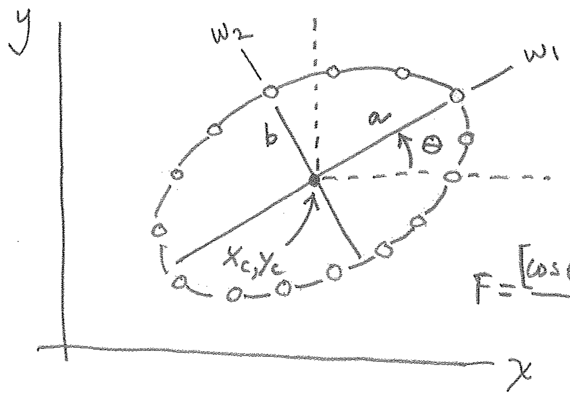
CE 597 Adj. Geospa. Obs. Homework 7

GLS & Constraints

Assigned Fri, 20 Nov., Due Fri 4 Dec.

1. 15 points are observed in x & y lying on an ellipse. Solve the LS problem to determine the parameters a, b, x_c, y_c, θ. (GLS)



$$\frac{w_1^2}{a^2} + \frac{w_2^2}{b^2} = 1$$

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x - x_c \\ y - y_c \end{pmatrix}$$

$$F = \frac{[\cos\theta\,(x-x_c) + \sin\theta\,(y-y_c)]^2}{a^2} + \frac{[\sin\theta\,(x-x_c) + \cos\theta\,(y-y_c)]^2}{b^2} - 1 = 0$$

$\boxed{\text{OR}}$

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0, \quad \text{where } A = 1$$

$$M_0 = \begin{bmatrix} F & D/2 & E/2 \\ D/2 & A & B/2 \\ E/2 & B/2 & C \end{bmatrix}, \quad M = \begin{bmatrix} A & B/2 \\ B/2 & C \end{bmatrix}$$

$\lambda_{1,2}$ eigenvalues of M with $|\lambda_1 - A| \le |\lambda_1 - C|$ & $|\lambda_2 - C| \le |\lambda_2 - A|$

$$a = \left[-|M_0| / |M| \cdot \lambda_1\right]^{1/2}$$

$$b = \left[-|M_0| / |M| \cdot \lambda_2\right]^{1/2}$$

$$x_c = (BE - 2CD)/(4AC - B^2)$$

$$y_c = (BD - 2AE)/(4AC - B^2)$$

$$\theta = \operatorname{arccot}((A-C)/B)/2$$

	x	y
1	58.02	37.08
2	54.62	40.27
3	47.99	41.82
4	38.83	41.33
5	28.70	38.80
6	19.54	34.63
7	12.86	29.95
8	10.15	25.14
9	11.19	21.14
10	16.33	18.67
11	24.59	18.15
12	34.32	19.84
13	44.09	23.07
14	52.27	27.57
15	57.02	32.62

$\sigma_x = \sigma_y = 0.1$

Do 2-sided global test @ $\alpha = .05$

find 60% conf. int. for $\mu_{\hat{x}_5}$

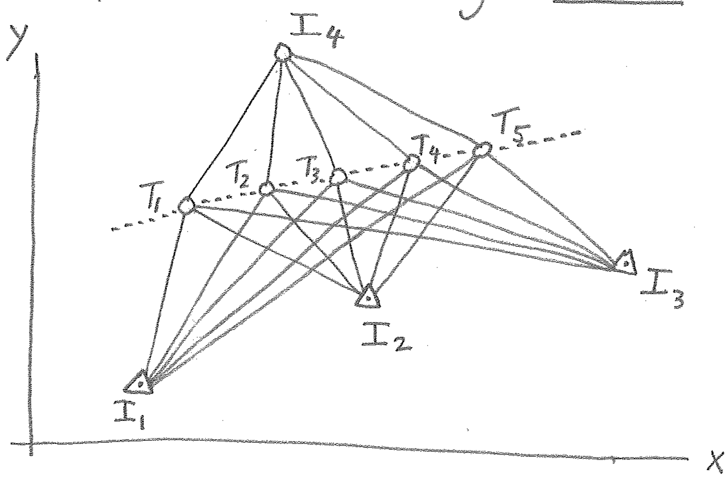
find 65% conf. int. for $\mu_{\hat{y}_5}$

Use any method for partial derivatives, choose either one of the two condition equations. Extra Credit: Do Both!

2. The trilateration network in the sketch has 4 instrument stations, and 5 target points (all 2D). Each target point is observed by range to each target point. Instrument stations 1,2,3 are fixed, instrument station 4 and all target points are unknown.

(a) Solve the network using I/O

(b) Solve the network using I/O and <u>constrain</u> all target points to lie along a <u>line</u>.



	x	y
△ I_1	11.00	7.00
△ I_2	47.00	22.00
△ I_3	80.00	28.00

2D range observations:

		Targets				
		1	2	3	4	5
Instr.	1	28.28	34.69	43.15	52.68	62.28
	2	34.40	27.42	23.68	24.62	29.34
	3	65.54	55.81	46.64	38.88	32.57
	4	20.68	17.06	19.95	27.09	36.12

$\sigma = 0.15$

(c) make 2-sided global test @ $\alpha = .05$

for (d) and (e), I forgot to specify - so let's use 90% probability for the conf. ellipses.

(d) for part (a) draw on <u>same plot</u> conf. ell. for T_3 and I_4

(e) on the <u>same plot</u> draw conf. ell. for T_5 for (a) and (b)

If reject H_0 on global test & <u>obvious blunder</u>, remove & re-do. Otherwise use σ_0^2 or $\hat{\sigma}_0^2$ as appropriate.