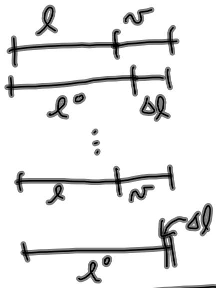
Nonlinear LS : Indirect Observations $\frac{n}{\frac{n_0}{r}}$ 11-1

$\hat{l} = G(x)$ ✓ , need exactly $u = n_0$ unknown parameters

$\underline{F(\hat{l}, x) = \hat{l} - G(x) = 0}$

Linearize (Taylor Series) $F(\hat{l}, x) \approx F(l^0, x^0) + \underline{\frac{\partial F}{\partial l}} \Delta l + \underline{\frac{\partial F}{\partial x}} \Delta x = 0$

$\Delta x, v, \Delta l$



$\hat{l} = l + v$ l: original obs.

$= l^0 + \Delta l$ l^0: current est.

as you proceed through iterations

$\Delta l \rightarrow$ zero

$v \rightarrow$ not zero

$l + v = l^0 + \Delta l = \boxed{\Delta l = (l - l^0) + v}$

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$F(\hat{l}, x) = \underbrace{F(l^0, x^0)}_{l^0 - G(x^0)} + \underbrace{\frac{\partial F}{\partial l}}_{= I_n} \Delta l + \underbrace{\frac{\partial F}{\partial x}}_{B} \Delta x = 0$ 11-2

Δl: $(l - l^0) + v$

$= l^0 - G(x^0) + l - l^0 + v + B\Delta = 0$

$= (l^0 - l^0) + \underbrace{l - G(x^0)} + v + B\Delta = 0$

$= \underline{F(l, x^0)} + v + B\Delta = 0$ mis-closure

$v + B\Delta = \underbrace{-F(l, x^0)}_{f}$

$\boxed{v + B\Delta = f}$ both B, f

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$$\underset{(n,1)}{V} + \underset{(n,\mu)}{B}\ \underset{(\mu,1)}{\Delta} = \underset{(n,1)}{f}$$

n obs. $\mu = n_0$
n_0 min.
r red.

11-3

$W = \sigma_0^2 \Sigma^{-1}$ need B, f, W

$$\Delta = (B^T W B)^{-1} B^T W f$$

$$X^0_{New} = X^0_{Old} + \Delta \leftarrow \text{re-eval } B, f, W$$

examine magnitude of Δ
when all |element of δ are < threshold

after convergence

$V = f - B\Delta$ (from last iteration)

$\hat{\ell} = \ell + v$

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Observations only - Nonlinear

n
n_0
r

11-4

$F(\hat{\ell}) = 0$ condition equations
r cond. eqns $c = r$

$$F(\hat{\ell}) \approx F(\ell^0) + \underbrace{\frac{\partial F}{\partial \ell}}_{A} \underbrace{\Delta \ell}_{(\ell - \ell^0 + v)} = 0$$

$$F(\ell^0) + A(\ell - \ell^0) + Av = 0$$

$$Av = \underbrace{-F(\ell^0) - A(\ell - \ell^0)}_{f}$$

$$\boxed{Av = f} \leftarrow$$

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11-5

$c = r$

$$\underset{(c,n)}{A}\ \underset{(n,1)}{v} = \underset{(c,1)}{f}$$

$$\left.\begin{array}{l} k = W_e f \\ v = Q A^T k \end{array}\right\}$$

$$\boxed{l_{new} = l + v}$$

re-evaluate A, f + next iteration

$\Delta l = l^0_{new} - l^0_{prev}$

test for convergence

$K = (A Q A^T)^{-1} f$

$v = Q A^T K$

$Q = W^{-1}$

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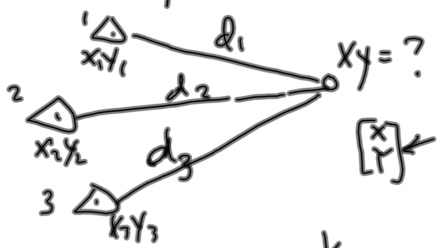
11-6

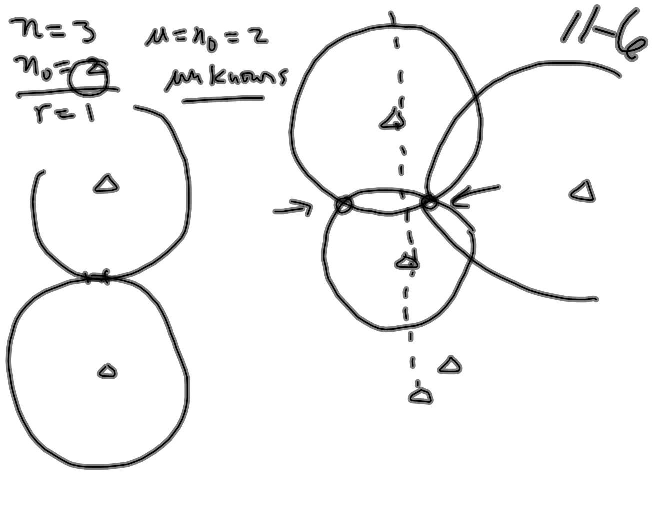
examples I/O



$n = 3$ $\quad u = n_0 = 2$

$n_0 = 2$ $\quad$ unknowns

$r = 1$



$\hat{d}_1 = [(\hat{x} - x_1)^2 + (\hat{y} - y_1)^2]^{1/2}$

$\hat{d}_2 = [(\hat{x} - x_2)^2 + (\hat{y} - y_2)^2]^{1/2}$

$d_3 = [(x - x_3)^2 + (y - y_3)^2]^{1/2}$

$d_i = [(x - x_i)^2 + (y - y_i)^2]^{1/2}$

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$$F_i = \underset{1\ \text{obs.}}{\underline{\underline{l_i}}} - \Big[\underbrace{(x - x_i)^2 + (y - y_i)^2}_{\text{(params)}}\Big]^{1/2} = 0$$

$$f = -F(l, x^0) \quad 11\text{-}7$$

$$-\left(d - [\;]^{1/2}\right)$$

$$B = \begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \\ \frac{\partial F_3}{\partial x} & \frac{\partial F_3}{\partial y} \end{bmatrix}$$

x y

$$\frac{\partial F_i}{\partial x} = -\frac{1}{2}[\cdot]^{-1/2}\, 2(x - x_i) = \boxed{\frac{-(x^0 - x_i)}{D_i^0}}$$

$$\frac{\partial F_i}{\partial y} = -\frac{1}{2}[\cdot]^{-1/2}\, 2(y - y_i) = \boxed{\frac{-(y^0 - y_i)}{D_i^0}}$$

$$\left[(x^0 - x_i)^2 + (y^0 - y_i)^2\right]^{1/2} = D_i^0$$

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