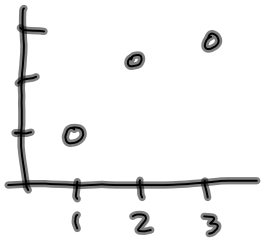

X	Y
1	1
2	2.2
3	3.0

$y = mx + b$ const obs.

$n = 3$ most common 3-1
$n_0 = n$ method for LS
$r = 1$ Indirect Observations

exactly n_0 unknowns (u)
write exactly n cond. eqn. w/each obs as function of unknowns

$$\left.\begin{aligned} \hat{y}_1 &= y_1 + v_1 = mx_1 + b \\ \hat{y}_2 &= y_2 + v_2 = mx_2 + b \\ \hat{y}_3 &= y_3 + v_3 = mx_3 + b \end{aligned}\right\} \text{n condition eqn's.}$$

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$$\begin{aligned} v_1 &= mx_1 + b - y_1 & v_1 &= m\cdot 1 + b - 1.0 \\ v_2 &= mx_2 + b - y_2 & v_2 &= m\cdot 2 + b - 2.2 \\ v_3 &= mx_3 + b - y_3 & v_3 &= m\cdot 3 + b - 3.0 \end{aligned}$$

$\Phi = \sum v_i^2$ 3-2

$= v_1^2 + v_2^2 + v_3^2$

$$\Phi = (m+b-1)^2 + (m\cdot 2 + b - 2.2)^2 + (m\cdot 3 + b - 3.0)^2$$

$$\frac{\partial \Phi}{\partial m} = 2(m+b-1) + 2(m\cdot 2 + b - 2.2)(2) + 2(m\cdot 3 + b - 3.0)(3) = 0$$

$$\frac{\partial \Phi}{\partial b} = 2(m+b-1) + 2(m\cdot 2 + b - 2.2) + 2(m\cdot 3 + b - 3.0) = 0$$

$$m + 4m + 9m + b + 2b + 3b = 1 + 4.4 + 9$$

$$m + 2m + 3m + b + b + b = 1 + 2.2 + 3.0$$

$$\boxed{\begin{aligned} 14m + 6b &= 14.4 \\ 6m + 3b &= 6.2 \end{aligned}}$$ normal equations

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3-3

$$\begin{bmatrix} 14 & 6 \\ 6 & 3 \end{bmatrix}\begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 14.4 \\ 6.2 \end{bmatrix}$$

normal equations
Symmetric
full rank

$Np = t$

$p = N^{-1}t$

manual sol'n 2x2 (exam)

gauss elim, matrix inverse, cramer's rule

$p = \begin{bmatrix} 1.0000 \\ 0.0667 \end{bmatrix}$ slope (m) intercept (b)

```
N = [14 6 ; 6 3] ;
t = [14.4 ; 6.2] ;
p = inv(N) * t
```

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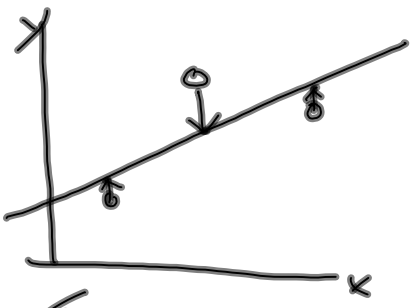
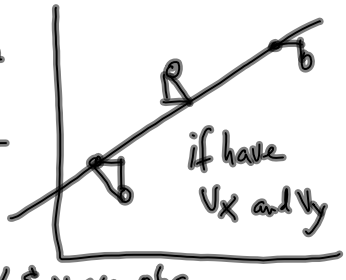
3-4

$v_1 = mx_1 + b - y_1 = 0.0667$
$v_2 = mx_2 + b - y_2 = -1.1333$
$v_3 = mx_3 + b - y_3 = 0.0667$

$\hat{y}_1 = y_1 + v_1 = 1.0667$
$\hat{y}_2 = y_2 + v_2 = 2.0667$
$\hat{y}_3 = y_3 + v_3 = 3.0667$





if both x & y are obs.

