Error Propagation for GLS

Stated in Ch. 9

$$\Delta = \boxed{}\,\ell + c_1$$

$$v = \boxed{}\,\ell + c_2$$

$$\hat{\ell} = \boxed{}\,\ell + c_3$$

$$v = \boxed{}\,K + c_4$$

$$K = \boxed{}\,\ell + c_5$$

$$Q_{\Delta\Delta} = N^{-1}$$

$$N = B^T W_e B, \quad W_e = Q_e^{-1}, \quad Q_e = AQA^T$$

$$Q_{vv} = Q A^T W_e (I - B N^{-1} B^T W_e) A Q$$

$$Q_{\hat{\ell}\hat{\ell}} = Q - Q A^T W_e (I - B N^{-1} B^T W_e) A Q$$

$$Q_{\hat{\ell}\hat{\ell}} = Q - Q_{vv}$$

$$\boxed{Q = Q_{vv} + Q_{\hat{\ell}\hat{\ell}}}$$

Initial Parameter Approx (HW #5)

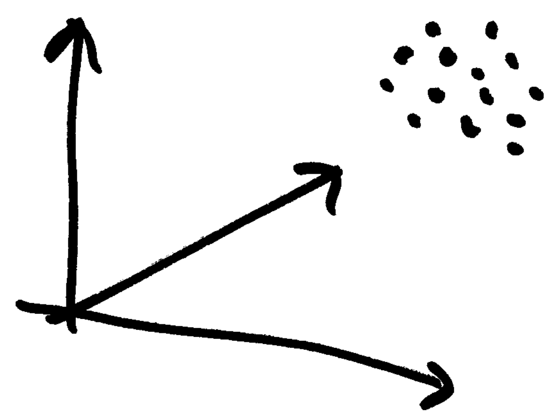
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} + \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

for initial approx : <u>assume</u> $\begin{pmatrix} X \\ Y \end{pmatrix}$ are constant

$$x - a_0 - a_1 X - a_2 Y = 0$$
$$y - b_0 - b_1 X - b_2 Y = 0$$

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} + \begin{pmatrix} -1 & -X & -Y & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -X & -Y \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix}$$

#2



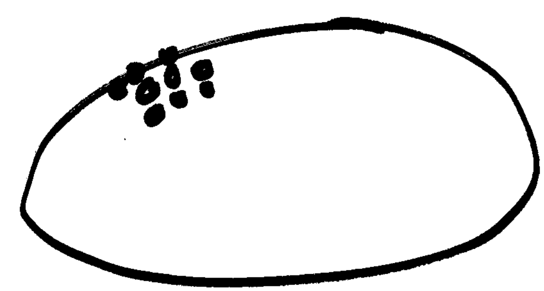
$$\frac{(x-x_0)^2 + (y-y_0)^2}{a^2} + \frac{(z-z_0)^2}{b^2} = 1$$

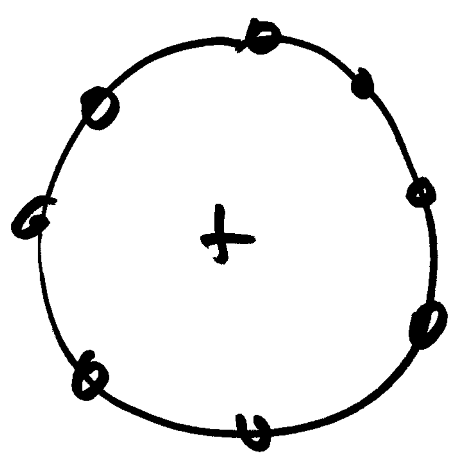
assume points are well-distributed Then use centroid

