L1 : 154×10.23 MHz = 1575.42 MHz

L2 : 120×10.23 MHz : 1227.60 MHz

Basic Clock 10.23 Mhz (L-Band)

C/A : coarse acquisition

1023 binary values

period 1 ms

50 Hz

L1, L2 Carrier

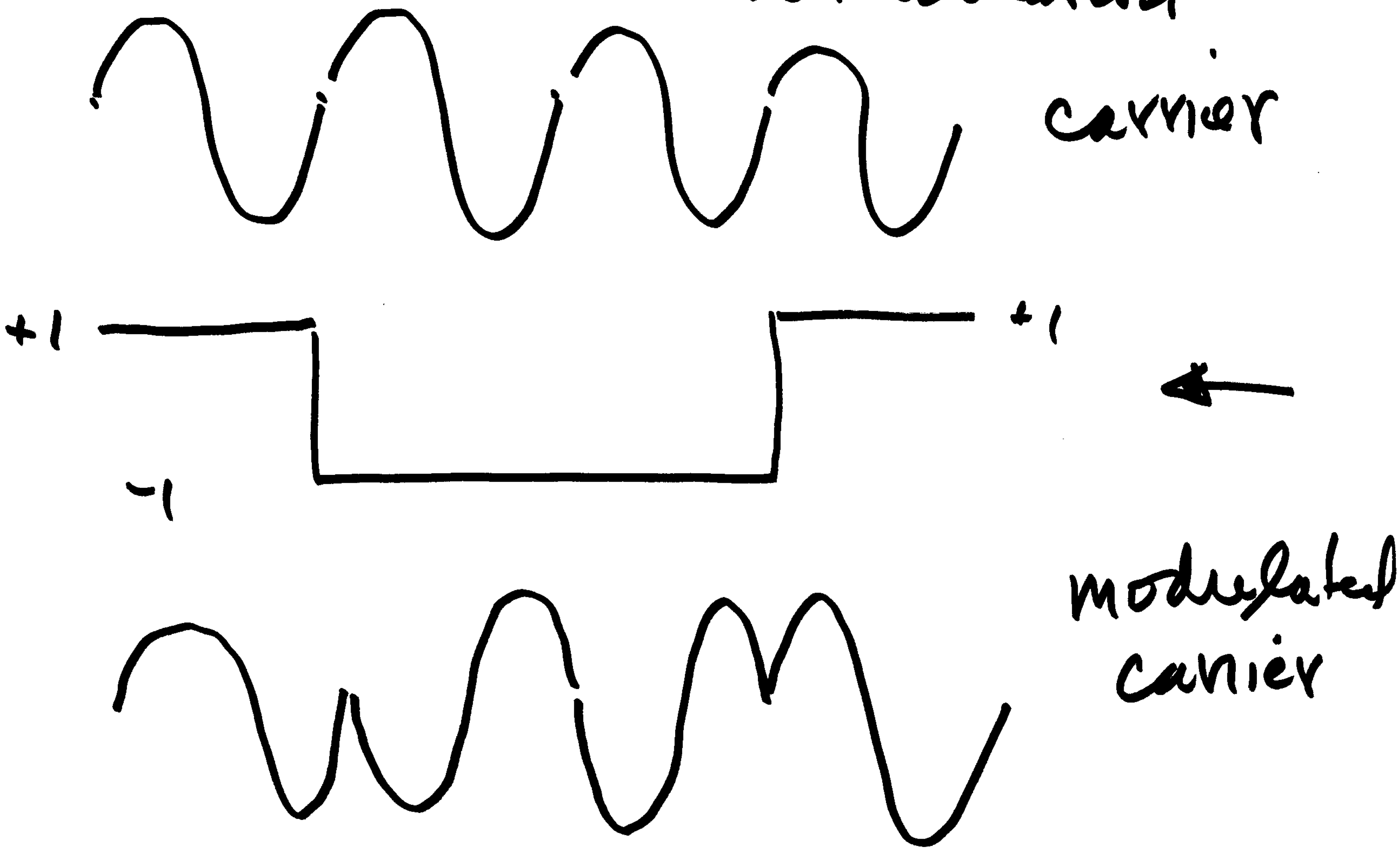
AM

FM

Phase Modulation

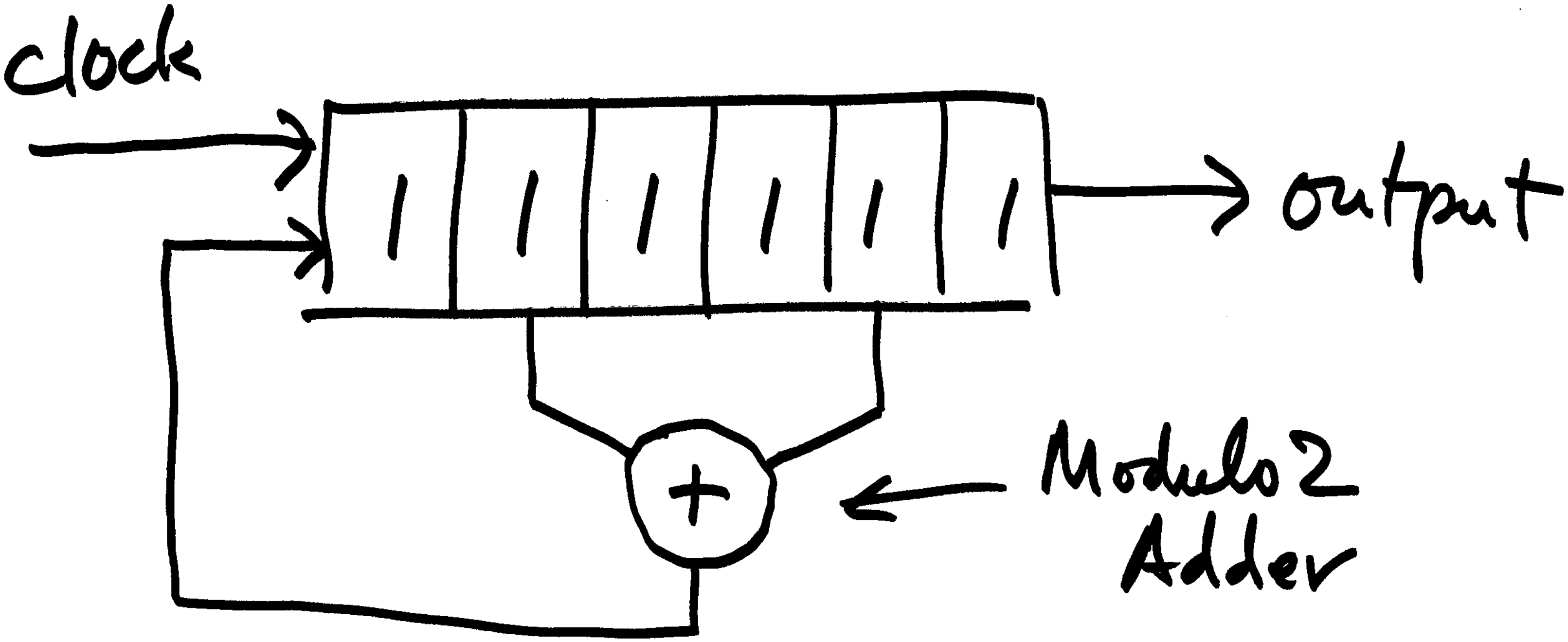
AM: amplitude modulation
FM: frequency modulation
PM: phase modulation (GPS uses this)

