$z_i = H x_i + v$ z: cov R (Brown + Hwang) 28-1
(obs)

$x_i = \Phi_{i-1} x_{i-1} + w_{i-1}$ cov Q

x_0, cov. P

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enter x_0, P_0^- 28-2

(gain) $K = P^- H^T (H P^- H^T + R)^{-1}$

$x = x^- + K(z - Hx^-)$

$P = (I - KH)P^-$

$x_{k+1} = \Phi x_k$

$P_{k+1}^- = \Phi P \Phi^T + Q$

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ϕ : state transition matrix

28-3

$$\vec{x} = \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix}$$

$$f(t+\Delta t) = f(t) + \frac{\partial f}{\partial t}\Delta t + \frac{\partial^2 f}{\partial t^2}\frac{\Delta t^2}{2}$$

$$\underbrace{\begin{aligned} x(t+\Delta t) &= x(t) + \dot{x}\Delta t + \ddot{x}\,\Delta t^2/2 \\ \dot{x}(t+\Delta t) &= \dot{x}(t) + \ddot{x}\Delta t \\ \ddot{x}(t+\Delta t) &= \ddot{x}(t) \end{aligned}}$$

$$\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix}_{(t+\Delta t)} = \begin{bmatrix} 1 & \Delta t & \Delta t^2/2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix}_{(t)}$$

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another approach to obtain STM

28-4

$$\vec{x} = \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} \qquad \frac{\partial x}{\partial t} = A\,x(t) + B\,w(t)$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \begin{pmatrix} \dot{x} \\ \ddot{x} \\ \dddot{x} \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} x \\ \dot{x} \\ \ddot{x} \end{pmatrix}$$

$$x_{k+1} = \phi_k x_k + w_k$$

$$\phi_k = e^{A\Delta t} = \begin{bmatrix} 1 & \Delta t & \Delta t^2/2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}$$

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Unified LS $\left[\begin{array}{ll} l \;\; n,1 \;\; , & v \;\; n,1 \\ x \;\; u,1 & r = n - n_0 \;, \; c = r + u \end{array}\right]$ 28-5

conventional LS

$\downarrow$

u new observations, new total obs $= n + u$

x, v_x, Q_{xx}

n_0 same, new redundancy

$n + u - n_0 = \underbrace{n - n_0}_{r} + u = r + u$

$\boxed{r_{new} = r_{old} + u}$

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conditions equations 28-6

$r_{old} + u + u = r_{old} + 2u = \underline{\underline{c_{old}}} + \underline{\underline{u}}$

$(c_{old}) \quad Av + B\Delta = f$

$(u) \quad \Delta = x + v_x$

Δ ; obs. value of param ; correction

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$$Av + B\delta = f \quad , \quad \delta = x + v_x \quad , \quad v_x - \delta = -x \qquad 28\text{-}7$$

$$v_x - \delta = -x$$

$$Av + 0 \cdot v_x + B\delta = f$$
$$0v + I v_x - I\delta = -x$$

$$\underbrace{\begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix}}_{\dot{A}} \underbrace{\begin{bmatrix} v \\ v_x \end{bmatrix}}_{\dot{v}} + \underbrace{\begin{bmatrix} B \\ -I \end{bmatrix}}_{\dot{B}} \delta = \underbrace{\begin{bmatrix} f \\ -x \end{bmatrix}}_{\dot{f}}$$

$$\dot{A}\dot{v} + \dot{B}\delta = \dot{f}$$

solve like conventional GLS problem

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$$(B^T W_e B)\,\delta = B^T W_e f \qquad 28\text{-}8$$
$$(N) \qquad\qquad (t)$$

$$\dot{l} = \begin{pmatrix} l \\ x \end{pmatrix} : \begin{bmatrix} Q & 0 \\ 0 & Q_{xx} \end{bmatrix} = \dot{Q} \,, \quad \dot{Q}_e = \dot{A}\dot{Q}\dot{A}^T$$

$$\dot{Q}_e = \begin{pmatrix} A & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} Q & 0 \\ 0 & Q_{xx} \end{pmatrix} \begin{pmatrix} A^T & 0 \\ 0 & I \end{pmatrix} = \begin{pmatrix} AQA^T & 0 \\ 0 & Q_{xx} \end{pmatrix} = \dot{Q}_e$$

$$\dot{W}_e = \begin{pmatrix} W_e & 0 \\ 0 & W_{xx} \end{pmatrix}$$

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$$\dot{N} = \dot{B}^T \dot{W}_e \dot{B}$$

28-9

$$[B^T \; -\underline{I}] \begin{pmatrix} W_e & 0 \\ 0 & W_{xy} \end{pmatrix} \begin{pmatrix} B \\ -\underline{I} \end{pmatrix}$$

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