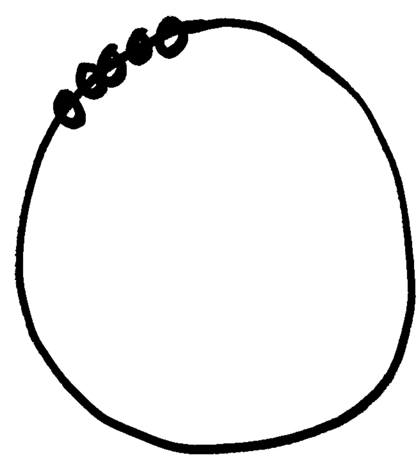
$$X_0 \approx \frac{\sum x_i}{n}$$

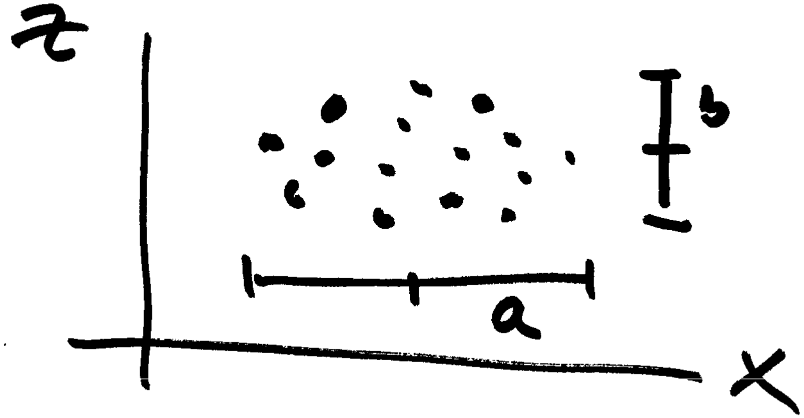
$$y_0 \approx \frac{\sum y_i}{n}$$

$$z_0 \approx \frac{\sum z_i}{n}$$







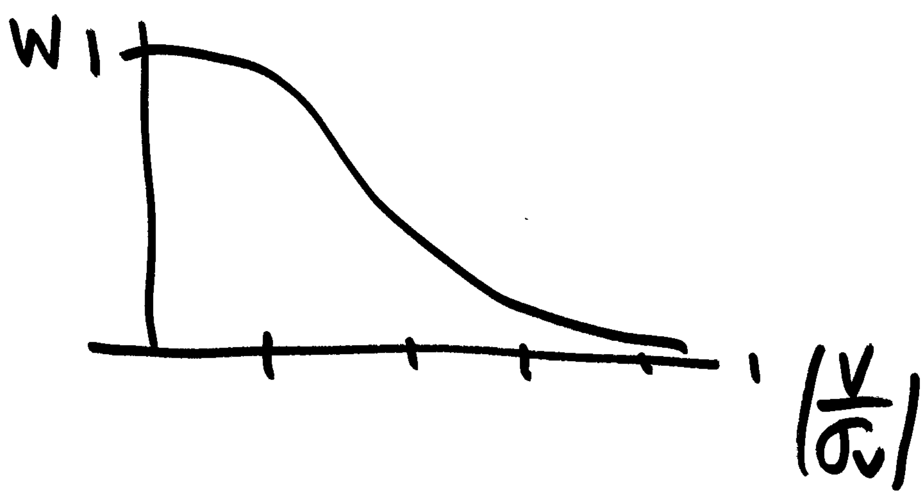


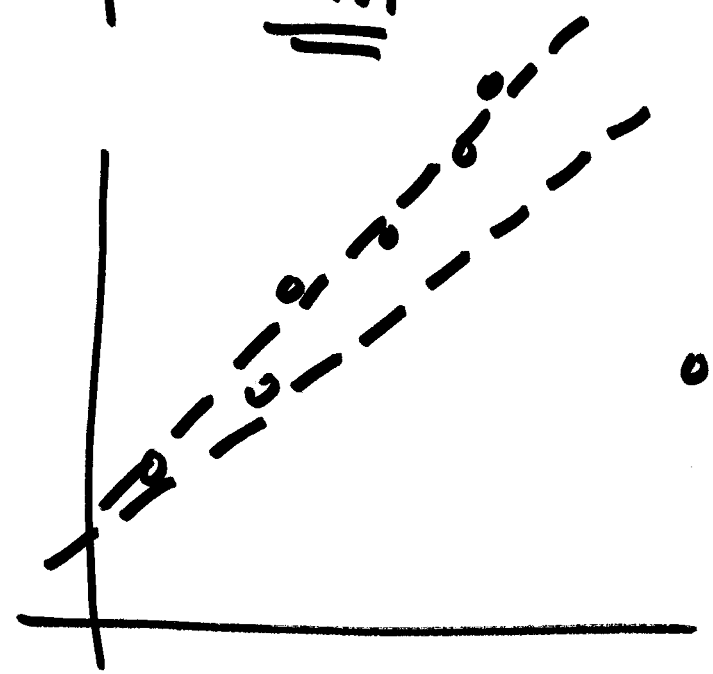
Blunder Detection, Outlier Detection, Gross Error Detection, Robust Estimation

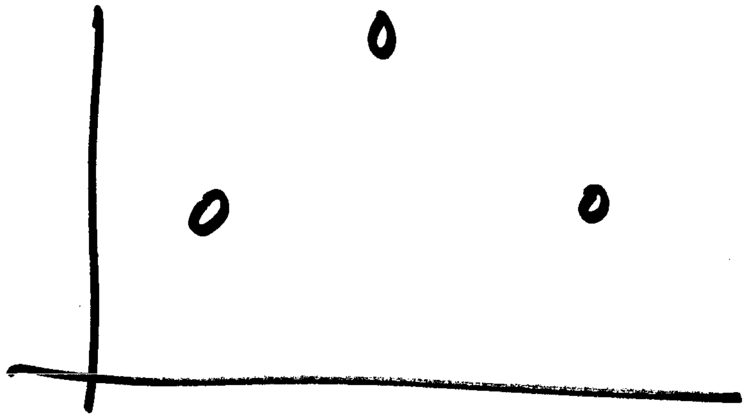
- IRLS
- L1-norm minimization
- Data Snooping

IRLS
- iterate to convergence
- modify weights by magn of residual
- std. residual $\left|\frac{v}{\sigma_v}\right|$ $\left|\frac{v_i}{\sigma_{v_i}}\right|$