Phase Modulation



Bi-Phase Shift Keying
= BPSK

C/A codes: Linear Feedback Shift Register
LFSR



$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 0$

$1 + 1 + 1 = 1$

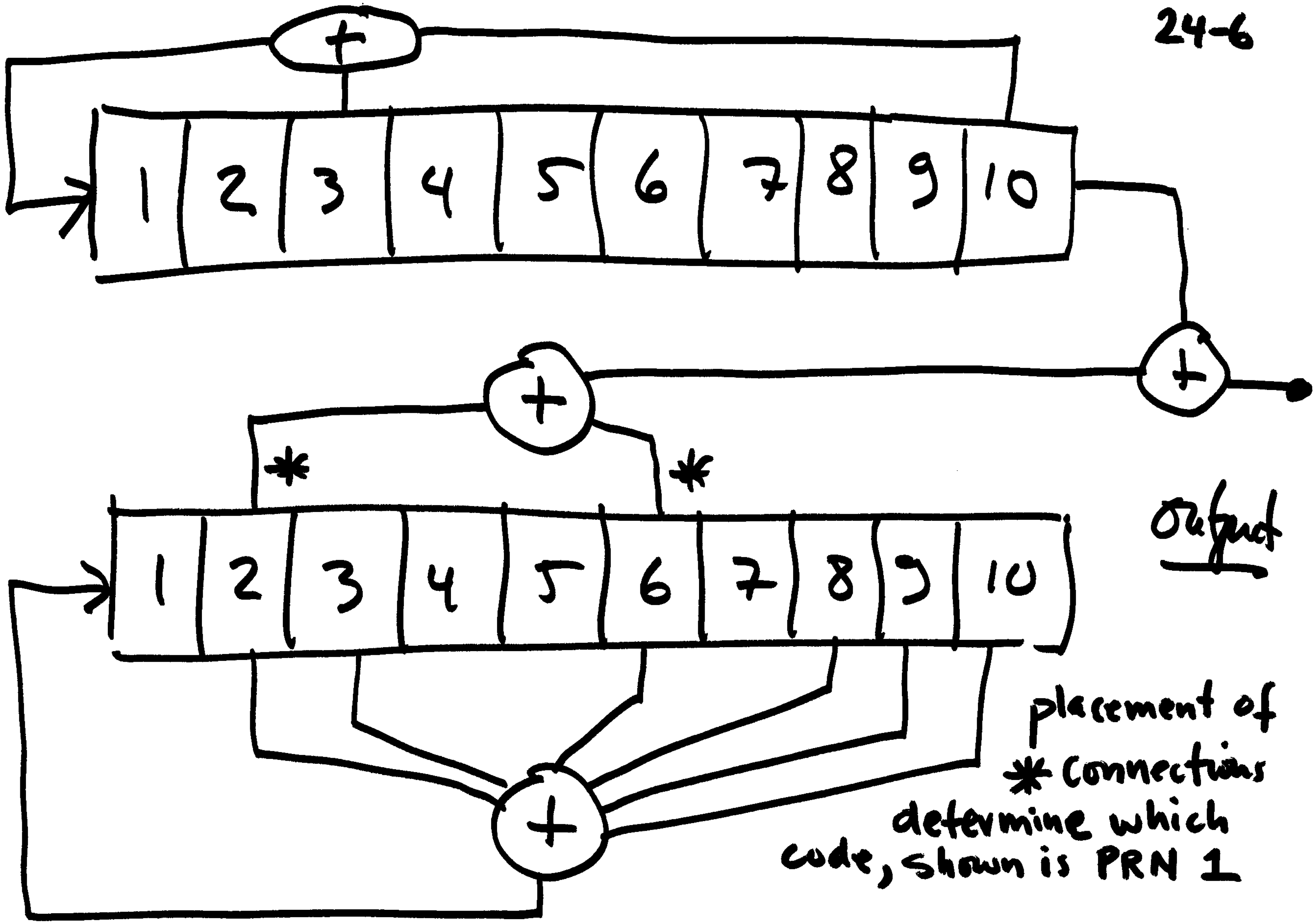
$1 + 1 + 1 + 1 = 0$

examples of modulo-2 addition

output of LFSR : sequence of 1's + 0's

appears Random, but it is not

called PRN code = pseudo random number code



Gold Codes, repeat after 1023

correlation coefficient:

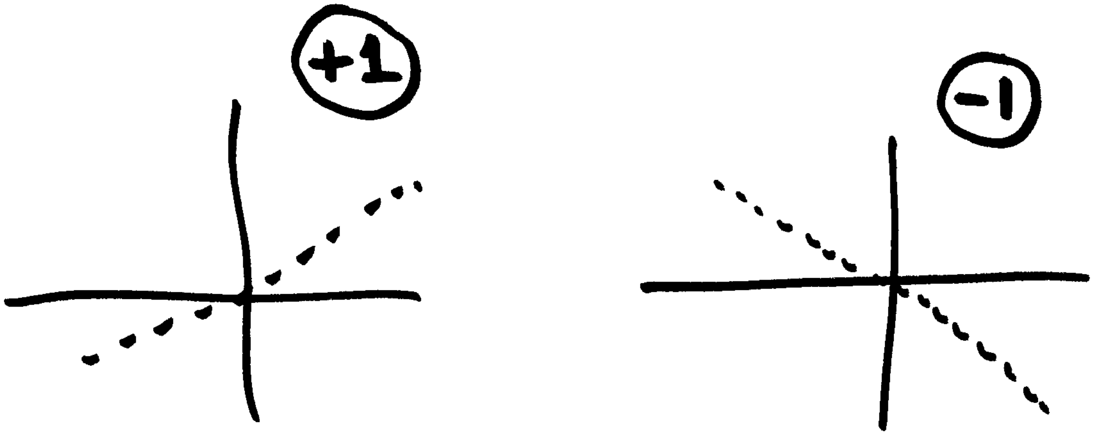
$$\Sigma = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix}$$

from covariance matrix

$$r_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

+1 2 RV's functionally dependent

−1 " "



Sample covariance: (from sample data)

$$S_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{N-1}$$

Sample correlation coefficient:

$$r_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{N-1 \cdot S_x \cdot S_y}$$

$$S_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N-1}}, \quad S_y = \sqrt{\frac{\sum (y_i - \bar{y})^2}{N-1}}$$

$$r_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\left[\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2 \right]^{1/2}}$$

applications: image matching
feature detection
target location
Radar match filtering
GPS acquisition

(many applications)

Time or Space domain approach (Slow)

Frequency Domain : <u>Correlation</u> (fast)