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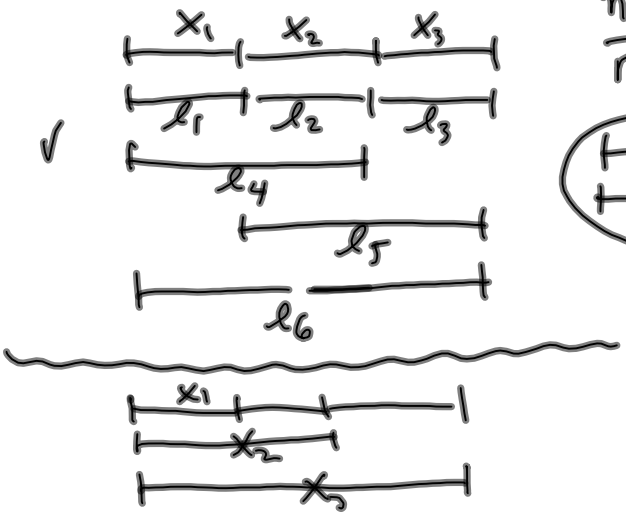
Examples (Indirect Obs.)



$n = 6$

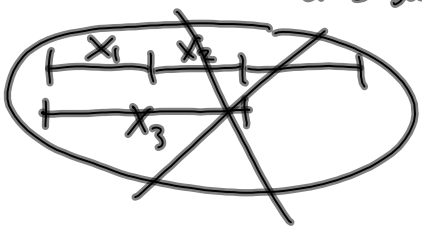
$n_0 = 3$

$r = 3$

If solve sys 3-5

Ind. Obs.

Select 3 unknowns



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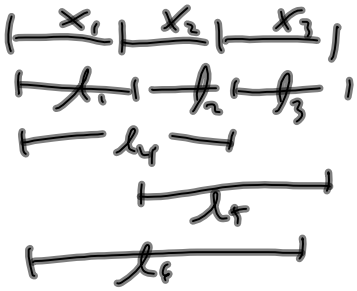

$\hat{l}_1 = x_1$

$\hat{l}_2 = x_2$

$\hat{l}_3 = x_3$

$\hat{l}_4 = x_1 + x_2$

$\hat{l}_5 = x_2 + x_3$

$\hat{l}_6 = x_1 + x_2 + x_3$

$v_1 = x_1 - l_1$

$\vdots$

$\Phi = \Sigma v_i^2$

3-6

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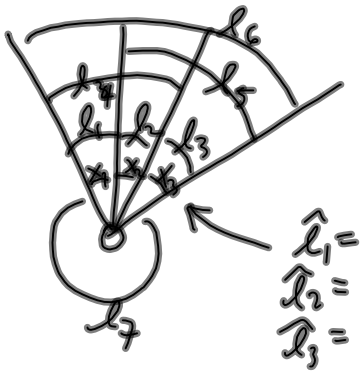

$n = 7$
$n_0 = 3$
$r = 4$

$\hat{\ell}_7 = 360^\circ - \hat{\ell}_6$

Cannot use with indirect obs

$\Rightarrow$ General LS mixed model

1-7

$\hat{\ell}_1 = x_1$
$\hat{\ell}_2 = x_2$
$\hat{\ell}_3 = x_3$
$\hat{\ell}_4 = x_1 + x_2$
$\hat{\ell}_5 = x_2 + x_3$
$\hat{\ell}_6 = x_1 + x_2 + x_3$
$\hat{\ell}_7 = 360 - x_1 - x_2 - x_3$

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Special Cases of LS

3-8

1. Indirect Observations ★
2. Observations only

1 & 2 are special cases of

General LS (mixed model)

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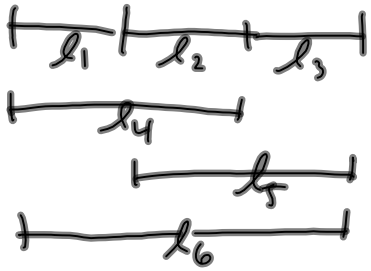
Observations only



$n = 6$
$n_0 = 3$
$r = 3$

need exactly r 3-g condition equations

$\hat{\ell}_1 + \hat{\ell}_2 = \hat{\ell}_4$

$\hat{\ell}_2 + \hat{\ell}_3 = \hat{\ell}_5$

$\hat{\ell}_1 + \hat{\ell}_2 + \hat{\ell}_3 = \hat{\ell}_6$

$\ell_4 + \ell_5 = \ell_1 + 2\ell_2 + \ell_3$

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