Coord. transf.: 8-1

$x' = ax + by + c$ 2D conformal

$y' = -bx + ay + d$ 4 parameters

a, b, c, d

physical parameters:

$$\left.\begin{array}{l}\text{rotation}\\ \text{scale}\\ \text{shift } x\\ \text{shift } y\end{array}\right\} \longleftrightarrow \underbrace{a, b, c, d}$$

rotation:

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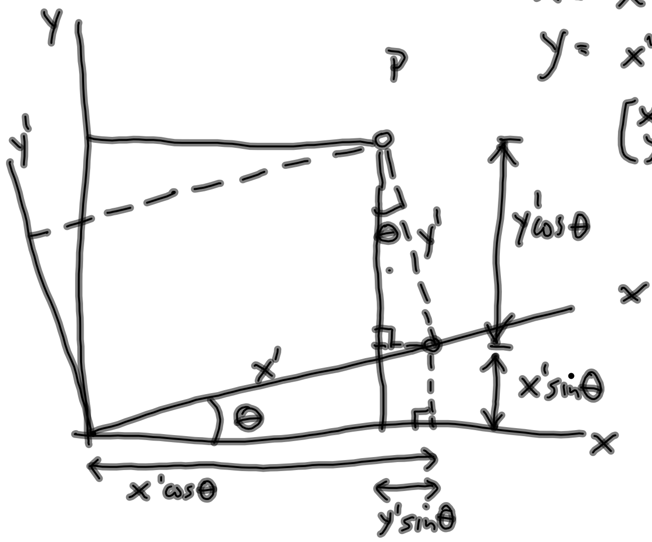
2D rotations 8-2

$x = x'\cos\theta - y'\sin\theta$

$y = x'\sin\theta + y'\cos\theta$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$



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8-3

$$\begin{pmatrix}x\\y\end{pmatrix} = \underbrace{\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}}_{\text{orthogonal matrix}}\begin{pmatrix}x'\\y'\end{pmatrix}$$

det +1 : rotation matrix
 −1 : reflection matrix

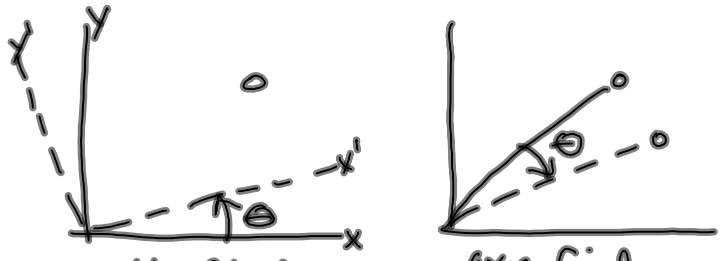
<u>inverse = transpose</u>

$$\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x'\\y'\end{pmatrix}$$

inner product (dot prod.)
any row w/ itself = 1
any col w/ itself = 1
inner product
any row with another row = 0
any col with another col = 0
determinant : ±1

8-4

$$\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$



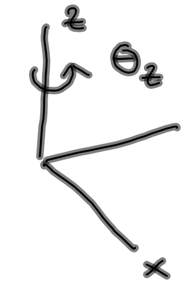
point fixed
rotate axes

axes fixed
point rotates
(opposite direction)

<u>2D rotation</u>

extend rotation to 3D

8-5



right hand

positive rotation by right hand rule

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta_z & \sin\theta_z & 0 \\ -\sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{pmatrix}}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$R_3, R_z, M_z, M_k, \ldots\ldots$

elementary rotation matrix

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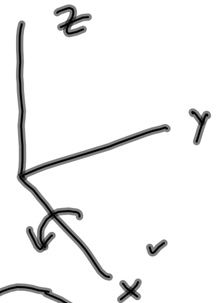
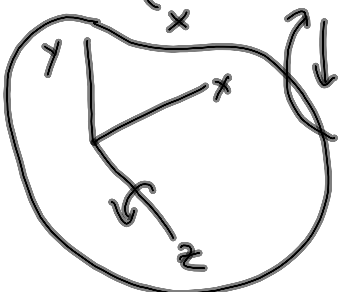
8-6



$$\begin{pmatrix} y' \\ z' \\ x' \end{pmatrix} = \begin{pmatrix} \cos\theta_x & \sin\theta_x & 0 \\ -\sin\theta_x & \cos\theta_x & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} y \\ z \\ x \end{pmatrix}$$



$$\begin{pmatrix} y' \\ z' \\ x' \end{pmatrix} = \begin{pmatrix} 0 & \cos\theta_x & \sin\theta_x \\ 0 & -\sin\theta_x & \cos\theta_x \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & \sin\theta_x \\ 0 & -\sin\theta_x & \cos\theta_x \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}}$$

R_1, R_x, M_x, M_ω

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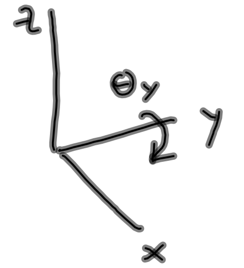
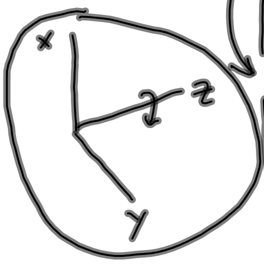
8-7



$$\begin{pmatrix} z' \\ x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta_y & \sin\theta_y & 0 \\ -\sin\theta_y & \cos\theta_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} z \\ x \\ y \end{pmatrix}$$

$$\begin{pmatrix} z' \\ x' \\ y' \end{pmatrix} = \begin{pmatrix} \sin\theta_y & 0 & \cos\theta_y \\ \cos\theta_y & 0 & -\sin\theta_y \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



$$\boxed{\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta_y & 0 & -\sin\theta_y \\ 0 & 1 & 0 \\ \sin\theta_y & 0 & \cos\theta_y \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}}$$

R_z, R_y, M_y, M_ϕ

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Elementary Rotation Matrices

8-8

R_1, R_2, R_3

$R = R_3 R_2 R_1$

$\quad \theta_z \; \theta_y \; \theta_x$

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