Parameter Constraints

26-1

$-Wv + A^T k = 0$

$B^T k + C^T k_c = 0$

$Av + B\delta = f$

$C\delta = g$

elim. v & k ⇒

$-N\delta + C^T k_c = -t$

$C\delta = g$

<u>assume N full rank</u>

→ continue elimination

$\delta = \underline{N^{-1}t + N^{-1}C^T k_c}$

$C\left[N^{-1}t + N^{-1}C^T k_c\right] = g$

$CN^{-1}C^T k_c = g - CN^{-1}t$

solve numerically for k_c

$k_c = (CN^{-1}C^T)^{-1}(g - CN^{-1}t)$

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26-2

$\boxed{\delta = N^{-1}t + N^{-1}C^T k_c}$

$k = W_e(f - B\delta)$

$v = QA^T k$

$\delta = \underbrace{N^{-1}t}_{\delta^0} + N^{-1}C^T(CN^{-1}C^T)^{-1}(g - C\underbrace{N^{-1}t}_{\delta^0})$

δ^0: solution for unconstrained prob.

$\delta = \delta^0 + N^{-1}C^T(CN^{-1}C^T)^{-1}(g - C\delta^0)$

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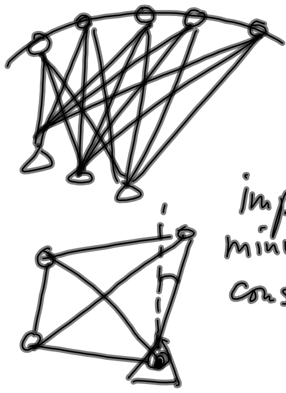
we had assumption N : full rank

26-3

(1) if constraints augment a solvable system

(2) if constraints are necessary to solve the problem (resolving datum defect)



impose minimal constr.

if case 2 : $-N\delta + C^T k_e = -t$

$C\delta = g$

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26-4

$$\begin{bmatrix} -N & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} \delta \\ k_e \end{bmatrix} = \begin{bmatrix} -t \\ g \end{bmatrix}$$

partially reduced normal equations

(1) if N full rank

$Q_{\delta\delta} = N^{-1}(I - C^T(CN^{-1}C^T)^{-1}CN^{-1})$

(2) if N not full rank

$$\begin{bmatrix} -N & C^T \\ C & 0 \end{bmatrix}^{-1} \rightarrow \begin{bmatrix} \alpha & \beta^T \\ \beta & \gamma \end{bmatrix}; \quad Q_{\delta\delta} = -\alpha$$

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derive $Q_{\Delta\Delta}$ for full rank case $\Delta = N^{-1}t + N^{-1}C^T k_c$ 26-5

$A\hat{\ell} + Bx = d$

$A(\ell + v) + Bx = d$

$Av + Bx = \underbrace{d - A\ell}_{f}$

$f = d - A\ell$

$Q_{ff} = (-A)\,Q_{\ell\ell}(-A^T)$

$\boxed{Q_{ff} = AQA^T = Q_e}$

$\Delta = N^{-1}t + N^{-1}C^T(CN^{-1}C^T)^{-1}(g - CN^{-1}t)$

$\Delta = [N^{-1} - N^{-1}C^T(CN^{-1}C^T)^{-1}CN^{-1}]t + \boxed{}$

$t = B^T W_e f$

$\Delta = \underbrace{[N^{-1} - N^{-1}C^T(CN^{-1}C^T)^{-1}CN^{-1}]B^T W_e}\, f$

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$Q_{\Delta\Delta} = [N^{-1} - N^{-1}C^T(CN^{-1}C^T)^{-1}CN^{-1}]\underbrace{B^T W_e Q_e W_e B}_{N}[N^{-1} - N^{-1}C^T(CN^{-1}C^T)^{-1}CN^{-1}]$

$Q_{\Delta\Delta} = [N^{-1} - N^{-1}C^T(CN^{-1}C^T)^{-1}CN^{-1}][I - C^T(CN^{-1}C^T)^{-1}CN^{-1}]$ 26-6

$Q_{\Delta\Delta} = N^{-1} - N^{-1}C^T(CN^{-1}C^T)^{-1}CN^{-1} - N^{-1}C^T(CN^{-1}C^T)^{-1}CN^{-1} +$

$N^{-1}C^T\underbrace{(CN^{-1}C^T)^{-1}CN^{-1}C^T}(CN^{-1}C^T)^{-1}CN^{-1}$

$+ N^{-1}C^T(CN^{-1}C^T)^{-1}CN^{-1}$ eq 9.15a

$Q_{\Delta\Delta} = N^{-1} - N^{-1}C^T(CN^{-1}C^T)^{-1}CN^{-1}$

$Q_{\Delta\Delta} = N^{-1}[I - C^T(CN^{-1}C^T)^{-1}CN^{-1}]$ p.216 OCS

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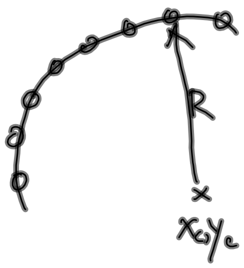
2 approaches to write constrained equations

