9-1

$$R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cw & sw \\ 0 & -sw & cw \end{bmatrix}$$

$$R_2 = \begin{bmatrix} c\phi & 0 & -s\phi \\ 0 & 1 & 0 \\ s\phi & 0 & c\phi \end{bmatrix}$$

$$R_3 = \begin{bmatrix} ck & sk & 0 \\ -sk & ck & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\theta_x = \omega$

$\theta_y = \phi$

$\theta_z = k$

$cw : \cos(\omega)$

$sw : \sin(\omega)$

$X_2 = R_3 R_2 R_1 X_1$

R_1 — primary

R_2 — secondary

R_3 — tertiary

$R = \underbrace{R_3 R_2 R_1}^{*}$

$X_2 = R X_1$

$X_2 = \underbrace{R_3 \underbrace{R_2 \underbrace{R_1 X}}}$

$R' = \underline{R_1 R_2 R_3}$

$\underbrace{R_1 \underbrace{R_2 \underbrace{R_3 X}}}$

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9-2

$R = R_3 R_2 R_1$

$M = M_k M_\phi M_\omega$ ✓

$[M_k][M_\phi][M_\omega]$

$$M = \begin{bmatrix} c\phi ck & cwsk + swS\phi ck & swsk - cws\phi ck \\ -c\phi sk & cwck - sws\phi sk & swck + cws\phi sk \\ s\phi & -swc\phi & cwc\phi \end{bmatrix}$$

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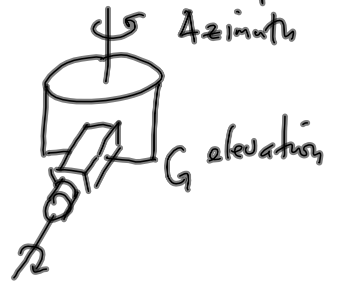
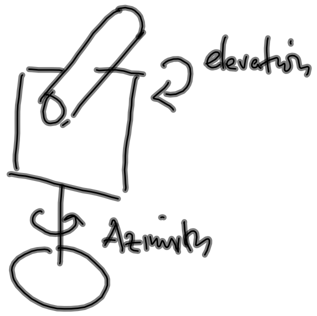
Mechanical Implementation of Sequential Rotations : Gimbal





9-3

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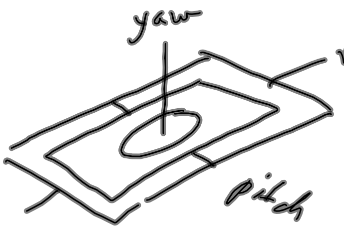
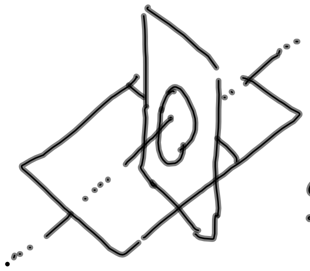


stabilized platform

problem if middle rotation is $\pm 90°$



put roll axis in line with new position of yaw axis

critical geometry

Gimbal Lock

model rotations : 9-4

sequential elem. rotations

axis-angle

quaternions (algebraic param.)

direction cosines

⋮

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2D conformal coord. transf. scale, rotation, x-shift, y-shift 9-5

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \underset{\text{scale}}{\lambda} \underset{\text{rotation}}{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}} \begin{pmatrix} x \\ y \end{pmatrix} + \underset{\text{2 shifts}}{\begin{pmatrix} t_x \\ t_y \end{pmatrix}}$$

4 parameter transf.

if x, y constant
linear in a, b, c, d

physical parameters

$$x' = \underbrace{\lambda\cos\theta}\, x + \underbrace{\lambda\sin\theta}\, y + t_x$$
$$y' = -\lambda\sin\theta\, x + \lambda\cos\theta\, y + t_y$$

$$\left.\begin{aligned} \lambda\cos\theta &= a \\ \lambda\sin\theta &= b \\ t_x &= c \\ t_y &= d \end{aligned}\right\} \quad \begin{aligned} x' &= ax + by + c \\ y' &= -bx + ay + d \\ \begin{pmatrix} x' \\ y' \end{pmatrix} &= \begin{pmatrix} a & b \\ -b & a \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} \end{aligned}$$

