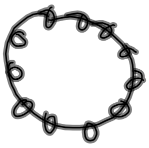
23-1

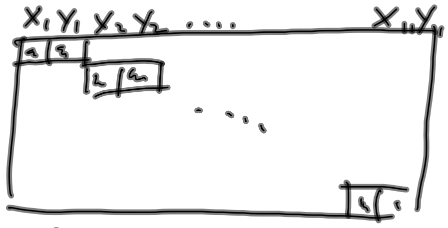


n. pts = 11
n = 22

$$F_i = \left[(x_i - x_c)^2 + (y_i - y_c)^2\right]^{1/2} - R = 0$$

A: $\frac{\partial F}{\partial \ell}$: ℓ for 1 point x_i, y_i

A: c, n , $c = r + u$



A : Block Diagonal

B: $\frac{\partial F}{\partial X_c, Y_c, R}$ $\quad \frac{\partial F_i}{\partial X_c} = \frac{-(x_i - x_c)}{[\,]^{1/2}}$

$$\frac{\partial F_i}{\partial y_c} = \frac{-(y_i - y_c)}{[\,]^{1/2}} \qquad \frac{\partial F_i}{\partial R} = -1$$

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23-2

$$B = \begin{bmatrix} X_c & Y_c & R \\ \frac{-(x_1 - x_c)}{(\,)^{1/2}} & \frac{-(y_1 - y_c)}{(\,)^{1/2}} & -1 \\ \frac{-(x_2 - x_c)}{(\,)^{1/2}} & \frac{-(y_2 - y_c)}{(\,)^{1/2}} & -1 \\ \vdots & \vdots & \vdots \\ \frac{-(x_{11} - x_c)}{(\,)^{1/2}} & \frac{-(y_{11} - y_c)}{(\,)^{1/2}} & -1 \end{bmatrix} \begin{matrix} \\ \leftarrow \text{point 1} \\ \leftarrow \text{point 2} \\ \vdots \\ \leftarrow \text{point 11} \end{matrix}$$

$$f = -F(\ell^0, x^0) - A(\ell - \ell^0)$$

$$f_i = -F_i(x_i^0, y_i^0, x_c^0, y_c^0, R^0) - \underset{1,2}{A_i} \underset{2,1}{\begin{pmatrix} x_i - x_i^0 \\ y_i - y_i^0 \end{pmatrix}} \quad \leftarrow$$

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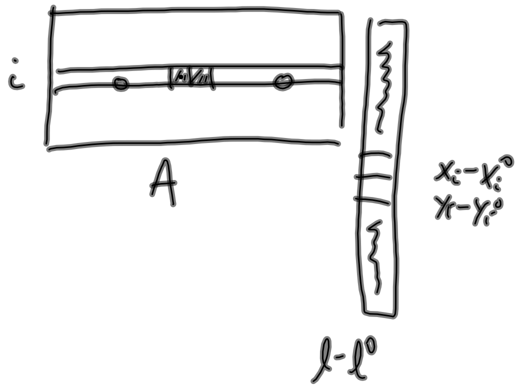
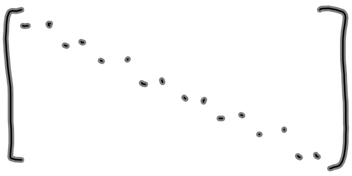


Spy(A) 23-3



A, B, f, W

$\Delta = (B^T W_e B)^{-1} (B^T W_e f) \ldots$

$Q_e = AQA^T \quad Q = W^{-1}$

$W_e = (AQA^T)^{-1}$

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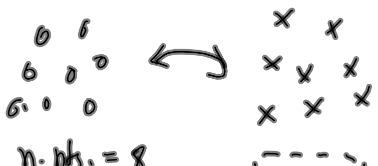
GLS for coordinate transformations 23-4

3D conformal coord. transf.

