Adjustment of Geospatial Observations
EXAM 1
25 October, 2012

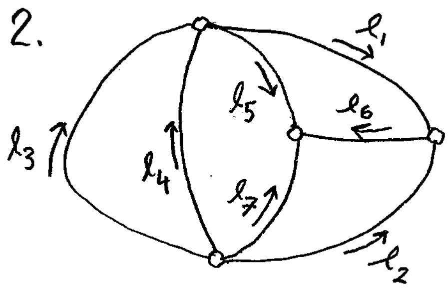
(1 page of notes allowed)

(name)

SHOW YOUR WORK

1. A 2D coordinate transformation includes 1 scale and 2 shifts. x, y are observations and X, Y are constants. Show the condition equations in matrix form as $v + B\Delta = f$. $\boxed{\text{DO NOT SOLVE}}$

#	x	y	X	Y
1	4.6	5.4	1	1
2	4.5	7.2	1	2
3	5.9	5.5	2	1
4	6.0	7.1	2	2

2. 

Show the condition equations for this level network in matrix form as $Av = f$.

("⟶" points up hill)

3. You have made a LS adjustment with $r = 2$, $\sigma_i = 0.5$ for $i = 1, 2, 3$.

$$W = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \text{ and you get residuals : } v = \begin{bmatrix} 0.3 \\ 0.2 \\ -0.6 \end{bmatrix}$$

show the 2-sided global test with level of significance $(\alpha) = 0.10$

4. $\sigma_{x_1} = 2$, $\sigma_{x_2} = 1$, $r_{x_1 x_2} = -0.5$

$$\begin{aligned} y_1 &= 3x_1 + 2x_2 \\ y_2 &= -x_2 \end{aligned} \quad , \quad z = y_1 - y_2$$

what is σ_z ?

useful fact : $r = \dfrac{\sigma_{xy}}{\sigma_x \sigma_y}$

Exam 1 Solution 30 October, 2012

1. $n = 8$, $n_0 = 3$, $r = 5$, $\mu = 3$, $c = 8$

$$\begin{bmatrix} \hat{x}_i \\ \hat{y}_i \end{bmatrix} = \underset{\text{scale}}{k} \begin{bmatrix} X_i \\ Y_i \end{bmatrix} + \underset{\text{shifts}}{\begin{bmatrix} t_x \\ t_y \end{bmatrix}}$$

$$v_{x_i} - kX_i - t_x = -x_i$$
$$v_{y_i} - kY_i - t_y = -y_i$$

$$\Rightarrow \begin{bmatrix} v_{x_1} \\ v_{y_1} \\ v_{x_2} \\ v_{y_2} \\ v_{x_3} \\ v_{y_3} \\ v_{x_4} \\ v_{y_4} \end{bmatrix} + \begin{bmatrix} -X_1 & -1 & 0 \\ -Y_1 & 0 & -1 \\ -X_2 & -1 & 0 \\ -Y_2 & 0 & -1 \\ -X_3 & -1 & 0 \\ -Y_3 & 0 & -1 \\ -X_4 & -1 & 0 \\ -Y_4 & 0 & -1 \end{bmatrix} \begin{bmatrix} k \\ t_x \\ t_y \end{bmatrix} = - \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4 \end{bmatrix}$$

$$v + B \cdot \Delta = f$$

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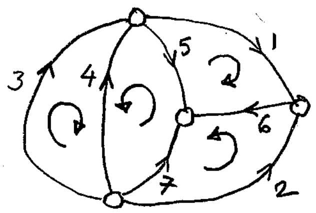
2. $n = 7$, $n_0 = 3$, $r = 4$, $c = r = 4$

$$\hat{\ell}_1 + \hat{\ell}_6 - \hat{\ell}_5 = 0$$
$$\hat{\ell}_2 + \hat{\ell}_6 - \hat{\ell}_7 = 0$$
$$\hat{\ell}_7 - \hat{\ell}_5 - \hat{\ell}_4 = 0$$
$$\hat{\ell}_3 - \hat{\ell}_4 = 0$$



$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix} = -A \cdot \begin{bmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \\ \ell_4 \\ \ell_5 \\ \ell_6 \\ \ell_7 \end{bmatrix}$$

$$A \cdot v = f$$

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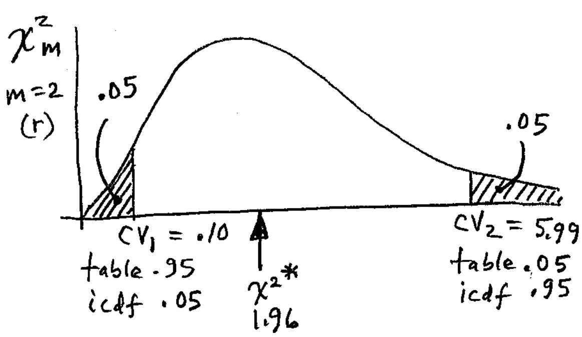
3. $\omega_i = \dfrac{\sigma_0^2}{\sigma_i^2}$, $4 = \dfrac{\sigma_0^2}{(0.5)^2}$, $\sigma_0^2 = 4 \times 0.25 = 1$

$$\chi^{2*} = \frac{v^TWv}{\sigma_0^2} = \begin{bmatrix} .3 & .2 & -.6 \end{bmatrix} \begin{bmatrix} 4 & & \\ & 4 & \\ & & 4 \end{bmatrix} \begin{bmatrix} .3 \\ .2 \\ -.6 \end{bmatrix} \div 1 = 1.96$$

$$\alpha_{\text{global test}} = \alpha_g = 0.10$$



$\Rightarrow$ Accept H_0
$\sigma^2 = \sigma_0^2$

possible test statistics

$$\frac{v^TWv}{\sigma_0^2} \sim \chi^2_r$$

$$\frac{r \cdot \hat{\sigma}_0^2}{\sigma_0^2} \sim \chi^2_r$$

$$v^T\Sigma^{-1}v \sim \chi^2_r$$

$$\frac{\hat{\sigma}_0^2}{\sigma_0^2} \sim \frac{\chi^2_r}{r} = F_{r,\infty}$$

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4. $-0.5 = \dfrac{\sigma_{x_1x_2}}{\sigma_{x_1}\sigma_{x_2}}$, $\sigma_{x_1x_2} = 2 \cdot 1 \cdot (-0.5) = -1 \Rightarrow \Sigma_{xx} = \begin{bmatrix} 4 & -1 \\ -1 & 1 \end{bmatrix}$

$\boxed{\text{2-step}}$

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \Sigma_{YY} = \begin{bmatrix} 3 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 28 & 1 \\ 1 & 1 \end{bmatrix}$$

$$Z = Y_1 - Y_2 = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}, \quad \sigma_Z^2 = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 28 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 27$$

$$\sigma_Z = \sqrt{27}$$

$\boxed{\text{1-step}}$ $Z = Y_1 - Y_2$

$Y_1 = 3X_1 + 2X_2$
$Y_2 = -X_2$

$$Z = 3X_1 + 2X_2 - (-X_2) = 3X_1 + 3X_2$$

$$Z = \begin{bmatrix} 3 & 3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$\sigma_Z^2 = \begin{bmatrix} 3 & 3 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \end{bmatrix} = 27$$

$$\sigma_Z = \sqrt{27}$$