7-1

$$W = \sigma_0^2 \begin{bmatrix} 1/\sigma_1^2 & & & \\ & 1/\sigma_2^2 & & \\ & & \ddots & \\ & & & 1/\sigma_n^2 \end{bmatrix}, \quad W = \sigma_0^2 \Sigma^{-1}$$

$$Q = \frac{1}{\sigma_0^2}\Sigma$$

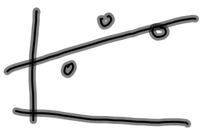
$$\Sigma = \sigma_0^2 Q$$

O/O : r condition equations

$$\begin{array}{r} n \\ -n_0 \\ \hline r \end{array} \qquad \text{I/O} \quad r + \underset{(n_0)}{u} = n \text{ cond. equs.}$$

Sep 10-10:23 AM

7-2



$$\begin{array}{r} n = 3 \\ n_0 = 2 \\ \hline r = 1 \end{array}$$

$y = mx + b$

$y_1 = mx_1 + b$

$y_2 = mx_2 + b$

$y_3 = mx_3 + b$

$b = y_1 - mx_1$

$y_2 = mx_2 + y_1 - mx_1$

$y_3 = mx_3 + y_1 - mx_1$

$y_2 = m(x_2 - x_1) + y_1$

$y_3 = m(x_3 - x_1) + y_1$

$y_2 - y_1 = m(x_2 - x_1)$

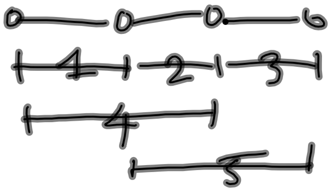
$m = (y_2 - y_1)/(x_2 - x_1)$

$$y_3 = \frac{(y_2 - y_1)}{(x_2 - x_1)} \cdot (x_3 - x_1) + y_1$$

$$\boxed{\frac{(y_3 - y_1)}{(x_3 - x_1)} = \frac{(y_2 - y_1)}{(x_2 - x_1)}}$$

Sep 10-10:23 AM

1D network



$n = 5$
$n_0 = 3$
$r = 2$

o/p : 2 cond eqn.

$\hat{l}_4 = \hat{l}_1 + \hat{l}_2$
$\hat{l}_5 = \hat{l}_2 + \hat{l}_3$

$\hat{l}_4 - \hat{l}_1 - \hat{l}_2 = 0$
$\hat{l}_5 - \hat{l}_2 - \hat{l}_3 = 0$

A, f, W 7-3

$$\begin{bmatrix} -1 & -1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_1+v_1 \\ l_2+v_2 \\ l_3+v_3 \\ l_4+v_4 \\ l_5+v_5 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$A \cdot \hat{l} = d$$

$A(l+v) = d$

$Av = \underbrace{d - Al}_{f}$

$\rightarrow \boxed{Av = f}$

Matrix LS derivations

$\frac{d}{dx} x^T A x = 2x^T A$ 7-4

$\Phi = v^T W v$, use Lagrange Mult.

$\Phi' = v^T W v - 2k_1(\cdots) - 2k_2(\cdots) - \cdots$

$\Phi' = \underset{1,n\ n,n\ n,1}{v^T W v} - 2\underset{1,r}{k^T}\underset{r,1}{(Av - f)}$ min. wrt v, k

$- 2(Av-f)^T k$

$\frac{\partial \Phi'}{\partial v} = \cancel{2}v^T W - \cancel{2}k^T A = 0$

$\frac{\partial \Phi'}{\partial k} = -\cancel{2}(Av-f)^T = 0$

$\rightarrow Wv - A^T k = 0$
$-(Av - f) = 0$

$$\left.\begin{aligned} -Wv + A^T k &= 0 \\ Av &= f \end{aligned}\right\} \quad \begin{bmatrix} -W & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} v \\ k \end{bmatrix} = \begin{bmatrix} 0 \\ f \end{bmatrix}$$

7-5

Can solve directly the → full normal equations for O/O

$n+r \times n+r$

or, elimination

$v = QA^T W_e f$

$Wv = A^T k$

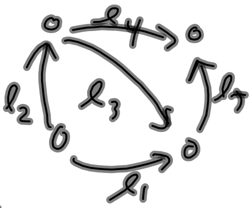
$v = QA^T k$

$\underbrace{AQA^T}_{Q_e} k = f$

$\boxed{k = W_e f}$

Example Matrix of Obs. Only

7-6



$l = \begin{bmatrix} 10 \\ 8 \\ 3 \\ 9 \\ 1 \end{bmatrix}$

$n = 5$

$n_0 = 3$

$r = 2$

$\hat{l}_3 - \hat{l}_1 + \hat{l}_2 = 0$

$\hat{l}_4 - \hat{l}_5 - \hat{l}_3 = 0$

$$(Av = f) \quad \underset{(A)}{\begin{bmatrix} -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 \end{bmatrix}} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix} - A \cdot l}_{f}$$

A ✓, f ✓, W ✓

$W = I_5$, $Q = I_5$

$$\left.\begin{aligned} k &= (AQA^T)^{-1} f \\ v &= QA^T k \end{aligned}\right\} \quad k = \begin{bmatrix} -.375 \\ -.125 \end{bmatrix}$$

$v = [.375\ -.375\ -.250\ -.125\ .125]^T$ ✓

Summary of matrix LS for I/O & O/O

7-7

I/O $\quad v + B\Delta = f \quad B, f, W \quad \Delta = (B^T W B)^{-1} B^T W f$

$(d - \ell) \qquad v = f - B\Delta$

O/O $\quad Av = f \quad A, f, W \quad k = (AQA^T)^{-1} f$

$(d - A\ell) \qquad v = QA^T k$

$\hat{\ell} = \ell + v$

Sep 10-10:24 AM

a few other linear models

7-8

curve fitting: $y = mx + b$

$y = a_0 + a_1 x + a_2 x^2$

$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n$

y: obs, x_i const.

high order polynomials: can be dangerous

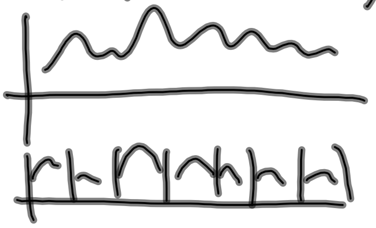
(1) low redundancy

(2) badly conditioned

Sep 10-10:24 AM

one strategy to deal with high order polyn. 7-9

adopt piecewise polynomial



consider continuity

function cont.

1^{st} deriv. cont.

2^{nd} deriv. cont.

spline

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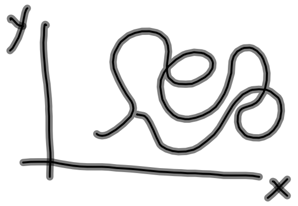

7-10

$x = f(s)$

$y = f(s)$

2D spline

$$y = a_0 + a_1 \cos x + a_2 \sin x + a_3 \cos 2x + a_4 \sin 2x + \cdots$$

y obs.

x constant

surface fitting $z = a_0 + a_1 x + a_2 y$

$z = a_0 + a_1 x + a_2 y + a_3 xy$

(bilinear)

Sep 10-10:24 AM

Coordinate transformations 7-11

$x = a_0 + a_1 X + a_2 Y$
$y = b_0 - a_2 X + a_1 Y$

2D conformal coord. transf.

$$\left.\begin{aligned} x &= a_0 + a_1 x + a_2 y \\ y &= b_0 + b_1 x + b_2 y \end{aligned}\right\} \text{affine transf.}$$

Sep 10-10:24 AM