7 parameter transf.

rigid body transf.



n. pts. = 8

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda \cdot M \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

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23-5

$$F_i = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} : \begin{pmatrix} x \\ y \\ z \end{pmatrix}_i - \lambda M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_i - \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\lambda, \omega, \varphi, K$
t_x, t_y, t_z
7 par

8 pts:

$n = 8 \times 3 \times 2 = 48$

$n_0 = 7 + 3 \times 8 = 31$

$r = 17$

$C = r + \mu = 17 + 7 = \underline{\underline{24}}$

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23-6

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_i - \lambda M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_i - \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\frac{\partial F}{\partial \ell}$ x_i, y_i, z_i

$X_i Y_i Z_i$

$\frac{\partial F_1}{\partial x_i} = 1 \quad \frac{\partial F_1}{\partial y_i} = 0 \quad \frac{\partial F_1}{\partial z_i} = 0$

$\frac{\partial F_2}{\partial x_i} = 0 \quad \frac{\partial F_2}{\partial y_i} = 1 \quad \frac{\partial F_2}{\partial z_i} = 0$

$\frac{\partial F_3}{\partial x_i} = 0 \quad \frac{\partial F_3}{\partial y_i} = 0 \quad \frac{\partial F_3}{\partial z_i} = 1$

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23-7

$$\begin{pmatrix}F_1\\F_2\\F_3\end{pmatrix} = \begin{pmatrix}x_i\\y_i\\z_i\end{pmatrix} - \lambda M \begin{pmatrix}X_i\\Y_i\\Z_i\end{pmatrix} - \begin{pmatrix}t_x\\t_y\\t_z\end{pmatrix}$$

$$\frac{\partial F_1}{\partial X_i} - \lambda m_{11} \qquad \frac{\partial F_1}{\partial Y_i} = -\lambda m_{12} \qquad \frac{\partial F_1}{\partial Z_i} = \lambda m_{13}$$

$$\frac{\partial F_2}{\partial X_i} = -\lambda m_{21} \qquad \frac{\partial F_2}{\partial Y_i} = -\lambda m_{22} \qquad \cdots$$

$$\cdots$$

$$(-\lambda M)$$

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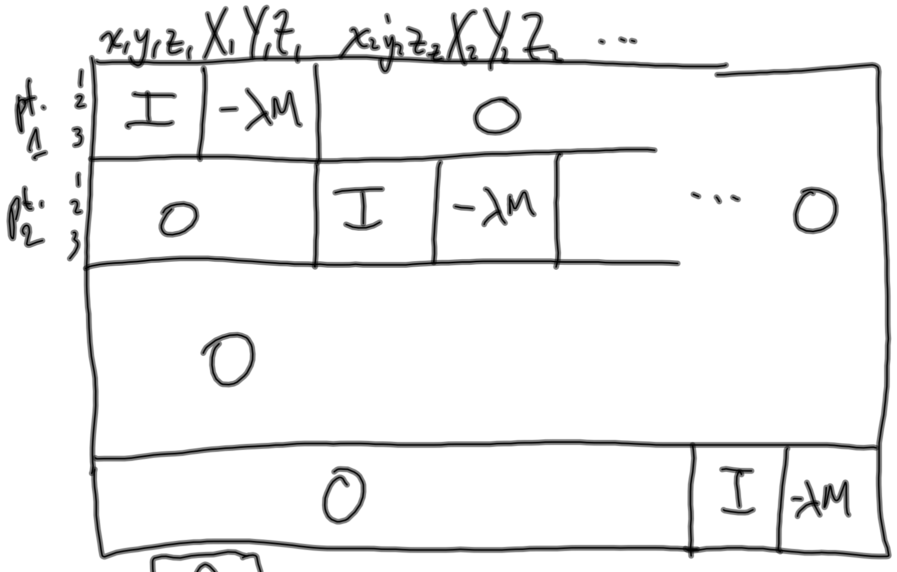
23-8



A c×n 24×48

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B: $\underline{\lambda}\ \underline{\omega}\ \underline{\varphi}\ \underline{K}\ t_x\ t_y\ t_z$

23-9

$$\frac{\partial F_1}{\partial \lambda} = -M_1 \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix}$$

$$\frac{\partial F_2}{\partial \lambda} = -M_2 \begin{pmatrix} \vdots \end{pmatrix}$$

$$\frac{\partial F_3}{\partial \lambda} = -M_3 \begin{pmatrix} \vdots \end{pmatrix}$$

$$\begin{bmatrix} \partial F_1/\partial \lambda \\ \partial F_2/\partial \lambda \\ \partial F_3/\partial \lambda \end{bmatrix} = -M \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix}$$

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$$M = M_K M_\varphi M_\omega \qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix}_i - \lambda M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} - \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} \leftarrow$$

