23 Oct 08

CE 597 (029) Adj. of Geospa. Obs.

**Exam 1**

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Name

( 75 minutes — 1 page of notes allowed )

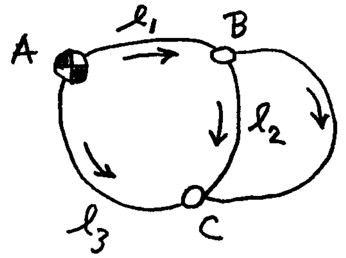
1. Is the following 3-parameter transformation <u>linear</u> or <u>nonlinear</u> ?

$$x = \cos\theta \cdot X + \sin\theta \cdot Y + t_x$$
$$y = -\sin\theta \cdot X + \cos\theta \cdot Y + t_y$$

$x, y$ are observations, $X, Y$ are constant, $\theta, t_x, t_y$ are the parameters we estimate in the adjustment.

2. $$\mu = 2x + y \qquad \Sigma_{\binom{x}{y}} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$
$$v = x - y$$

what is the standard deviation $\sigma_\mu$ ? what is the covariance $\sigma_{\mu v}$ ?

3. 


In the given level network, point A is fixed at elevation = 0.0. Show $n, n_0, r$ and show the condition equations in the form $Av = f$.

| | |
|---|---|
| $l_1$ | 9.9 |
| $l_2$ | 10.3 |
| $l_3$ | 21.1 |
| $l_4$ | 10.1 |

4. Show condition equations for problem 3 in the form $V + B\Delta = f$.

5. A weight matrix is given as $W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.
If $\sigma_0 = 2$ what is $\sigma$ for observation number 3 ?

6. A condition equation for the indirect observation method is given as
$$F = \hat{l} - [4x^2 + 3y] = 0$$
where $l$ is an observation and $x, y$ are parameters.
$l = 12$, and parameter approximations are $x \approx 1$, $y \approx 2$.
Show the linearized condition equation in the form $V + B\Delta = f$.
(Show numerical values.)

1. $x = \cos\theta \cdot X + \sin\theta \cdot Y + t_x$
$y = -\sin\theta \cdot X + \cos\theta \cdot Y + t_y$

<u>nonlinear</u>: parameter $\theta$ argument of trig function (more than <u>scalar</u> multiple)

2. $u = 2x + y$
$v = x - y$

$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}, \quad \Sigma_{\binom{x}{y}} = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$

$$\Sigma_{\binom{u}{v}} = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 7 & 5 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} = \begin{bmatrix} 19 & 2 \\ 2 & 4 \end{bmatrix}$$

$\left\{ \text{application of error propagation: } Y = AX, \ \Sigma_{XX} \quad \Sigma_{YY} = A\,\Sigma_{XX}A^T \right\}$

$\sigma_u = \sqrt{19} = 4.36, \quad \sigma_{uv} = 2$

3. $n = 4$
$n_0 = 2$
$r = 2$

<u>obs. only</u>

$\hat{\ell}_1 + \hat{\ell}_2 - \hat{\ell}_3 = 0 \qquad \ell_1 + v_1 + \ell_2 + v_2 - \ell_3 - v_3 = 0$

$\hat{\ell}_2 - \hat{\ell}_4 = 0 \qquad \ell_2 + v_2 - \ell_4 - v_4 = 0$

$v_1 + v_2 - v_3 = -(\ell_1 + \ell_2 - \ell_3) = -(9.9 + 10.3 - 21.1) = +0.9$

$v_2 - v_4 = -(\ell_2 - \ell_4) = -(10.3 - 10.1) = -0.2$

$$\underset{A}{\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}} \underset{v}{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}} = -\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} 9.9 \\ 10.3 \\ 21.1 \\ 10.1 \end{bmatrix} = \begin{bmatrix} 0.9 \\ -0.2 \end{bmatrix} = f$$

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4. <u>indirect obs.</u> parameters $u = n_0 = 2$ B, C elevations of 2 unknown points

$\ell_1 + v_1 = B \qquad v_1 - B = -\ell_1 = -9.9$

$\ell_2 + v_2 = C - B \qquad v_2 - C + B = -\ell_2 = -10.3$

$\ell_3 + v_3 = C \qquad v_3 - C = -\ell_3 = -21.1$

$\ell_4 + v_4 = C - B \qquad v_4 - C + B = -\ell_4 = -10.1$

$$\underset{v}{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}} + \underset{B}{\begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & -1 \\ 1 & -1 \end{bmatrix}} \underset{\Delta}{\begin{bmatrix} B \\ C \end{bmatrix}} = \underset{f}{\begin{bmatrix} -9.9 \\ -10.3 \\ -21.1 \\ -10.1 \end{bmatrix}}$$

5. $W = \begin{bmatrix} 1 & & \\ & 2 & \\ & & 4 \end{bmatrix}$, $\sigma_0 = 2$, $\sigma_0^2 = 4$, $W = \begin{bmatrix} 4/4 & & \\ & 4/2 & \\ & & 4/1 \end{bmatrix}$

$\sigma_3^2 = 1$, $\sigma_3 = \sqrt{1} = 1$

6. $F = \hat{\ell} - [4x^2 + 3y] = 0$

$\ell = 12$
$x \approx 1$, $x^\circ$
$y \approx 2$, $y^\circ$

F is a non linear equation

$B = \begin{bmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \end{bmatrix}$
$f = (-F)$
$\Big\}$ all evaluated at current approximations

$\partial F/\partial x = -8x$, $\partial F/\partial y = -3$

$F^\circ = 12 - [4 \cdot 1^2 + 3 \cdot 2]$

$F^\circ = 12 - [4 + 6] = 2$

$$v + \begin{bmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = -F^\circ$$

$-8x \quad -3 \qquad -(2)$
$(1)$
$-8 \quad -3$

the unknowns that we solve for in the non linear iterations are $\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$

$$\boxed{v + \begin{bmatrix} -8 & -3 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = -2}$$

with 1 equation and 2 unknown parameters you cannot <u>solve</u> a LS problem – only write the linearized condition equation, with numbers <u>where possible</u>.