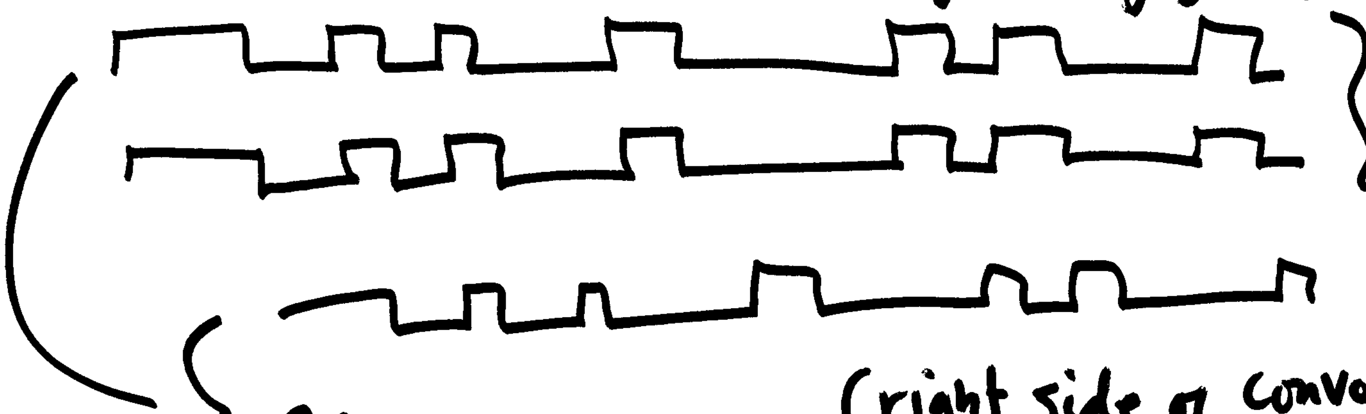
<u>Convolution</u> (")

g, h: time or space domain, G, H fourier transforms

<u>Convolution</u> $g * h \longleftrightarrow G(f)\,H(f)$

<u>Correlation</u> $\mathrm{Corr}(g,h) \longleftrightarrow G(f)\,H^{*}(f)$

(* here means complex conjugate)



CC = 1

CC = 0

(right side of convolution and correlation expressions are element-wise multiplication)