23-10

$$\begin{pmatrix} \frac{\partial F_1}{\partial \omega} \\ \frac{\partial F_2}{\partial \omega} \\ \frac{\partial F_3}{\partial \omega} \end{pmatrix} = -\lambda M_K M_\varphi \frac{\partial M_\omega}{\partial \omega} \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix}, \quad \frac{\partial M_\omega}{\partial \omega} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin\omega & \cos\omega \\ 0 & -\cos\omega & -\sin\omega \end{bmatrix}$$

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23-11

$$\frac{\partial F}{\partial \varphi} = -\lambda M_\kappa \frac{\partial M_\varphi}{\partial \varphi} M_\omega \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}$$

$$M_\varphi = \begin{bmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{bmatrix} \qquad \frac{\partial M_\varphi}{\partial \varphi} = \begin{bmatrix} -\sin\varphi & 0 & -\cos\varphi \\ 0 & 0 & 0 \\ \cos\varphi & 0 & -\sin\varphi \end{bmatrix}$$

$$\frac{\partial F}{\partial \kappa} = -\lambda \frac{\partial M_\kappa}{\partial \kappa} M_\varphi M_\omega \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}$$

$$M_\kappa = \begin{bmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \frac{\partial M_\kappa}{\partial \kappa} = \begin{bmatrix} -\sin\kappa & \cos\kappa & 0 \\ -\cos\kappa & -\sin\kappa & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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23-12

$$\frac{\partial F}{\partial T} = -I$$

B-matrix

	λ	ω	φ	κ	t_x t_y t_z
1	$-M\begin{pmatrix} x \\ y \\ z \end{pmatrix}_1$	$-\lambda M_\kappa M_\varphi \frac{\partial M_\omega}{\partial \omega}\begin{pmatrix} x \\ y \\ z \end{pmatrix}_1$	...	...	$\begin{matrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{matrix}$
2	$-M\begin{pmatrix} x \\ y \\ z \end{pmatrix}_2$	$-\lambda M_\kappa M_\varphi \frac{\partial M_\omega}{\partial \omega}\begin{pmatrix} x \\ y \\ z \end{pmatrix}_2$	...	...	$\begin{matrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{matrix}$
	⋮	⋮	⋮	⋮	⋮

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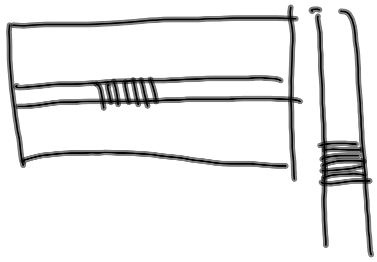
23-13

$$f = -F^\circ - A(l - l^\circ)$$

$$\underset{3\times 1}{f_{point\ i}} = \underset{3\times 1}{-F_i^\circ} - \underset{3\times 6}{A_i} \begin{bmatrix} x_1 - x_1^\circ \\ y_1 - y_1^\circ \\ z_1 - z_1^\circ \\ X_1 - X_1^\circ \\ Y_1 - Y_1^\circ \\ Z_1 - Z_1^\circ \end{bmatrix}$$



A, B, f, w

:

Δ, k, v

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23-14

special case for 7-par. transf.

angle are very small

$$M = M_K M_\varphi M_\omega = ?$$

$$M_\omega = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \omega \\ 0 & -\omega & 1 \end{bmatrix}$$

$$M_\varphi = \begin{bmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -\varphi \\ 0 & 1 & 0 \\ \varphi & 0 & 1 \end{bmatrix}$$

$$M_K = \begin{bmatrix} \cos K & \sin K & 0 \\ -\sin K & \cos K & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & K & 0 \\ -K & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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23-15

Small angle approx: datum transformations

$$\begin{bmatrix} 1 & 0 & -\varphi \\ 0 & 1 & \omega \\ \varphi & -\omega & 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & -\varphi \\ 0 & 1 & 0 \\ \varphi & 0 & 1 \end{bmatrix}}_{M_\varphi} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \omega \\ 0 & -\omega & 1 \end{bmatrix}}_{M_\omega}$$

$$\begin{bmatrix} 1 & K & -\varphi \\ -K & 1 & \omega \\ \varphi & -\omega & 1 \end{bmatrix} = \begin{bmatrix} 1 & K & 0 \\ -K & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -\varphi \\ 0 & 1 & \omega \\ \varphi & -\omega & 1 \end{bmatrix}$$

↑ Small angle representation of Rotation Matrix

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