probability distributions 18-1

N, MVN, t, χ^2, F

t distr.

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$$

$$\frac{\bar{X}-\mu}{s/\sqrt{n}} \sim t_n$$

$$\frac{\hat{X}-\mu}{S_{\hat{x}}} \sim t_r$$

$$\frac{\hat{X}-\mu}{\sigma_{\hat{x}}} \sim z$$

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18-2

$$f_n(t) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n\pi}\,\Gamma\left(\frac{n}{2}\right)}\left(1+\frac{t^2}{n}\right)^{-\frac{n+1}{2}}$$

as $n \to \infty$ $t \to z$

large n, replace t by z : N(0,1)

F distribution : 2 χ^2 : χ^2_m χ^2_n

$$\mu = \frac{\chi^2_m/m}{\chi^2_n/n} \sim F_{m,n}$$

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$$f_{m,n}(u) = \frac{\Gamma\left(\frac{m+n}{2}\right)}{\Gamma\left(\frac{m}{2}\right)\Gamma\left(\frac{n}{2}\right)}\left(\frac{m}{n}\right)^{m/2} \cdot \frac{u^{(m-2)/2}}{\left[1+\left(\frac{m}{n}\right)u\right]^{\frac{m+n}{2}}}$$

18-3

common use of F: compare 2 variance estimates

$$\left.\begin{aligned} \frac{(n_1-1)S_1^2}{\sigma_1^2} &\sim \chi^2_{n_1-1} \\ \frac{(n_2-1)S_2^2}{\sigma_2^2} &\sim \chi^2_{n_2-1} \end{aligned}\right\} \quad \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F_{n_1-1,n_2-1}$$

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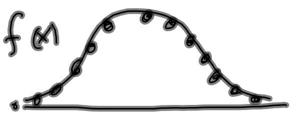
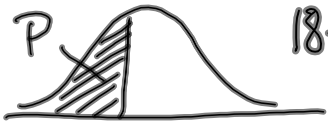
density function $f(x)$



cumulative distr. function

P



x

18-4

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

pdf, cdf, icdf

$y = pdf('norm', x, \mu, \sigma)$

matlab specific

1. matlab
2. tables
3. sci. calc.

$P = cdf('norm', x, \mu, \sigma)$

$x = icdf('norm', P, \mu, \sigma)$

'chi2', 't', 'f'

n $\quad$ n $\quad$ n_1, n_2

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18-5

σ_0^2 prior, a priori

$\hat{\sigma}_0^2$ post adjustment, a posteriori

$\hookrightarrow \boxed{\hat{\sigma}_0^2 = \frac{v^T W v}{r}} \rightarrow \frac{r \cdot \hat{\sigma}_0^2}{\sigma_0^2} \sim \chi_r^2$

$$\boxed{\frac{v^T W v}{\sigma_0^2} \sim \chi_r^2}$$

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18-6

When you choose σ_0^2 (reference variance)
choosing an observation to assign $w = \underline{1}$

$$w_i = \frac{\sigma_0^2}{\sigma_i^2}$$

after adjustment we have residuals
which may be <u>consistent</u> or <u>not</u>
with σ_i^2

$$\frac{v^T W v}{\sigma_0^2} \sim \chi_r^2 \longleftarrow \text{Global Test}$$

result of test <u>DOES NOT</u> depend
on choice of σ_0^2

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Indirect Obs $Q_{\Delta\Delta} = N^{-1} = (B^T W B)^{-1}$ 18-7

$\Sigma_{\Delta\Delta} = \sigma_0^2 Q_{\Delta\Delta}$

↑

either pre or post value
depending on result of
Global Test

$\underline{Q_{\hat{\ell}\hat{\ell}}}$, Q_{vv}

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I/O : $Q_{\hat{\ell}\hat{\ell}}$ $\vec{y} = A\vec{x} + \vec{b}$ ← constants; A ← constant $\vec{x} : \Sigma_{xx}$, Q_{xx} 18-8

$\hat{\ell} = \ell + v$, $v + B\Delta = f$, $v = f - B\Delta$

$\hat{\ell} = \ell + f - B\Delta$, $f = d - \ell$, $\hat{\ell} = \ell + d - \ell - B\Delta$

$\hat{\ell} = d - B\Delta$, $\Delta = (B^T W B)^{-1} B^T W (d - \ell)$

$\hat{\ell} = d - B(B^T W B)^{-1} B^T W (d - \ell)$

$$\hat{\ell} = \underbrace{d - B(B^T W B)^{-1} B^T W d}_{\text{Const. vector}} + \underbrace{B(B^T W B)^{-1} B^T W}_{\text{const. matrix}} \underbrace{\ell}_{\text{R.V.}}$$

