numerical approximations of derivatives 13-1

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

take very small value for Δx

$$\frac{df}{dx} \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

eval @ current value of x

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$$\frac{\partial f}{\partial x_i} = \lim_{\Delta x_i \to 0} \frac{f(x_1, x_2, \ldots, x_i+\Delta x_i, \ldots, x_n) - f(x_1, x_2, \ldots, x_n)}{\Delta x_i}$$ 13-2

$$\frac{\partial f}{\partial x_i} \approx \frac{f(\cdots) - f(\cdots)}{\Delta x_i}$$

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function f, vector x, dx

↳ small perturbations

13-3

```
for i=1:n
    df_dx(i) = (f(x(i)+dx(i)) - f(x(i)))/dx(i);
end
```

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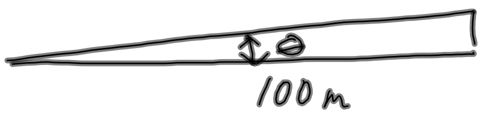
13-4



.01 m = 1 cm

$\sigma_{dist} = .01\ m$

$\sigma_{ang} = \text{atan}(.01/100) = .0001\ R$ (.0057 deg.)

choose $\sigma_0^2 = (.01)^2 = .0001$

$W_d = \frac{(.01)^2}{(.01)^2} = 1$

$W_a = \frac{(.01)^2}{(.0001)^2} = 10{,}000$

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do this again, dist units Km 13-5

$\sigma_d = .00001$ Km

$\sigma_a = .0001$ Rad

$\sigma_0^2 = (.0001)^2$

$$w_d = \frac{(.0001)^2}{(.00001)^2} = 100$$

$$w_a = \frac{(.0001)^2}{(.0001)^2} = 1$$

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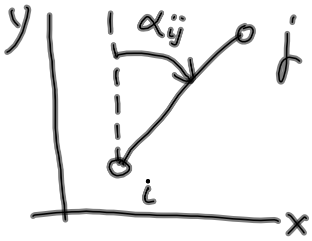
linearization of some famous condition equations 13-6

azimuth observations (clockwise angle from north)



$$\alpha_{ij} = \operatorname{atan}\left(\frac{x_j - x_i}{y_j - y_i}\right)$$

$$\boxed{F_\alpha = \alpha_{ij} - \operatorname{atan}\left(\frac{\Delta x}{\Delta y}\right) = 0}$$

$$\frac{\partial F_\alpha}{\partial x_i} = -\frac{1}{1+\left(\frac{\Delta x}{\Delta y}\right)^2}\cdot\frac{-1}{\Delta y}\cdot\frac{\Delta y}{\Delta y}$$

$$\left.\frac{d}{dx}\operatorname{atan}(u) = \frac{1}{1+u^2}\cdot\frac{du}{dx}\right\} \quad \frac{\partial F_\alpha}{\partial x_i} = \frac{\Delta y}{\Delta x^2 + \Delta y^2}$$

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$$F_\alpha = \alpha_{ij} - \operatorname{atan}\left(\frac{\Delta x}{\Delta y}\right) = 0$$

$\Delta x = x_j - x_i$
$\Delta y = y_j - y_i$

$$\frac{\partial F_\alpha}{\partial x_j} = \frac{-\Delta y}{\Delta x^2 + \Delta y^2} = \frac{-\Delta y}{D_{ij}^2}$$

$$\frac{\partial F_\alpha}{\partial y_i} = -\frac{1}{1+\left(\frac{\Delta x}{\Delta y}\right)^2}\cdot(-1)\frac{\Delta x}{\Delta y^2}(-1) = \frac{-\Delta x}{D_{ij}^2}$$

$$\frac{\partial F_\alpha}{\partial y_j} = \frac{\Delta x}{D_{ij}^2}$$

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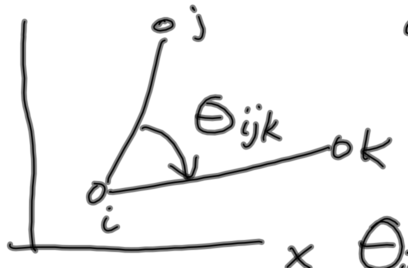

