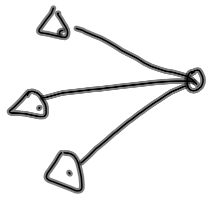

? $F_i = \hat{l}_i - \left[(x-x_i)^2 + (y-y_i)^2\right]^{1/2} = 0$ 12-1

observed − computed

$\frac{\partial F_i}{\partial x} = \frac{-(x^0 - x_i)}{D_i^0}$ , $\frac{\partial F_i}{\partial y} = \frac{-(y^0 - y_i)}{D_i^0}$

$f = -F_i(l, x^0)$

1. 9, 14
2. 8, 12
3. 10, 10

obs: 10.2, 11.0, 9.5

$W = I_3$

$$B = \begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \\ \frac{\partial F_3}{\partial x} & \frac{\partial F_3}{\partial y} \end{bmatrix}, \quad f = \begin{bmatrix} -F_1 \\ -F_2 \\ -F_3 \end{bmatrix}$$

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$$B = \begin{bmatrix} -.9983 & .1386 \\ -.9985 & -.0544 \\ -.9607 & -.2775 \end{bmatrix} \quad f = \begin{bmatrix} -.1024 \\ .0163 \\ -.1319 \end{bmatrix}$$

$x^0 = 19$
$y^0 = 12.6$

12-2

$$\Delta = (B^T W B)^{-1} B^T W f = \begin{bmatrix} .0672 \\ .0925 \end{bmatrix}$$

$$\begin{pmatrix} x^0 \\ y^0 \end{pmatrix} + \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{matrix} 19.067 \\ 12.692 \end{matrix}$$

$$\Delta_2 = \begin{bmatrix} -.0004 \\ .0013 \end{bmatrix}$$

$$\Delta_3 \begin{bmatrix} 1.16e-6 \\ 1.34e-5 \end{bmatrix}$$

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Sample matlab code for terminating iterations

12-3

all (v) : true if all elements of v are true  true = 1  false = 0

```
n_iter = 0
keep_going = 1 ;
while ( keep_going == 1 )
    [ LS code B,f,W.. δ, update x0 ]
    if all( abs(del) < 1e-06 ))
        keep_going = 0 ;
        disp ('we have converged') ;
        v = f - B * del ;
        % print other things
    end
    if ( n_iter > 10 )
        keep_going = 0
        disp ('we did not converge') ;
    end
    n_iter = n_iter + 1 ;
end
```

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debug strategies

12-4

1. stop after iteration 1
   examine f = -F
   should be small
2. compare derivatives with numerical approx.

observation only problem

Non Linear

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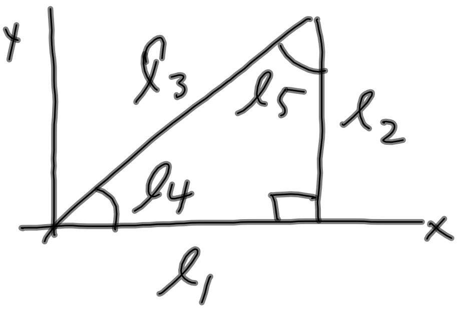


$n = 5$
$n_0 = 2$
$r = 3$

12-5

$\ell_1^2 + \ell_2^2 = \ell_3^2$

$\ell_4 + \ell_5 = \pi/2$ (Radians)

$\ell_4 = \text{atan}(\ell_2/\ell_1)$

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$F_1 = \ell_1^2 + \ell_2^2 - \ell_3^2 = 0$

$F_2 = \ell_4 + \ell_5 - \pi/2 = 0$

$F_3 \quad \ell_4 - \text{atan}(\ell_2/\ell_1) = 0$

$\frac{\partial F}{\partial \ell} = A$

$\ell^0 = \ell$

$\frac{d}{dx}\text{atan}(u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$

12-6

$\partial F_1/\partial \ell_1 = 2\ell_1, \quad \partial F_1/\partial \ell_2 = 2\ell_2, \quad \partial F_1/\partial \ell_3 = -2\ell_3$