$$Q_{vv} \rightarrow \Sigma_{vv} = \begin{bmatrix} \sigma_{v_1}^2 & & & \\ & \sigma_{v_2}^2 & & \\ & & \ddots & \\ & & & \sigma_{v_n}^2 \end{bmatrix}$$







advantages of IRWLS

- easy to implement
- works well if high redundancy

L1-norm minimization

LS obj. funct. $\phi_2 = V_1^2 + V_2^2 + \cdots + V_n^2$

L2-norm

$d^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$

$\phi_1 = |V_1| + |V_2| + \cdots + |V_n|$

L1-norm

1 parameter case:

L2 : sample mean

L1 : sample median

20, 21, 21, 19, 20

mean = L2 or LS estimate

20.2

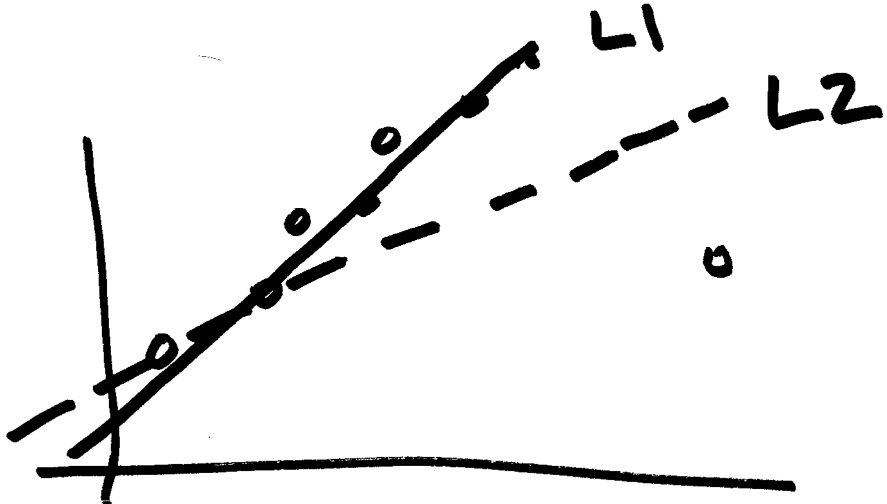
what if blunder ?

