CE 597 Adj. of Geospatial Obs.

$\boxed{\text{Exam 1}}$

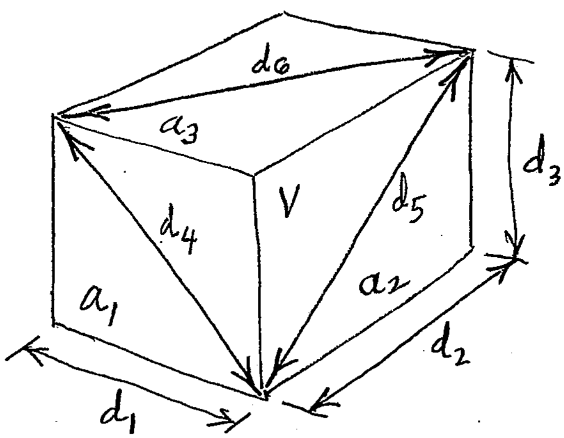
Name ______________________

75 minutes, 1 page of notes allowed, 17 Oct. 2013

1. Rotation matrix $M = M_y(\phi) \cdot M_x(\omega) = \begin{bmatrix} \square & \square & -0.1730 \\ \square & 0.9962 & \square \\ \square & \square & \square \end{bmatrix}$

What are ω and ϕ ? (only m_{13} and m_{22} are given)

2. We need the dimensions of a rectangular box. All sides meet at 90°. Lengths (d), areas (a), and volume (v) are observed as shown in the sketch. Give n, n_0, and r, and show condition equations for <u>observations</u> <u>only</u>.



a_i : area of rectangular face

v : volume of box

3. $\Sigma^{-1} = \begin{bmatrix} 1/6 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$, $w_1 = 1/2$, what is w_3 ?

w_i are weights of the respective elements of a 3x1 random vector.

4. The relationship of observation, l, and parameters X, y, and θ is

$$\tfrac{1}{2}X^2 + X\cos\theta - l = y$$

(a) show the condition equation suitable for Indirect Observations

(b) For the B matrix we need $\partial F/\partial X$

(i) find it analytically, then get numerical value

(ii) find it by numerical approximation.

use $l = -0.52$, $X^\circ = 1$, $y^\circ = 2$, $\theta^\circ = 0.2$ Rad.

5. $F(x) = x^2 - \cos(x)$

We need to find a root of $F(x)$. Make 1 iteration. use $x^\circ = 0.75$. Show the refined value of the root.

Useful Info:

$$M_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix}, \quad M_y = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$M_z = \begin{bmatrix} \cos k & \sin k & 0 \\ -\sin k & \cos k & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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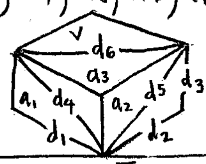
1. $$M = \begin{bmatrix} c\phi & 0 & -s\phi \\ 0 & 1 & 0 \\ s\phi & 0 & c\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\omega & s\omega \\ 0 & -s\omega & c\omega \end{bmatrix} = \begin{bmatrix} \square & \square & -s\phi c\omega \\ \square & c\omega & \square \\ \square & \square & \square \end{bmatrix}$$

$\cos(\omega) = .9962$, $\underline{\underline{\omega = 5^\circ}}$, $-\sin(\phi)\cos(\omega) = -.1730$, $\underline{\underline{\phi = 10^\circ}}$

2. $n = 10$, $n_0 = 3$, $\underline{\ \ }$ $r = 7$

$d_1, d_2, d_3, d_4, d_5, d_6, a_1, a_2, a_3, V$



$d_1 \cdot d_3 = a_1$ $\quad d_1^2 + d_3^2 = d_4^2$

$d_2 \cdot d_3 = a_2$ $\quad d_2^2 + d_3^2 = d_5^2$

$d_1 \cdot d_2 = a_3$ $\quad d_1^2 + d_2^2 = d_6^2$

$d_1 \cdot d_2 \cdot d_3 = V$

3. $$\Sigma^{-1} = \begin{bmatrix} 1/6 & & \\ & 1/3 & \\ & & 1/2 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 6 & & \\ & 3 & \\ & & 2 \end{bmatrix},$$

$\sigma_1^2 = 6$, $w_1 = 1/2 = \dfrac{\sigma_0^2}{\sigma_1^2} \Rightarrow \sigma_0^2 = 3$

$\sigma_3^2 = 2$, $w_3 = \dfrac{\sigma_0^2}{\sigma_3^2} = 3/2 = \underline{\underline{1\tfrac{1}{2}}}$

4. $F(\ell, x, \theta) = \ell - \tfrac{1}{2}x^2 - x\cos\theta + y = 0$ (arranged for Ind. Obs.)

(a) $\partial F/\partial x = -2 \cdot \tfrac{1}{2} \cdot x - \cos\theta = -x - \cos\theta$, $\underbrace{\ell = -.52,\ x^0 = 1,\ y^0 = 2,\ \theta^0 = 0.2\ \text{Rad}}$

$= -1 - \cos(0.2\,R) = \underline{\underline{-1.98}}$

(b) numerical approx:

$$\partial F/\partial x \approx \frac{F(\ell, x+\Delta x, \theta) - F(\ell, x, \theta)}{\Delta x} = \frac{\left[\ell - \tfrac{1}{2}(x+\Delta x)^2 - (x+\Delta x)\cos\theta + y\right] - \left[\ell - \tfrac{1}{2}x^2 - x\cos\theta + y\right]}{\Delta x}$$

let $\Delta x = .001$, $\dfrac{-.002047 - -.000066}{.001} = \underline{\underline{-1.98}}$

5. $F(x) = x^2 - \cos(x)$, $x^0 = 0.75$, use newton iteration to find root, which is solution of $F(x) = 0$

$F(x) \approx F(x^0) + \dfrac{dF}{dx} \cdot \Delta x = 0$, $\Delta x = \dfrac{-F(x^0)}{dF/dx}$

$-F(0.75) = -\left[0.75^2 - \cos(0.75)\right] = .169188$

$\left.dF/dx\right|_{x=0.75} = 2x + \sin(x) = 2 \cdot 0.75 + \sin(0.75) = 2.181638$

$\Big\}$ $\underline{\underline{\Delta x = 0.077}}$

$x^1 = x^0 + \Delta x = 0.75 + 0.077 = \underline{\underline{0.827}}$