24-11

Cross-corr i,k @ lag $= m$

$$r_{ik}(m) = \sum_{l=0}^{1022} c^i(l)\, c^k(l+m) \approx 0$$

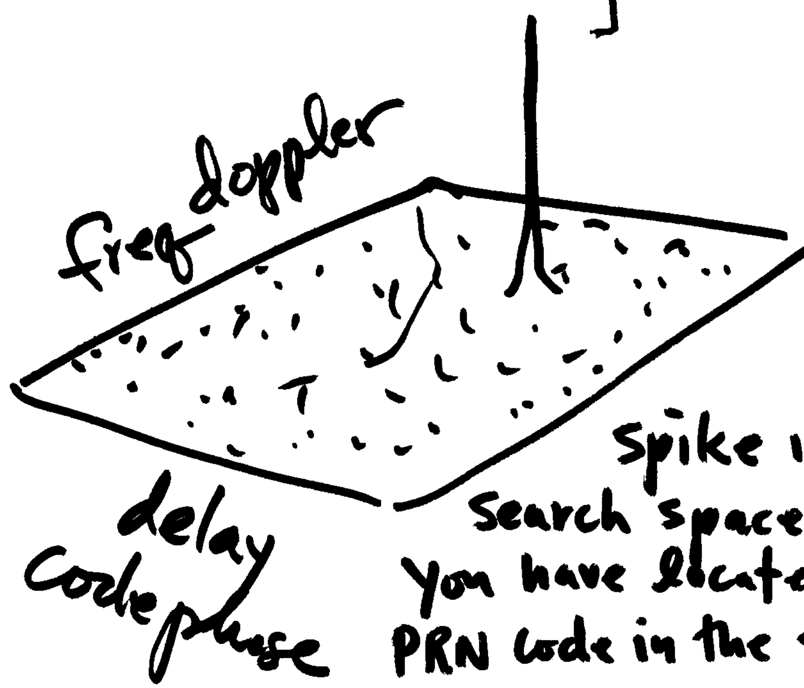
auto-corr @ lag m

$$r_{kk}(m) = \sum_{l=0}^{1022} c^k(l)\, c^k(l+m) \approx 0$$

