confidence circle r ← 24-1

confidence ellipse a, b, θ

USG agencies
USGS
NGS + contractors
NGA

CE / LE
CE50
CE90
CE99

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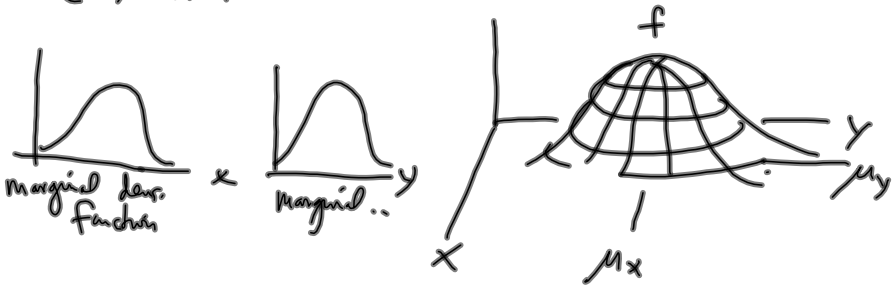
given covariance matrix (2×2) 24-2

MVN (n = 2)

$$f(\vec{x}) = \frac{1}{(2\pi)^{n/2}\sqrt{|\Sigma|}} \exp\left\{-\frac{1}{2}(x-\mu)^T \vec{\Sigma}(x-\mu)\right\}$$

$$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \quad \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$$

$\vec{x}: \begin{pmatrix} x \\ y \end{pmatrix}$



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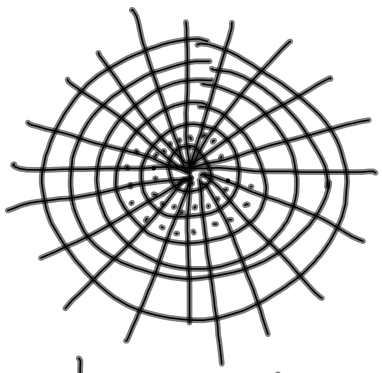
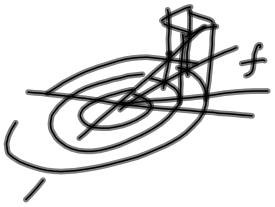
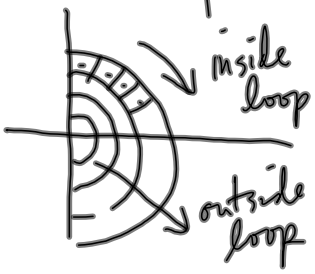
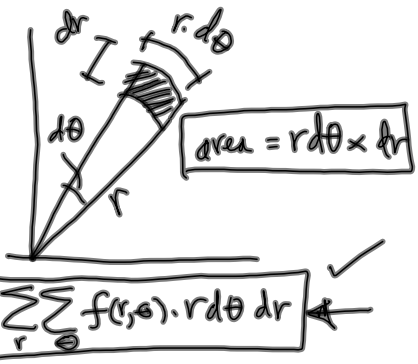




$x = r\cos\theta$
$y = r\sin\theta$ 24-3





$$\sum_r \sum_\theta f(r,\theta)\cdot r\,d\theta\,dr$$

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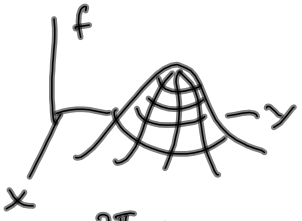

$$\iint_{circle} f(x,y)\,dx\,dy$$

$x = r\cos\theta$
$y = r\sin\theta$ 24-4

change of variable from $xy \to r\theta$

$$\int_0^{2\pi}\int_0^R f(r\cos\theta, r\sin\theta)\,|J|\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^R f\cdot r\cdot dr\cdot d\theta$$

$$J = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix}$$

$$r\cos^2\theta + r\sin^2\theta = r(\cos^2\theta + \sin^2\theta) = r$$

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24-5

Parameter Constraints:

observation equations, condition equations

have uncertainty

constraint equations

no uncertainty

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24-6

Common example: control points

replace every occurrence of $X_1 = 1000$ ...

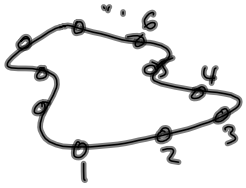
enforce constraint by substitution

examples:

$z_1 = z_2$
$z_2 = z_3$
$z_3 = z_4$
⋮

OR

$z_1 = z_2$
$z_1 = z_3$
$z_1 = z_4$
⋮



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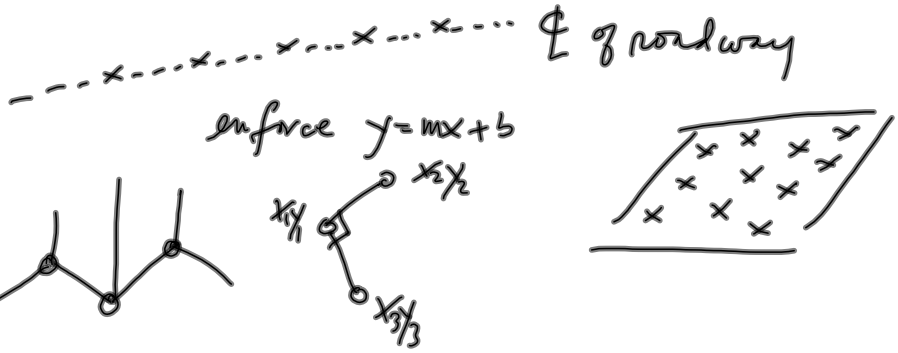

$$\begin{pmatrix} x_2-x_1 \\ y_2-y_1 \end{pmatrix} \cdot \begin{pmatrix} x_3-x_1 \\ y_3-y_1 \end{pmatrix} = 0$$

$$(x_2-x_1)(x_3-x_1) + (y_2-y_1)(y_3-y_1) = 0$$

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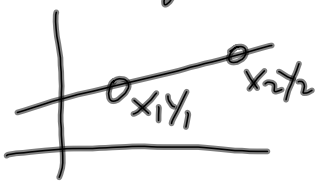
24-8

Constructing equations

$$c_1x + c_2y + c_3 = 0$$



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