20, 21, 100, 19, 20

mean, L2, LS estimate, samplemean

36 parameter corrupted by blunder

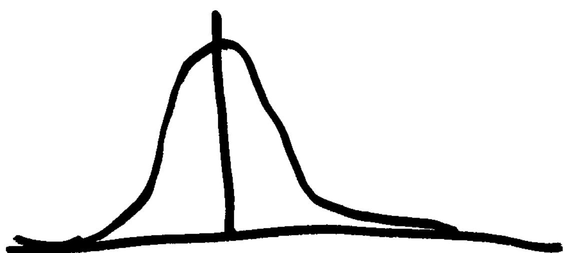
19
20
20 ←   median = 20
21
100



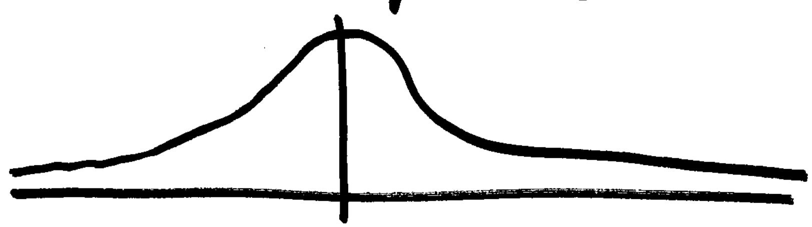
good strategy:

use L1 for QA, preliminary solution
data editing

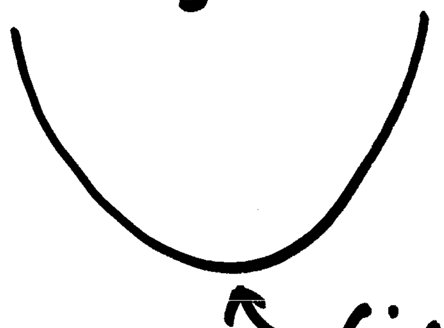
use L2 for final solution, estimate
error propagation



normal distr.

