$$x = \frac{P_a(X,Y,Z)}{P_b(X,Y,Z)}$$

$$y = \frac{P_c(X,Y,Z)}{P_d(X,Y,Z)}$$

78 parameters

$$x \cdot P_b(X,Y,Z) = P_a(X,Y,Z)$$

$$y \cdot P_d(X,Y,Z) = P_c(X,Y,Z)$$

xy: image coordinates

XYZ: ground coords

$x,y \rightarrow l,s \rightarrow r,c$

$XYZ \rightarrow \phi \lambda h$

instead of directly using XYZ or xy

$$X' = (X - \bar{X}) / (0.5 \cdot X_{range})$$
$$Y' = (Y - \bar{Y}) / (0.5 \cdot Y_{range})$$
$$Z' = (Z - \bar{Z}) / (0.5 \cdot Z_{range})$$

$$x' = (x - \bar{x}) / (0.5 \cdot x_{range})$$
$$y' = (y - \bar{y}) / (0.5 \cdot y_{range})$$

Support data:

78 coefficients

+

5 offsets

5 scale factors

estimate RPC coefficients ?

39 GCP's + 78 equations + solved uniquely

or more GCP's + LS - <u>poor</u> so<u>lution</u>

1. build physical model

   estimate parameters of physical model

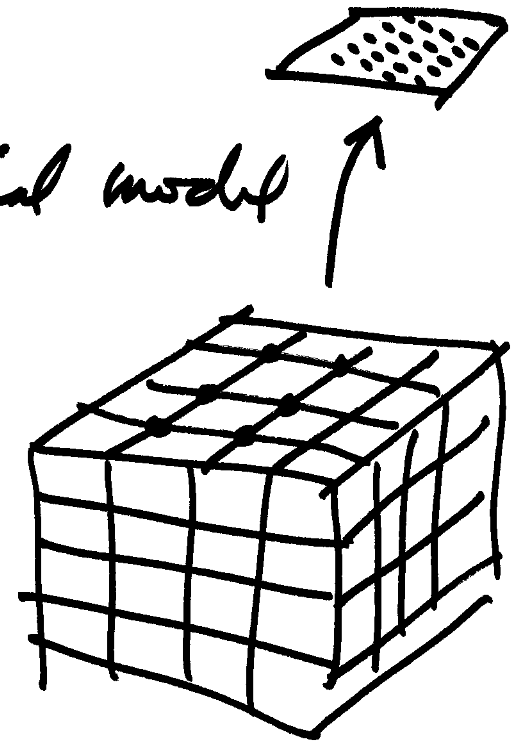
   (a) navigation data

   (b) GCP's

2. build synthetic grid (3D)

3. project each grid point into the image, via physical model

hundreds of pairs : obj. + image point



4. pseudo linear RPC – solve big LS regression problem, estimate 78 parameters
confirm that residuals are << 1 pixel

5. 78 × 78 normal equations still singular

$N\Delta = t, \quad \Delta = \cancel{N^{-1} t}$

<u>overparameterized</u>

use pseudo inverse pinv(N)

$\Delta = \text{pinv}(N) * t$

choosing Δ vector with minimum length individual parameters have no meaning but they still project accurately from Ground to Image

---

$$\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix} \quad \lambda, \theta, t_x, t_y$$

Nonlinear in 4 phys. parameters

$$\lambda \cos\theta = a, \quad \lambda \sin\theta = b$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix}$$

8 par, 78-par. RPC, intersection
pseudo LS good / usable results

---

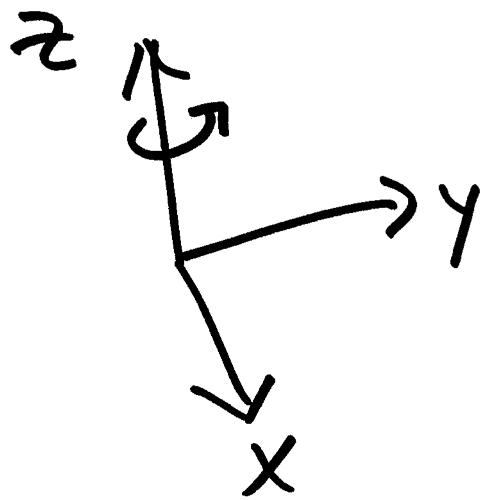
estimate matrix elements rather
the 3 angles ⇒ poor solution,
only for generation of approximations