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18-9

$$\hat{\ell} = \vec{vect} + \underbrace{B(B^TWB)^{-1}B^TW}\,\underbrace{\ell}$$

$$Q_{\hat{\ell}\hat{\ell}} = B(B^TWB)^{-1}B^TW \cdot Q \cdot \left[B(B^TWB)^{-1}B^TW\right]^T$$

$$B(B^TWB)^{-1}\underbrace{B^TW \cdot \underbrace{Q \cdot W}_{I}}\,\underbrace{B(B^TWB)^{-1}B^T}$$

$$Q_{\hat{\ell}\hat{\ell}} = B(B^TWB)^{-1}B^T = \boxed{BN^{-1}B^T = Q_{\hat{\ell}\hat{\ell}}}$$

$$\hat{\ell} = d - B\Delta, \quad Q_{\hat{\ell}\hat{\ell}} = (-B)N^{-1}(-B)^T = \underline{BN^{-1}B^T}$$

$$\Sigma_{\hat{\ell}\hat{\ell}} = \sigma_0^2 Q_{\hat{\ell}\hat{\ell}}$$

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18-10

$$v: \quad Q_{vv}, \quad \underline{v = \text{const}\,\vec{vect} + \square \cdot \ell}$$

$$v = f - B\Delta, \quad v = d - \ell - B\underline{N^{-1}B^TW(d-\ell)}$$

$$v = d - \ell - BN^{-1}B^TWd + BN^{-1}B^TW\ell$$

$$v = \underbrace{(I - BN^{-1}B^TW)d}_{\text{const. vector}} + \underbrace{(BN^{-1}B^TW - I)}_{\text{const. matrix}}\underbrace{\ell}_{RV}$$

$$Q_{vv} = (BN^{-1}B^TW - I)\,Q\,[BN^{-1}B^TW - I]^T$$

$$= (BN^{-1}B^T - Q)(WBN^{-1}B^T - I)$$

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18-11

$(BN^{-1}B^T - Q)(WBN^{-1}B^T - I) \quad Q_{vv}$

$BN^{-1}\underbrace{B^TWBN^{-1}}B^T + Q - BN^{-1}B^T - BN^{-1}B^T$

$Q + \underbrace{BN^{-1}B^T - BN^{-1}B^T} - BN^{-1}B^T$

$\boxed{Q_{vv} = Q - BN^{-1}B^T}$ $\quad Q_{vv} = Q - Q_{\hat{\ell}\hat{\ell}}$

$Q_{\hat{\ell}\hat{\ell}} = Q - Q_{vv}$

$\boxed{Q_{\hat{\ell}\hat{\ell}} = BN^{-1}B^T}$ $\quad Q = Q_{\hat{\ell}\hat{\ell}} + Q_{vv}$

all cofactor/covariance matrices for I/O

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Observations Only $\quad Av = f,\ k = W_e f,\ v = QA^Tk$ 18-12

$v = \underbrace{QA^TW_e d}_{\text{const. vector}} - \underbrace{QA^TW_eA}_{\text{const. matrix}}\underbrace{\ell}_{\text{R.V.}}$ $\quad \boxed{Q_{\hat{\ell}\hat{\ell}} = Q - Q_{vv}}$

$\boxed{Q_{vv} = QA^TW_eAQ}$

$\hat{\ell} = \underbrace{QA^TW_e d}_{\text{const. vect}} + \underbrace{(I - QA^TW_eA)}_{\text{Const. matrix}}\underbrace{\ell}_{\text{R.V.}}$

$\boxed{Q_{\hat{\ell}\hat{\ell}} = Q - QA^TW_eAQ}$

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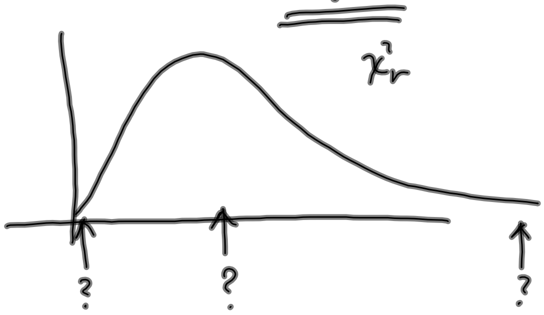
Global Test

only possible if we have $\Sigma_{\ell\ell}$ 18-B

not possible if "obs. are equally weighted" (W=I)

test statistic $\underline{\underline{\frac{v^T W v}{\sigma_0^2}}} \sim \chi^2_r$

χ^2_r



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