fast space or time domain correlation for +1, -1 data

GPS acquisition

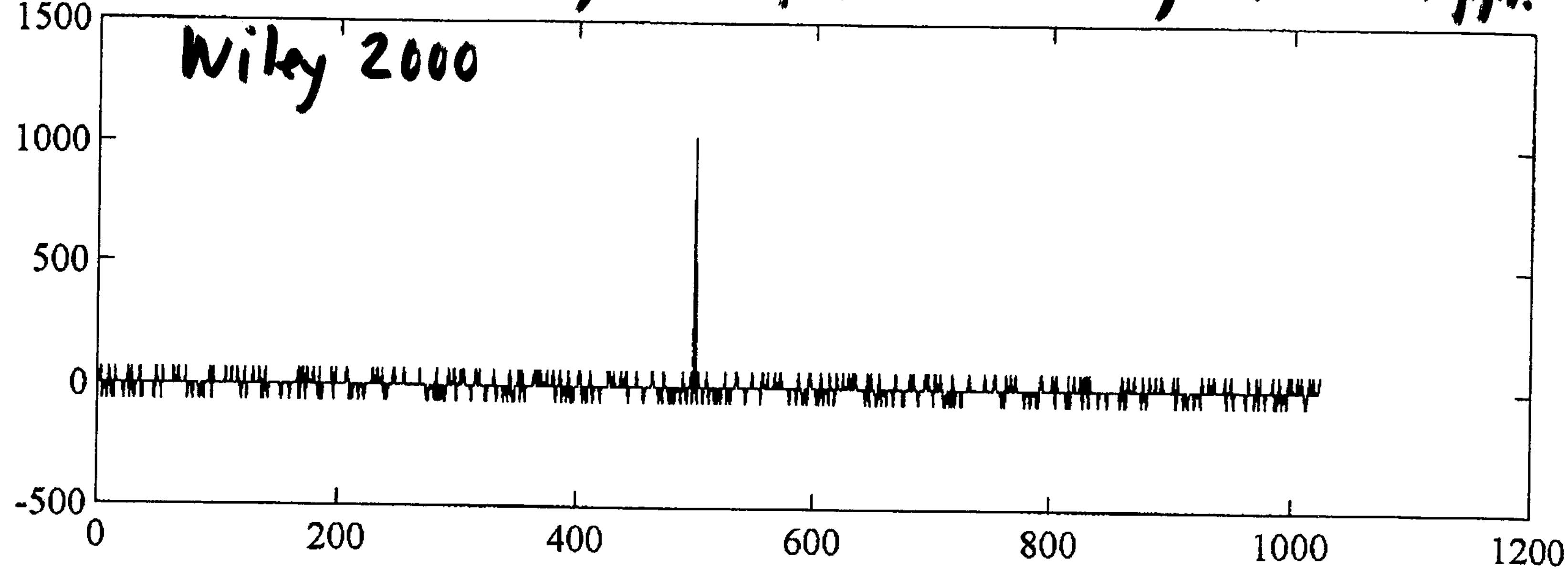
search in delay + freq. (doppler)



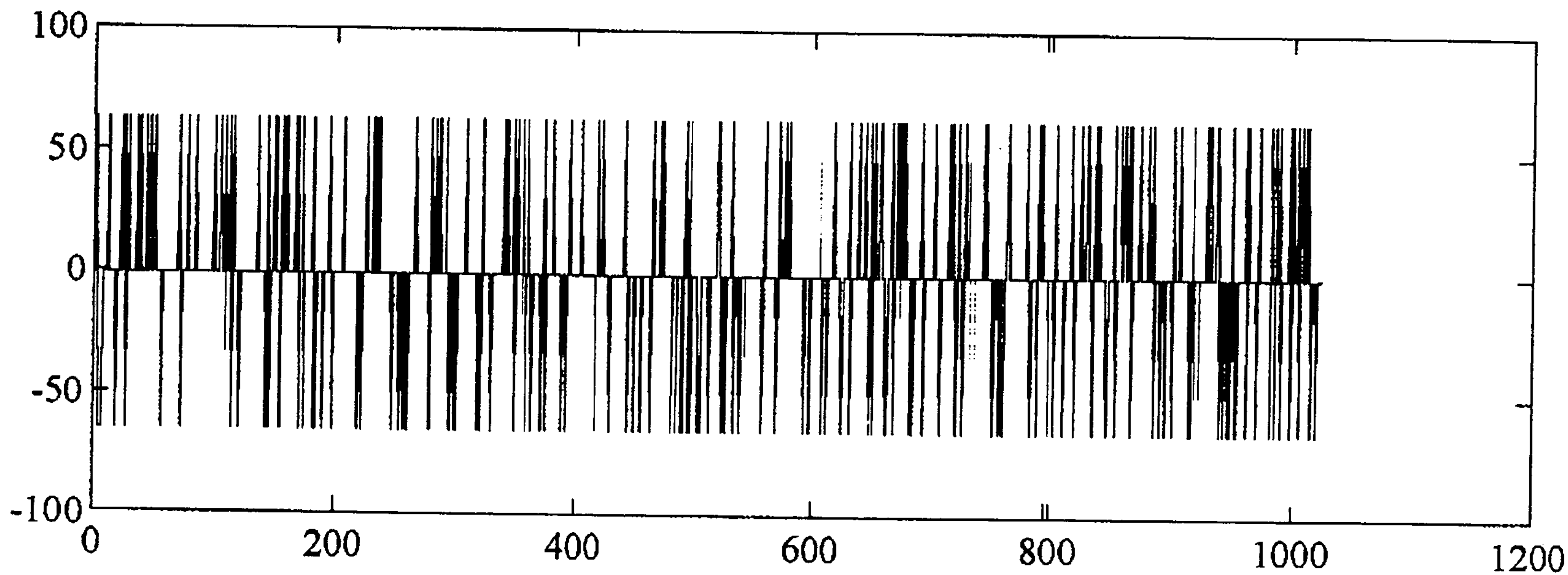
Spike in the search space means you have located that PRN code in the signal