3D rotations + parameters

applications 7 parameter transformation

register Point Clouds from LIDAR

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda \underbrace{\underline{\underline{M}}}_{3\times 3} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{elementary rotation matrix}} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

call angle
K

elementary rotation matrix

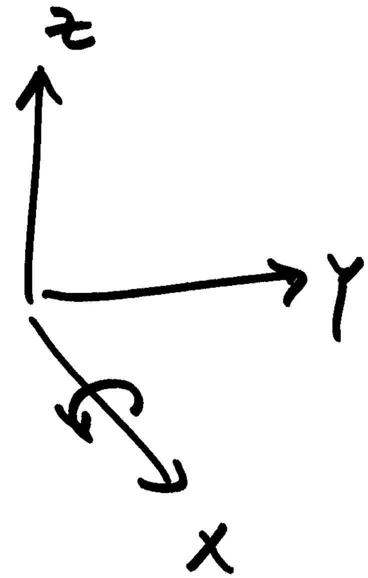
M_3, R_3, M_K, M_z

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos K & \sin K & 0 \\ -\sin K & \cos K & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

angles K, ω, ϕ : <u>Euler Angles</u>

applied in a sequential manner

rotate @ X axis
ω



$$\begin{pmatrix} y \\ z \\ x \end{pmatrix} = \begin{pmatrix} \cos\omega & \sin\omega & 0 \\ -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} Y \\ Z \\ X \end{pmatrix}$$

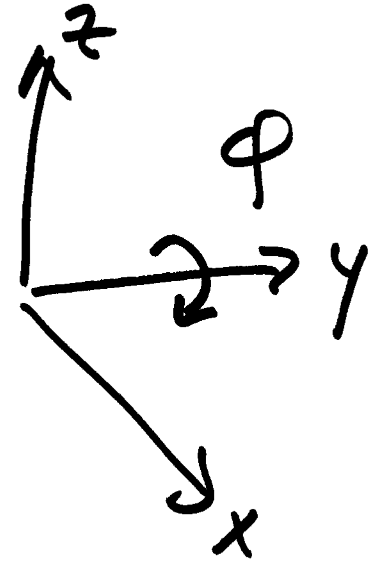
permute $YZX \to XYZ$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{pmatrix}}_{} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$M_{[illegible]}, R_{[illegible]}, M_\omega, M_x$

rotation about Y

$$\begin{pmatrix} Z \\ X \\ Y \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} Z \\ X \\ Y \end{pmatrix}$$



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \cancel{\cos}\,\sin\phi & 0 & \cos\phi \end{pmatrix}}_{M_2,\, R_2,\, M_\varphi,\, M_Y} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$M = M_\kappa M_\varphi M_\omega$$

elementary rotation matrices

$$M' = M_\omega M_\varphi M_\kappa$$

magnitude, sign, + order of rotations are important

$$M = M_\kappa M_\phi M_\omega$$

$$\begin{bmatrix} \cos\phi\cos\kappa & \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa & \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa \\ -\cos\phi\sin\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa & \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa \\ \sin\phi & -\sin\omega\cos\phi & \cos\omega\cos\phi \end{bmatrix} *$$

$$m_{31} = \sin\phi \qquad \frac{m_{21}}{m_{11}} = \frac{-\sin\kappa\cos\phi}{\cos\kappa\cos\phi}$$

$$\phi = \sin^{-1}(m_{31}) \qquad \kappa = \tan^{-1}\left(-\frac{m_{21}}{m_{11}}\right)$$

$$\frac{m_{32}}{m_{33}} = \frac{-\sin\omega\cos\phi}{\cos\omega\cos\phi} \qquad -90^\circ < \phi < +90^\circ$$

$$\omega = \tan^{-1}\left(-\frac{m_{32}}{m_{33}}\right)$$

* see lecture 22 + 23 for more readable version

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

$$F = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \lambda \underline{M} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} - \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} = 0$$

$$\frac{\partial F}{\partial \omega} :$$

$$F = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \lambda M_\kappa M_\varphi \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} - \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

$$\frac{\partial F}{\partial \omega} = -\lambda M_k M_\varphi \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\sin\omega & \cos\omega \\ 0 & -\cos\omega & -\sin\omega \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\frac{\partial F}{\partial \varphi}$$

$$F = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \lambda M_k \begin{pmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{pmatrix} M_\omega \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} - \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

$$\frac{\partial F}{\partial \varphi} = -\lambda M_k \begin{pmatrix} -\sin\varphi & 0 & -\cos\varphi \\ 0 & 0 & 0 \\ \cos\varphi & 0 & -\sin\varphi \end{pmatrix} M_\omega \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$