angle observation cond. equation 13-8

angle meas. at i, from j to k
CW direction

$$\theta_{ijk} = az_{ik} - az_{ij}$$

$$\theta_{ijk} = \tan^{-1}\left(\frac{\Delta x_{ik}}{\Delta y_{ik}}\right) - \tan^{-1}\left(\frac{\Delta x_{ij}}{\Delta y_{ij}}\right)$$

$$\boxed{F_\theta = \theta_{ijk} - \tan^{-1}\left(\frac{\Delta x_{ik}}{\Delta y_{ik}}\right) + \tan^{-1}\left(\frac{\Delta x_{ij}}{\Delta y_{ij}}\right) = 0}$$

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$$\frac{\partial F_\theta}{\partial x_i} = \frac{\Delta y_{ik}}{D_{ik}^2} - \frac{\Delta y_{ij}}{D_{ij}^2}$$ $\Delta x_{ij} = x_j - x_i$ 13-9

$\Delta y_{ij} = y_j - y_i$

$$\frac{\partial F_\theta}{\partial x_j} = \frac{\Delta y_{ij}}{D_{ij}^2}, \quad \frac{\partial F_\theta}{\partial x_k} = -\frac{\Delta y_{ik}}{D_{ik}^2}$$

$$\frac{\partial F_\theta}{\partial y_i} = \frac{-\Delta x_{ik}}{D_{ik}^2} + \frac{\Delta x_{ij}}{D_{ij}^2}$$

$$\frac{\partial F_\theta}{\partial y_j} = -\frac{\Delta x_{ij}}{D_{ij}^2}, \quad \frac{\partial F_\theta}{\partial y_k} = \frac{\Delta x_{ik}}{D_{ik}^2}$$

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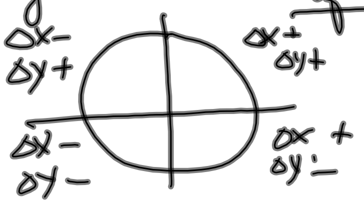
those partial deriv. become elements of B-matrix 13-10

RHS vector $f = -F(\ell, x^0)$

few pitfalls in evaluating this

$\operatorname{atan}\left(\frac{\Delta x}{\Delta y}\right)$, $\tan^{-1}\left(\frac{\Delta x}{\Delta y}\right)$

① get in correct quadrant



atan2(Δx, Δy)

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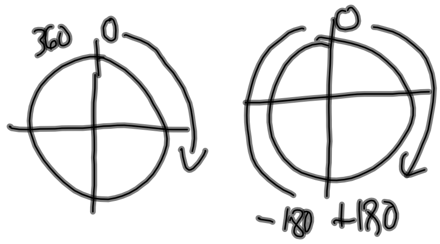
(2) Sums or differences of angles, can get out of range

$0 - 360, \quad 0 - 2\pi$

$-180 - +180, \quad -\pi - +\pi$

(3)



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Distance or range

$$d_{ij} = \left[(x_j - x_i)^2 + (y_j - y_i)^2\right]^{1/2}$$

$$F_d = d_{ij} - \left[(x_j - x_i)^2 + (y_j - y_i)^2\right]^{1/2} = 0$$

$$\frac{\partial F_d}{\partial x_i} = \frac{\Delta x}{D_{ij}} \qquad \frac{\partial F_d}{\partial y_i} = \frac{\Delta y}{D_{ij}}$$

$$\frac{\partial F_d}{\partial x_j} = \frac{-\Delta x}{D_{ij}} \qquad \frac{\partial F_d}{\partial y_j} = \frac{-\Delta y}{D_{ij}}$$

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$f = -F_d(\ell, x^0)$

13-13

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