$\partial F_2/\partial \ell_4 = 1, \quad \partial F_2/\partial \ell_5 = 1$

$\left.\begin{array}{l} \partial F_3/\partial \ell_4 = 1, \quad \frac{\partial F_3}{\partial \ell_1} = \frac{-1}{1+\frac{\ell_2^2}{\ell_1^2}} \cdot \frac{-\ell_2}{\ell_1^2} = \frac{\ell_2}{\ell_1^2+\ell_2^2} \end{array}\right\} @\, \ell^0$

$\frac{\partial F_3}{\partial \ell_5} = \frac{-1}{1+\frac{\ell_2^2}{\ell_1^2}} \cdot \frac{1}{\ell_1}\left(\frac{\ell_1}{\ell_1}\right) = \frac{-\ell_1}{\ell_1^2+\ell_2^2}$

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$$A = \begin{bmatrix} 2l_1^\circ & 2l_2^\circ & -2l_3^\circ & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ \frac{l_2^\circ}{l_1^{\circ 2}+l_2^{\circ 2}} & \frac{-l_1^\circ}{l_1^{\circ 2}+l_2^{\circ 2}} & 0 & 1 & 0 \end{bmatrix} \quad W, \dots \quad 12\text{-}7$$

$(3,5)$

$(c,n)$

$(r,n)$

$$f = -F - A(l - l^\circ)$$

$$f = \left[\begin{bmatrix} -(l_1^2 + l_2^2 - l_3^2) \\ -(l_4 + l_5 - \pi/2) \\ -(l_4 - \tan^{-1}(l_1/l_2)) \end{bmatrix} - A * (l - l^\circ)\right]$$

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$$l = \begin{bmatrix} 10.1 \\ 7.4 \\ 12.5 \\ 36^\circ.22 \\ 53^\circ.78 \end{bmatrix} \quad \left.\begin{matrix} \sigma_d = 0.1 \\ \sigma_a = 0.005R \end{matrix}\right\} W \qquad 12\text{-}8$$

$$\Delta l : \begin{bmatrix} -.0074 \\ -.0075 \\ 0.1 \\ 0 \\ 0 \end{bmatrix}, \underset{}{\overset{1e\text{-}05}{\begin{bmatrix} -.6 \\ .5 \\ .8 \\ -.0018 \\ .0018 \end{bmatrix}}}, \overset{1e\text{-}09}{\begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}}, \overset{1e\text{-}14}{\begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}}$$

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$\sigma_d = 0.1$ $\sigma_0 = 0.1, \sigma_0^2 = 0.01$ $w_d = \frac{(0.1)^2}{(0.1)^2} = \underline{1}$ 12-9

$\sigma_a = .005R$

$w_a = \frac{(0.1)^2}{(.005)^2} = 400$

$$W = \begin{bmatrix} 1 & & & & \phi \\ & 1 & & & \\ & & 1 & & \\ & & & 400 & \\ \phi & & & & 400 \end{bmatrix}$$

$A, f, W$

$K = (AQA^T)^{-1} f$

$Q = W^{-1}$

$v = QA^T k$

update $l^0 = l + v$

$\Delta l = l^0_{new} - l^0_{prev.}$

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12-10

$$V = \begin{bmatrix} -.0074 \\ -.0075 \\ .0104 \\ 0 \\ 0 \end{bmatrix}$$

Computing derivatives

1. analytically ✓
2. numerical approximation
3. Symbolic processing

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Symbolic

12-11

$$\boxed{\begin{aligned} F_x &= x_p + dx + foc*(u/w) ; \\ F_y &= y_p + dy + foc*(v/w) ; \end{aligned}}$$

$x_0, y_0, k_1, k_2, k_3, \omega, \varphi, k$

$$M_\omega = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix}, M_\varphi, M_k$$

$$M = M_k * M_\varphi * M_\omega$$

$$C = \begin{bmatrix} cx \\ cy \\ cz \end{bmatrix}$$

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12-12

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = M * c$$

$$x_p = x - x_0$$
$$y_p = y - y_0$$
$$r = \sqrt{x_p^2 + y_p^2}$$
$$dr = k_1 * r^3 + k_2 * r^5 + k_3 * r^7$$
$$dx = dr * xp/r$$
$$dy = dr * yp/r$$

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