(1) write constraints in terms of point coordinates
(determinant equations)

(2) introduce added parameters only in constr. equations

26-7



introduce 3 new parameters

X_c, Y_c, R

$$\left[(X_1-X_c)^2+(Y_1-Y_c)^2\right]^{1/2}-R=0$$

$\vdots$

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with constraints & added parameters : s' constr. equations

$Av + B\Delta = f$ $\quad \Delta$: regular params (u) $\quad c+s' =$

$D_1\Delta + D_2\Delta' = h$ $\quad \Delta'$: added parameters (q) $\quad r+u+q$

$$\Phi = v^TWv - 2k^T(Av+B\Delta-f) - 2k_c^T(D_1\Delta+D_2\Delta'-h)$$

$$\begin{bmatrix} -W & A^T & 0 & 0 & 0 \\ A & 0 & B & 0 & 0 \\ 0 & B^T & 0 & D_1^T & 0 \\ 0 & 0 & D_1 & 0 & D_2 \\ 0 & 0 & 0 & D_2^T & 0 \end{bmatrix} \begin{bmatrix} v \\ k \\ \Delta \\ k_c \\ \Delta' \end{bmatrix} = \begin{bmatrix} 0 \\ f \\ 0 \\ h \\ 0 \end{bmatrix}$$

26-8

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$$P = D_1 N^{-1} D_1^T$$
$$R = D_2^T P^{-1} D_2$$

26-9

$$Q_{\hat{\delta}\hat{\delta}} = N^{-1}\left[I - D_1^T P^{-1} D_1 N^{-1} + D_1^T P^{-1} D_2 R^{-1} D_2^T P^{-1} D_1 N^{-1}\right]$$

$Q_{\delta'\delta'} = R^{-1}$ chapter 9, OLS

other topics: Sequential LS ←
Unified LS
Blunder detection

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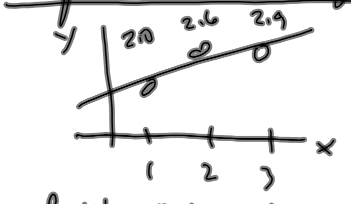
Sequential LS

26-10

- Batch mode LS
- Sequential LS

simplest application

sequential formation of NE



$$l_1 + v_1 = m x_1 + s$$
$$\vdots$$

$$v_1 - m x_1 - s = -l_1$$

$$v_1 + \begin{bmatrix} -x_1 & -1 \end{bmatrix} \begin{bmatrix} m \\ s \end{bmatrix} = -l$$

$$v + B\delta = f$$

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$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ -2 & -1 \\ -3 & -1 \end{bmatrix}\begin{pmatrix} m \\ b \end{pmatrix} = \begin{bmatrix} -2.0 \\ -2.8 \\ -2.9 \end{bmatrix}$$

$$\begin{bmatrix} 1+4+9 & 1+2+3 \\ 1+2+3 & 1+1+1 \end{bmatrix} = N$$

$$= \begin{bmatrix} 14 & 6 \\ 6 & 3 \end{bmatrix}$$

26-11

$v + B\delta = f$

$N = B^T W B,\ t = B^T W f$

$\delta = N^{-1} t,\ W = I$

$N = B^T B,\ t = B^T f$

$$\begin{bmatrix} -1 & -2 & -3 \\ -1 & -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ -2 & -1 \\ -3 & -1 \end{bmatrix} =$$

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$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix} + \cdots$$

$N_1 \qquad N_2 \qquad N_3$

26-12

$V + B\delta = f$

$v_1 + b_1\delta = f_1,\ N_1 = b_1^T b_1$

$v_2 + b_2\delta = f_2,\ N_2 = b_2^T b_2$

$\vdots \qquad \vdots$

$v_n + b_n\delta = f_n,\ N_n = b_n^T b_n$

$t_1 + t_2 + t_3 = t$

$b_1^T f_1 \quad b_2^T f_2 \quad b_3^T f_3$

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if W : diagonal

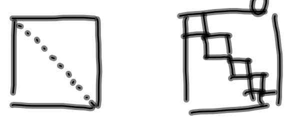
26-B

$N_1 = b_1^T w_1 b_1 \qquad t_1 = b_1^T w_1 f_1$

$N_2 = b_2^T W_2 b_2 \qquad t_2 = b_2^T W_2 f_2$

$\vdots \qquad\qquad\qquad \vdots$

if W : block diagonal



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