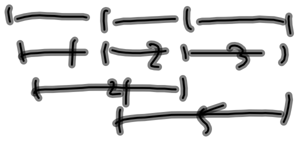

$v_4 - v_1 - v_2 = 0.4$

$v_5 - v_2 - v_3 = -0.2$

5-1

$n = 5$
$n_0 = 3$
$r = 2$

Observations only
write $c = r = 2$
condition equations
(only have obs & constants)

Sep 3-10:23 AM

$$\phi' = v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 + \underline{2}\lambda_1(v_4 - v_1 - v_2 - 0.4) + \underline{2}\lambda_2(v_5 - v_2 - v_3 + 0.2)$$ 5-2

$\partial\phi'/\partial v_1 = \cancel{2}v_1 - \cancel{2}\lambda_1 = 0$

$\partial\phi'/\partial v_2 = \cancel{2}v_2 - \cancel{2}\lambda_1 - \cancel{2}\lambda_2 = 0$

$\partial\phi'/\partial v_3 = \cancel{2}v_3 - \cancel{2}\lambda_2 = 0$

$\partial\phi'/\partial v_4 = \cancel{2}v_4 + \cancel{2}\lambda_1 = 0$

$\partial\phi'/\partial v_5 = \cancel{2}v_5 + \cancel{2}\lambda_2 = 0$

$\partial\phi'/\partial \lambda_1 = \underline{\cancel{2}}(v_4 - v_1 - v_2 - 0.4) = 0$

$\partial\phi'/\partial \lambda_2 = \underline{\cancel{2}}(v_5 - v_2 - v_3 + 0.2) = 0$

$v_1 = \lambda_1 \quad v_4 = -\lambda_1$
$v_2 = \lambda_1 + \lambda_2 \quad v_5 = -\lambda_2$
$v_3 = \lambda_2$

2 ways to proceed
(1) 7 equations for 7 unknowns
(2) elimination
solve for v's in terms of λ's
solve for λ's
solve for v's

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5-3

FULL NORMAL EQUATIONS

$$\begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & \lambda_1 & \lambda_2 \end{array}$$

$$\left[\begin{array}{ccccc:cc} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ \hdashline -1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 & 0 \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.4 \\ -0.2 \end{bmatrix} \longleftarrow Nx = d$$

I_n, CE, O_r

N = [I CE';
CE Or];

$Nx = d$

x = inv(N)* d

$$\left[\begin{array}{c|c} I_n & CE^T \\ \hline CE & O_r \end{array}\right] \left[\;\cdot\;\right] = \begin{bmatrix} 0 \\ - \end{bmatrix} \longleftarrow \text{misclosures}$$

5-4

solution vector :

finish $l + v \rightarrow \hat{l}$

$$\begin{bmatrix} -0.175 \\ -0.050 \\ 0.125 \\ 0.175 \\ -0.125 \\ \hdashline -0.175 \\ 0.125 \end{bmatrix} \begin{array}{l} \longleftarrow v\text{'s} \\ \\ \longleftarrow \lambda\text{'s} \end{array}$$

$$\begin{bmatrix} -3 & -1 \\ -1 & -3 \end{bmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 0.4 \\ -0.2 \end{pmatrix}$$

REDUCED NORMAL EQNS.

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{bmatrix} -0.175 \\ 0.125 \end{bmatrix}$$

then solve for v's by subst

after solving for v's in terms of λ's

$-3\lambda_1 - \lambda_2 = 0.4$

$-\lambda_1 - 3\lambda_2 = -0.2$

either by full normal equations or reduced normal equations $\}$ ⇒ get same result 5-5

Method	Size of N.E.	
I/O	$n_0 \times n_0$	98×98
O/O, LM, full	$n+r \times n+r$	102×102
O/O, LM, elim	$r \times r$	2×2

Suppose: $n = 100$
$n_0 = 98$
$r = 2$

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level networks (1 Dimensional Network) 5-6