24-12

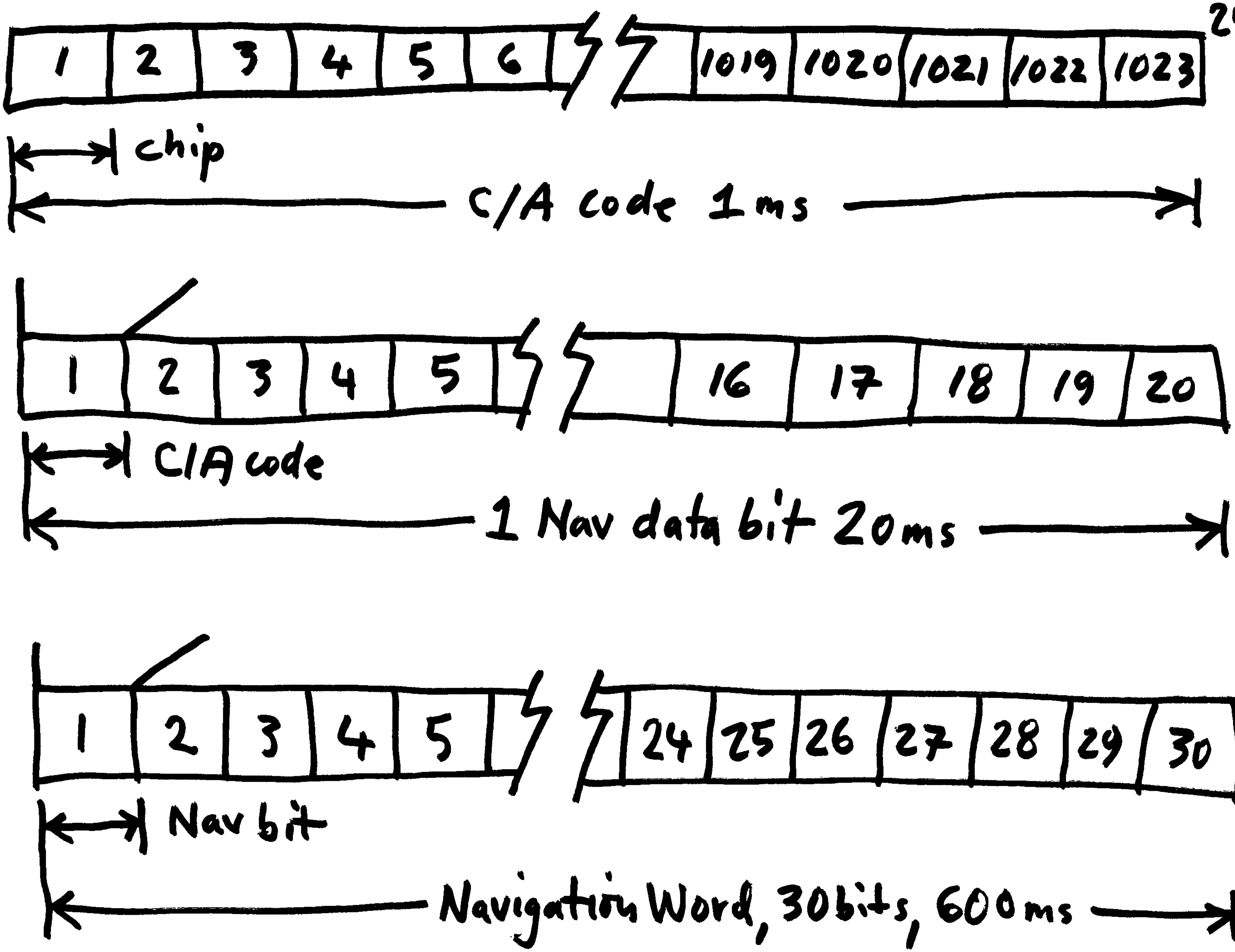
Taken from Tsui, Fund. of Glob. Pos. Rec., Software Appr. Wiley 2000



