Unified LS $\quad Av + B\Delta = f \quad$ (Linear) $\quad$ 29-1

$$\underbrace{v_x - \Delta = -x}$$

$$\Delta = x + v_x$$

obs. residual

$$\dot{A}\dot{v} + \dot{B}\Delta = \dot{f}$$

$$\dot{Q}_e = \begin{bmatrix} Q_e & 0 \\ 0 & Q_{xx} \end{bmatrix} \quad : \quad \Sigma_{xx} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix} \quad \Sigma = \sigma_0^2 Q$$

$$\dot{W}_e = \begin{bmatrix} W_e & 0 \\ 0 & W_{xx} \end{bmatrix}$$

$$\dot{N} = \underbrace{\begin{bmatrix} B^T & -I \end{bmatrix}}_{\dot{B}^T} \begin{bmatrix} W_e & 0 \\ 0 & W_{xx} \end{bmatrix} \underbrace{\begin{bmatrix} B \\ -I \end{bmatrix}}_{\dot{B}}$$

29-2

$$\begin{bmatrix} B^T W_e & -W_{xx} \end{bmatrix} \begin{bmatrix} B \\ -I \end{bmatrix} = \begin{bmatrix} B^T W_e B + W_{xx} \end{bmatrix}$$

$$\dot{t} = \dot{B}^T \dot{W}_e \dot{f} = \begin{bmatrix} B^T W_e & -W_{xx} \end{bmatrix} \begin{bmatrix} f \\ -x \end{bmatrix} = \begin{bmatrix} B^T W_e f + W_{xx} \cdot x \end{bmatrix}$$

$$\begin{bmatrix} \underbrace{B^T W_e B} + W_{xx} \end{bmatrix} \Delta = \begin{bmatrix} \underbrace{B^T W_e f} + W_{xx} \cdot x \end{bmatrix}$$

add weight matrix for parameters $\qquad$ add new term

Unified LS for nonlinear case: 29-3

$F(\ell, x) = 0 \quad \ell^0, x^0$

linearize and eqn: $Av + B\delta = \underbrace{-F(\ell^0, x^0) - A(\ell - \ell^0)}_{f_0}$

$\ell + v = \ell^0 + \delta\ell$

$x + v_x = x^0 + \delta, \quad v_x - \delta = x^0 - x = f_x$

$$Av + B\delta = f_0$$
$$v_x - \delta = f_x$$

$$\begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} v \\ v_x \end{bmatrix} + \begin{bmatrix} B \\ -I \end{bmatrix}\delta = \begin{bmatrix} f_0 \\ f_x \end{bmatrix} \quad \cdots$$

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$\dot{N}\delta = \dot{t}$ 29-4

$$\boxed{\left[B^T W_e B + W_{xx}\right]\delta = \left[B^T W_e f_0 - W_{xx} f_x\right]}$$

U.L.S. for nonlinear problems

$f_x = x^0 - x$

current value, original value

$\Sigma_{xx}, Q_{xx}, W_{xx}$

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Equivalent techniques in other fields

29-5

- Bayesian Estimation : framework for estimation where you use prior knowledge & distributions
- Ridge Regression $\hat{\beta} = [X^T X + \lambda I]^{-1} X^T y$

  $y \approx X\beta$
- Tikhonov Regularization

  $Ax \approx b$

  $P = W$ $\qquad \hat{x} = [A^T P A + Q][A^T P b + Q x_0]$

  $Q : w_{xx}, x_0$ $\qquad$ (1943)

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Blunder Detection or Robust Estimation

29-6

$Q : W^{-1}$

(I/O) $Q_{vv} = Q - Q_{\hat{\ell}\hat{\ell}} = \underline{Q - B N^{-1} B^T}$

$\underset{n,n}{Q_{vv}} \underset{n,n}{W} = \underset{n,n}{\overline{W}}$: diagonal elements

$w_{ii} = r_i$ redundancy number

$\sum_{i=1}^{n} r_i = r$

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29-7

properties of r_i

$$0 \leq r_i \leq 1 \qquad u_i = 1 - r_i$$

$$0 \leq u_i \leq 1$$

r_i : the fraction of the error in l_i which is revealed in the residual v_i

u_i : the fraction of the error in l_i which is "absorbed" by the parameter estimation

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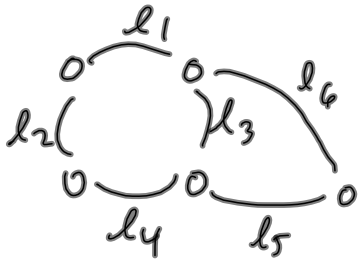
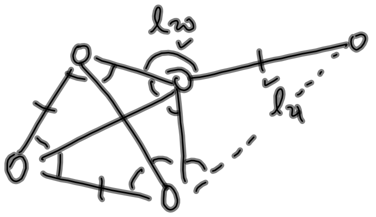
29-8



$\cancel{r_5 = 0}$



$\cancel{r_{20} = 0}$

$\cancel{r_{21} = 0}$

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- IRLS iteratively reweighted LS 29-9
- Data snooping
- L1 norm minimization

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