Δ elev $1 \to 2$: $r_1 - r_2$

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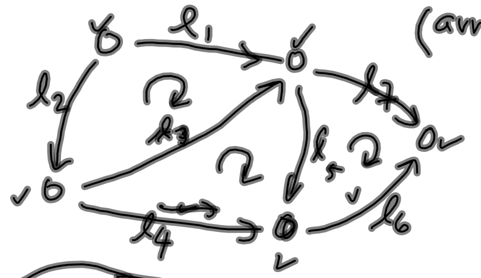


(arrow points uphill)

5-7

$n=7$
$n_0=4$
$r=3$

Obs. only : need $c=r=3$

$$\left.\begin{aligned} -\hat{l}_2+\hat{l}_1-\hat{l}_3&=0\\ \hat{l}_3+\hat{l}_5-\hat{l}_4&=0\\ \hat{l}_7-\hat{l}_6-\hat{l}_5&=0 \end{aligned}\right\}$$

Sep 3-10:24 AM

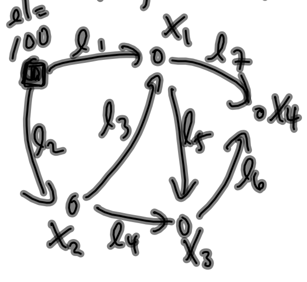
Same level net Indirect Obs

5-8

$n=7$
$n_0=4$
$r=3$

choose $u=n_0=4$ unknowns (parameters)

write $c=n=7$ cond. eqns : 1 obs. per equation + params + constants



$$\begin{aligned} \hat{l}_1&=x_1-100\\ \hat{l}_2&=x_2-100\\ \hat{l}_3&=x_1-x_2\\ \hat{l}_4&=x_3-x_2\\ \hat{l}_5&=x_3-x_1\\ \hat{l}_6&=x_4-x_3\\ \hat{l}_7&=x_4-x_1 \end{aligned}$$

$v_1 = x_1-100-l_1$

$\vdots$

$\phi=v_1^2+v_2^2+\dots$

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(5 problems in vector/matrix notation + solutions

5-9

(Ind. Obs.) example curve fitting

$\hat{l} = a_0 + a_1 x + a_2 x^2$

l : obs
x : constant
a : unknowns

$l + v = a_0 + a_1 x + a_2 x^2$

✓ $v - a_0 - a_1 x - a_2 x^2 = -l$

kept v + params on left
obs → right

$$v + \begin{bmatrix} -1 & -x & -x^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = -l$$

$$v + B \cdot \Delta = \underbrace{d - l}_{f}$$

$$\boxed{v + B\Delta = f}$$

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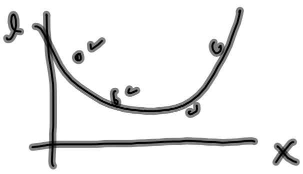

express cond. eqns in vector/matrix form.

5-10

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} + \begin{bmatrix} -1 & -x_1 & -x_1^2 \\ -1 & -x_2 & -x_2^2 \\ -1 & -x_3 & -x_3^2 \\ -1 & -x_4 & -x_4^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -l_1 \\ -l_2 \\ -l_3 \\ -l_4 \end{bmatrix}$$

$$v + B \cdot \Delta = f$$

$$\boxed{\begin{aligned} Ax &= l + v \\ Ax &= l - v \end{aligned}}$$ other notations!

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5-11

$V + B\Delta = f$

$\boxed{V = f - B\Delta}$ ←

$\Phi = V^T V$ $\quad [v_1\ v_2\ v_3\ \cdots]\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \end{bmatrix} = v_1^2 + v_2^2 + v_3^2 + \cdots$

$\Phi = V^T W V$ ↔ $w_1 v_1^2 + w_2 v_2^2 + w_3 v_3^2 + \cdots$

weighted problem

Sep 3-10:24 AM

5-12

$$\Phi = V^T W V = [v_1\ v_2\ v_3]\begin{bmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$= w_1 v_1^2 + w_2 v_2^2 + w_3 v_3^2$$

$\Phi = V^T W V$ subs. expr. for V

$$\Phi = (f - B\Delta)^T W (f - B\Delta)$$

$$f^T W f + \Delta^T B^T W B \Delta - f^T W B \Delta - \Delta^T B^T W f$$

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