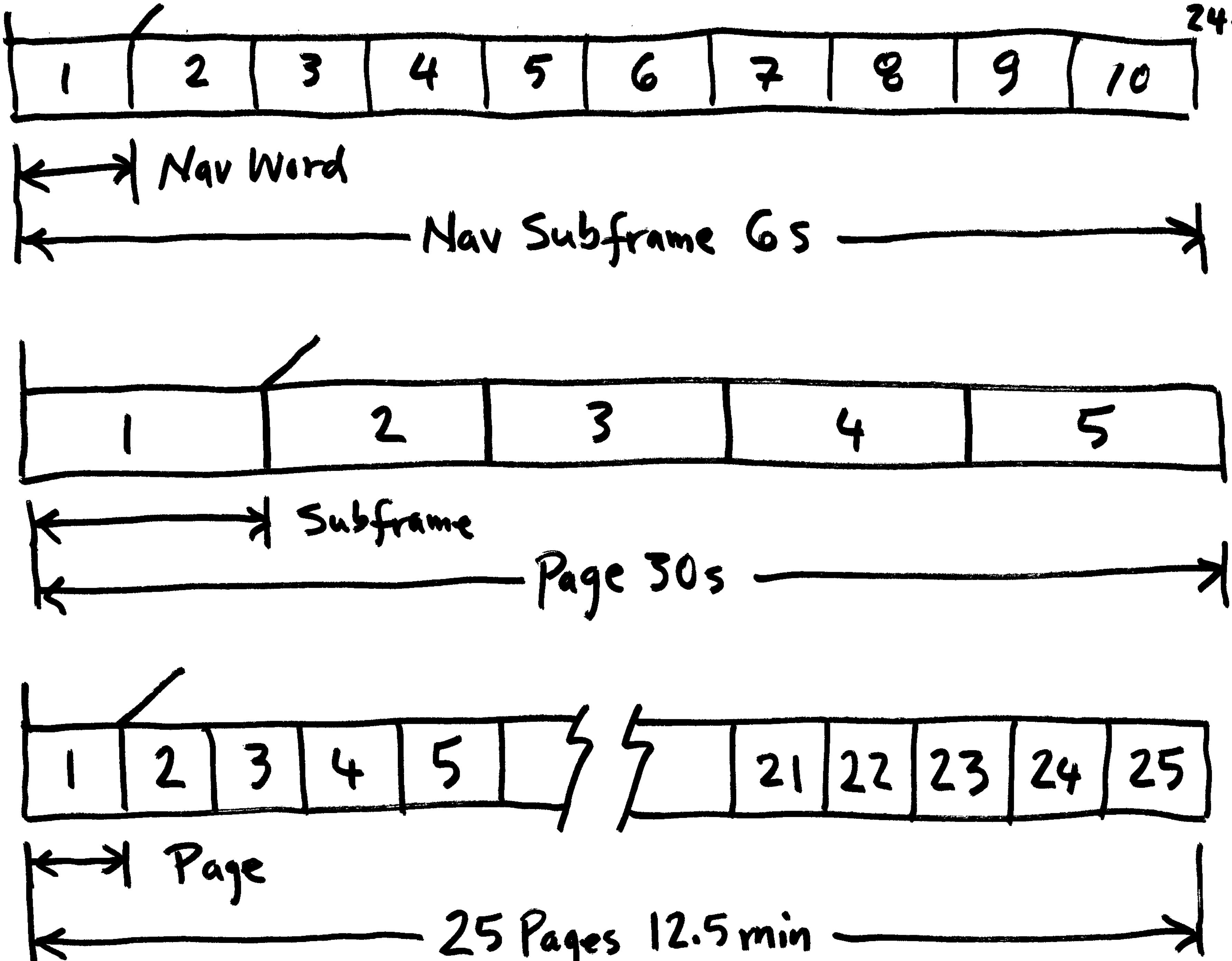
(a) Autocorrelation of satellite 19.



(b) Cross correlation of satellites 19 and 31.



— FORMAT OF NAV DATA —



- FORMAT OF NAV DATA -

tracking - refine + track
delay + doppler

extract Nav Message

TLM - unique for searching

HOW - TOW - time of transmission of next subframe

TOW = "time of week", resets each week
critical number to construct pseudorange observable