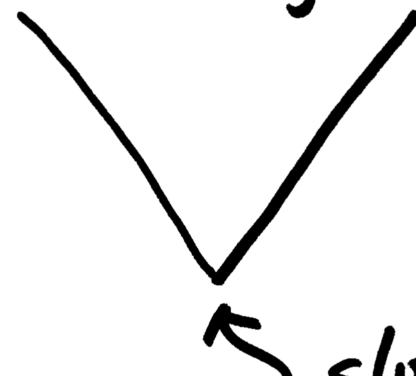


LS obj. function



find minimum by calculus

L1 obj. function



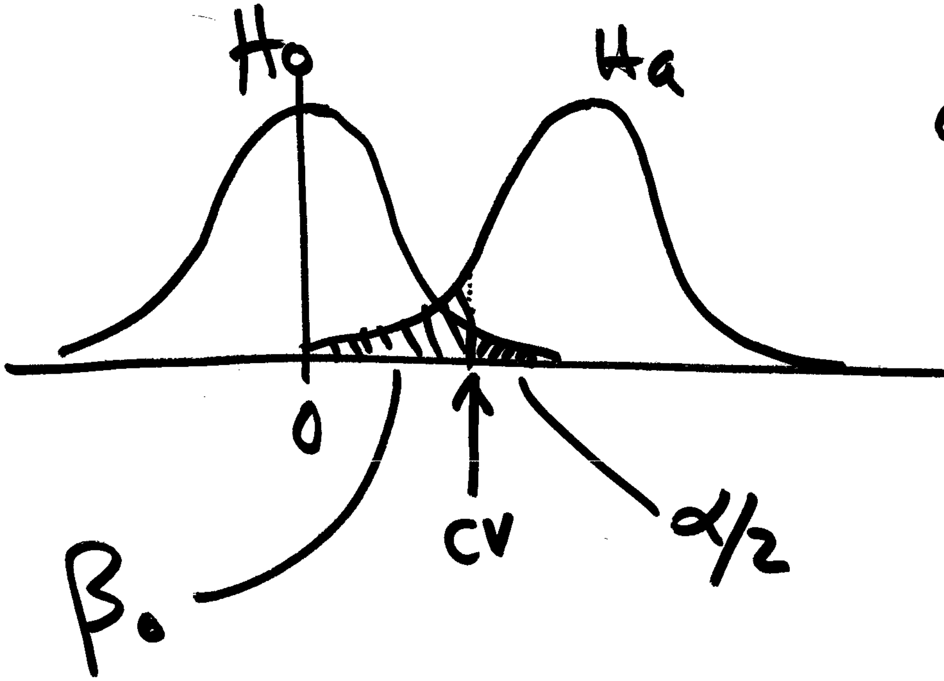
slope discontinuity @ minimum

L1 estimation - cannot use calculus
must search
L1 reformulated ~~to~~ into LP linear program.
intelligent search : simplex method
SLOW + more complicated

Data Snooping (Baarda)

$$\left|\frac{v_i}{\sigma_{v_i}}\right| \sim N(0,1)$$

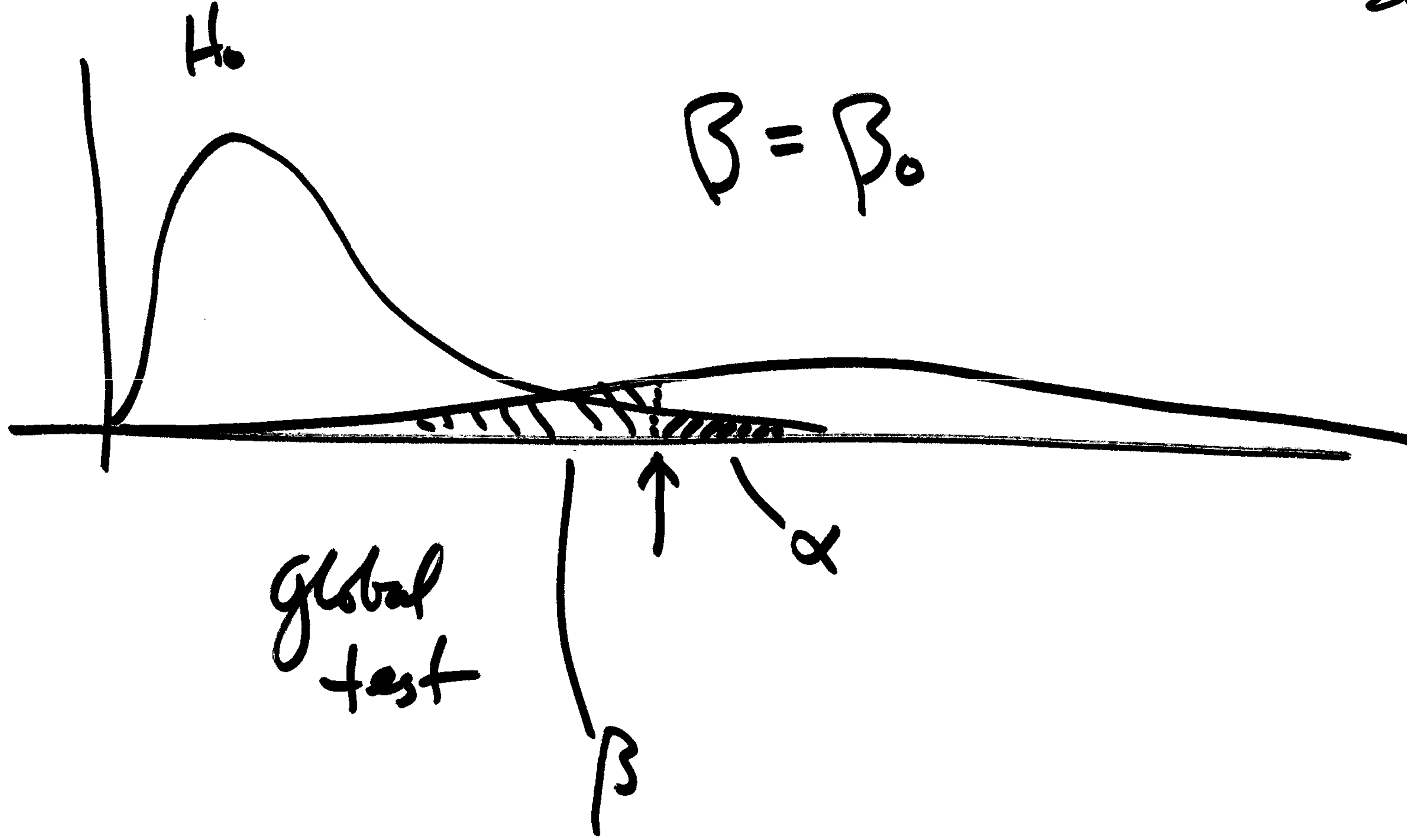
if $\left|\frac{v_i}{\sigma_{v_i}}\right| > 3$ reject observation



