10.713	2.978	7.745	3.179