Sat. position

a : semi major axis

e : eccentricity

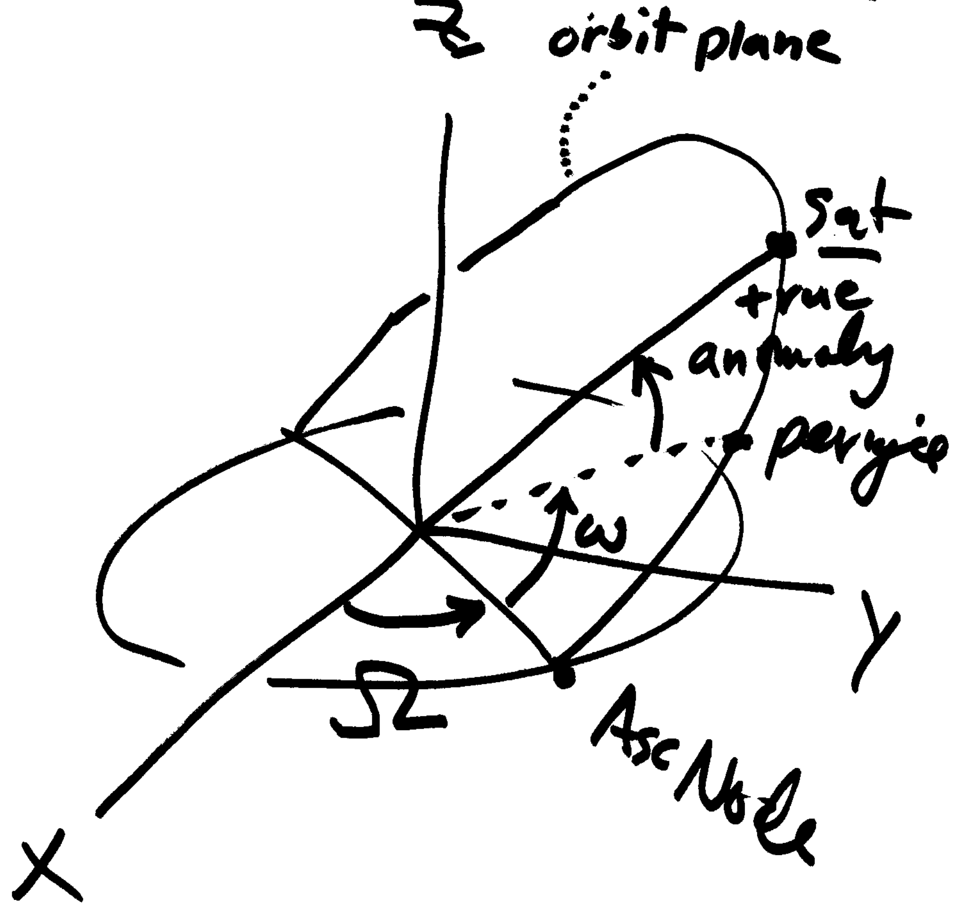
ω : arg of perigee

Ω : right asc. of asc. node

i : inclination

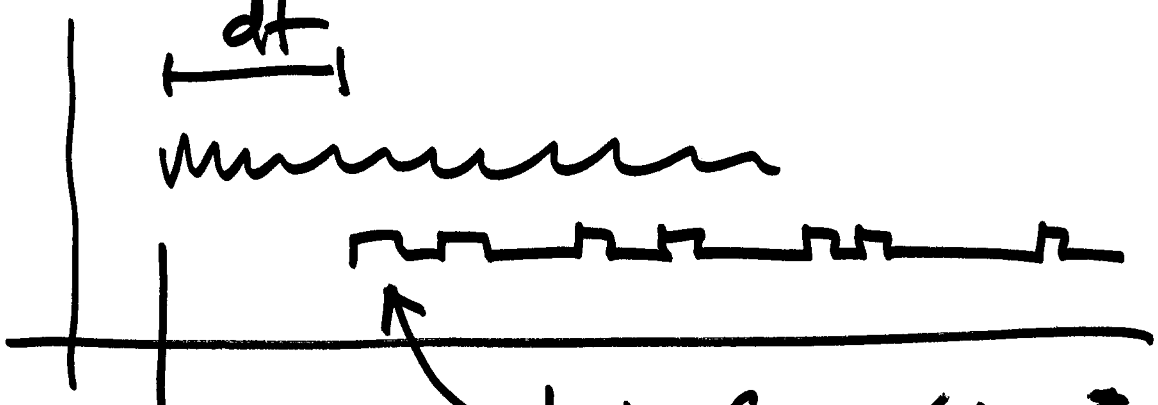
ν ~~M~~ : true anomaly

mean
true
eccentric



XYZ
earth centered fixed
ECEF

RINEX receiver independent exchange format



time ref. of reception

start of frame (TOW decoded prior frame!)

travel time : $\left(\text{time of reception} + dt\right) - \underset{\text{Transmit Time}}{\text{(TOW)}}$

$\parallel$

$\Delta t \longrightarrow$, $pr = c \cdot \Delta t$ **pseudorange**

raw pr must be corrected for known satellite clock error

3D ranging model

X, Y, Z + additional term
clock bias (of receiver)

require min. of 4 satellites
for unique solution

$$\Sigma_{\begin{pmatrix} x \\ y \\ z \\ b \end{pmatrix}} = \begin{bmatrix} \sigma_x^2 & & & \\ & \sigma_y^2 & & \\ & & \sigma_z^2 & \\ & & & \sigma_b^2 \end{bmatrix}$$

$$GDOP = \frac{1}{\sigma}\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 + \sigma_b^2}$$

σ : std dev. of PR's

~~HDOP~~

$$PDOP : \frac{1}{\sigma}\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}$$

$$HDOP : \frac{1}{\sigma}\sqrt{\sigma_x^2 + \sigma_y^2}$$

$$VDOP : \frac{1}{\sigma}\sqrt{\sigma_z^2}$$

these usually computed in local coordinates & used for planning or "pre-analysis"

if you choose $\sigma_0 = \sigma_{PR}$, then

$$PDOP = \begin{cases} \frac{1}{\sigma_0} \cdot \sigma_0 \sqrt{q_{xx} + q_{yy} + q_{zz}} \\ \sqrt{q_{xx} + q_{yy} + q_{zz}} \end{cases}$$