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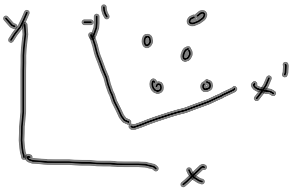

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix}$$ 9-6

x, y constant
x', y' observed

$$n = 5 \times 2 = 10$$
$$n_0 = 4$$
$$r = 6$$

$$\hat{x}_1' = ax_1 + by_1 + c$$
$$\hat{y}_1' = -bx_1 + ay_1 + d$$
$$\hat{x}_2' = ax_2 + by_2 + c$$
$$\hat{y}_2' = -bx_2 + ay_2 + d$$
$$\vdots$$
$$\hat{x}_5' = ax_5 + by_5 + c$$
$$\hat{y}_5' = -bx_5 + ay_5 + d$$

Indirect Obs.

10 cond. eqn ✓
4 par a, b, c, d ✓

Least Squares
⇓
$$v + B\Delta = f$$

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9-7

$$v_{x_1} - ax_1 - by_1 - c = -x_1'$$
$$v_{y_1} + bx_1 - ay_1 - d = -y_1'$$
$$\vdots$$

par. vector

$$\begin{bmatrix} v_{x_1} \\ v_{y_1} \\ v_{x_2} \\ v_{y_2} \\ \vdots \\ v_{x_5} \\ v_{y_5} \end{bmatrix} + \begin{array}{c} \begin{array}{cccc} (a) & (b) & (c) & (d) \end{array} \\ \begin{bmatrix} -x_1 & -y_1 & -1 & 0 \\ -y_1 & x_1 & 0 & -1 \\ -x_2 & -y_2 & -1 & 0 \\ -y_2 & x_2 & 0 & -1 \\ & \vdots & & \\ -x_5 & -y_5 & -1 & 0 \\ -y_5 & x_5 & 0 & -1 \end{bmatrix} \end{array} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -x_1' \\ -y_1' \\ -x_2' \\ -y_2' \\ \vdots \\ -x_5' \\ -y_5' \end{bmatrix}$$

$$v + B \quad \Delta = f$$

$$\underline{v + B\Delta = f}$$

$$\Delta = (B^T W B)^{-1} B^T W f \checkmark$$

$$v = f - B\Delta \checkmark$$

$$\hat{\ell} = \ell + v$$

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9-8

$$\left.\begin{array}{l} a = \lambda \cos\theta \\ b = \lambda \sin\theta \end{array}\right\}$$

$$a^2 + b^2 = \lambda^2\cos^2\theta + \lambda^2\sin^2\theta$$
$$= \lambda^2(\underbrace{\cos^2\theta + \sin^2\theta}_{=1})$$

$$a^2 + b^2 = \lambda^2$$

$$\boxed{\lambda = \sqrt{a^2 + b^2}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{b}{a}\right)} \checkmark$$

* may need 2 argument arctangent

atan, atan2

atan(a/b), $\underline{\text{atan2}(b, a)}$

recover physical parameters after using the linear params

$$\tan\theta = \left(\frac{\sin\theta}{\cos\theta}\right)$$

$$t_x = c$$
$$t_y = d$$

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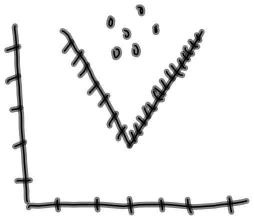
2D affine coordinate transf 6 parameter transf. 9-9

$$x' = a_0 + a_1 x + a_2 y$$
$$y' = b_0 + b_1 x + b_2 y$$

$$6\begin{cases}\text{general rotation}\\ \text{non-orthogonality angle}\\ \text{2 scale factors}\\ \text{2 shifts}\end{cases}$$



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3D conformal coord transf. 7 parameter transf / rigid transf. 9-10

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \underbrace{\lambda}_{\text{1 scale}} \cdot \underbrace{M}_{\text{3 rotations}} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \underbrace{\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}}_{\text{3 shifts}}$$

only handled with physical parameters
non-linear

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