α_0: level of significance test
: prob. of Type 1 error

~~Type 2 prob~~ α_0: prob. of Type 1 error
reject Ho when true

β_0: probability of Type 2 error
~~accept the~~ Accept Ho when false


H_0
$\beta = \beta_0$
β
α
global test

indirect observation

$$Q = W^{-1}$$

$$Q_{vv} = Q - Q_{\hat{\ell}\hat{\ell}}$$

$$\underset{n,n}{Q_{vv} W = \overline{W}}$$

$\bar{w}_{ii} = r_i$ redundancy number

$$\sum_{i=1}^{n} r_i = r$$

$$u_i = 1 - r_i$$

$$0 \le r_i \le 1$$

$$0 \le u_i \le 1$$

r_i : the fraction of the error in l_i
that is "revealed" in the residual v_i

u_i : the fraction of the error in l_i
that is "absorbed" by the parameter
estimation

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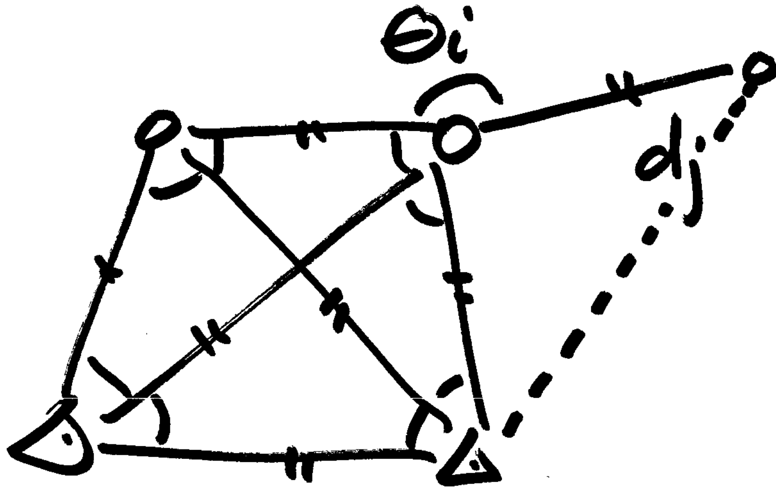
when r_i large + u_i small
then observation error is well revealed in v_i
when u_i large + r_i small
the observation error is hidden in par. estimate



$r_i = 0$

$r_j = 0$