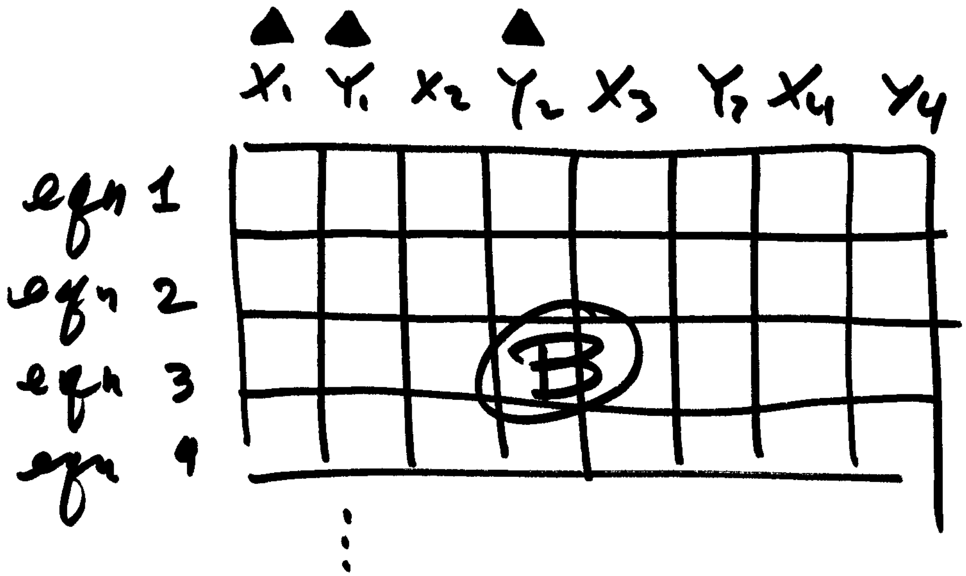


column_index =

$$[1 \;\; 3 \;\; 5 \;\; 7]$$

i = point #

B(row, column_index(i)) =

B(row, column_index(i)+1) =

form B with <u>all</u> <u>points</u> + <u>all</u> <u>obs</u> prior to forming N — eliminate columns of control points

some MATLAB routines to help organize the B-matrix:

$$B2 = \underline{elim\_col}(B, [1\ 2\ 4])$$

$$N2 = B2^T \cdot W \cdot B2$$

$$t2 = B2^T \cdot W \cdot f$$

$$\underline{\Delta 2} = N2^{-1} \cdot t2$$

$\left\{ \begin{array}{l} B2, \Delta 2, Q2dd : \\ \quad \text{reduced size} \\ B, \Delta, Qdd : \\ \quad \text{full size} \end{array} \right\}$

$$\Delta = ins\_zerv(\Delta 2, [1\ 2\ 4])$$

$$P = \begin{bmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \\ X_3 \\ Y_3 \\ X_4 \\ Y_{np} \end{bmatrix}, \quad P = P + \Delta, \quad \Delta = \begin{bmatrix} 0 \\ 0 \\ - \\ 0 \\ - \\ - \\ - \\ - \end{bmatrix}$$

$N2$, $N2^{-1} = Q2dd$

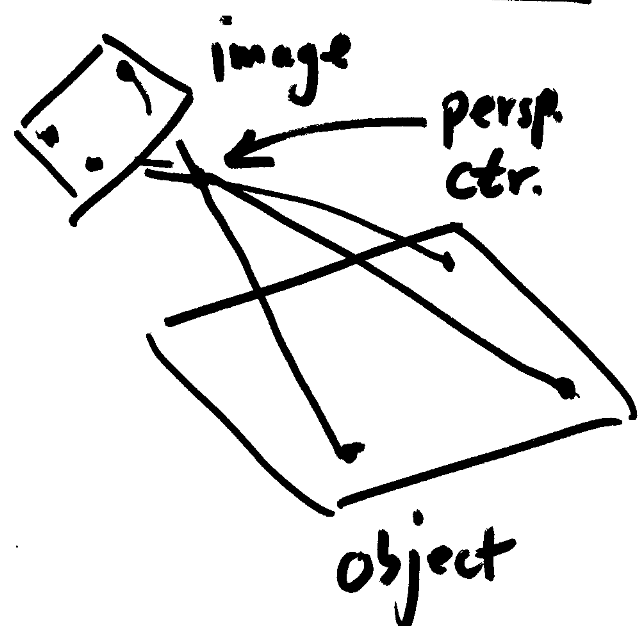
$\underline{Qdd} =$ ins_zerm(Q2dd, [1 2 4])

Pseudo LS

8 parameter transform
plane to plane perspective

xy image coords (obs)

XY ground coords. (fixed const.)



$$
\left.
\begin{aligned}
x &= \frac{a_0 + a_1 X + a_2 Y}{1 + c_1 X + c_2 Y} \\
y &= \frac{b_0 + b_1 X + b_2 Y}{1 + c_1 X + c_2 Y}
\end{aligned}
\right\}
\begin{array}{l}
\text{8 parameter} \\
\text{transformation} \\
\text{(2D)}
\end{array}
$$

$$x + c_1 x X + c_2 x Y = a_0 + a_1 X + a_2 Y$$

$$y + c_1 y X + c_2 y Y = b_0 + b_1 X + b_2 Y$$

$$x = a_0 + a_1 X + a_2 Y - c_1 \underline{x X} - \underline{c_2 x} Y$$

$$y = b_0 + b_1 X + b_2 Y - c_1 \underline{y} X - c_2 \underline{y} Y$$

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} + \begin{bmatrix} -1 & -X & -Y & 0 & 0 & 0 & xX & xY \\ 0 & 0 & 0 & -1 & -X & -Y & yX & yY \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix}$$

$$v + B\Delta = f$$

each point contributes 2 equations

min of 4 points, can solve for a, b, c

like linear Indirect Observation Model

n points

$$\begin{bmatrix} V_{x_1} \\ V_{y_1} \\ V_{x_2} \\ V_{y_2} \\ \vdots \\ V_{x_n} \\ V_{y_n} \end{bmatrix} + \begin{bmatrix} -1 & -X_1 & -Y_1 & 0 & 0 & 0 & x_1X_1 & x_1Y_1 \\ 0 & 0 & 0 & -1 & -X_1 & -Y_1 & y_1X_1 & y_1Y_1 \\ -1 & -X_2 & -Y_2 & 0 & 0 & 0 & x_2X_2 & x_2Y_2 \\ 0 & 0 & 0 & -1 & -X_2 & -Y_2 & y_2X_2 & y_2Y_2 \\ & & & & \vdots & & & \\ -1 & -X_n & -Y_n & 0 & 0 & 0 & x_nX_n & x_nY_n \\ 0 & 0 & 0 & -1 & -X_n & -Y_n & y_nX_n & y_nY_n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -X_1 \\ -Y_1 \\ -X_2 \\ -Y_2 \\ \vdots \\ -X_n \\ -y_n \end{bmatrix}$$

$$v + B\Delta = f$$

$a_0, a_1, a_2, b_0, b_1, b_2, c_1, c_2$ = unknown parameters

2 ray intersection

$$\begin{pmatrix} x - x_0 \\ y - y_0 \\ -f \end{pmatrix} = \lambda M \begin{pmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{pmatrix}$$

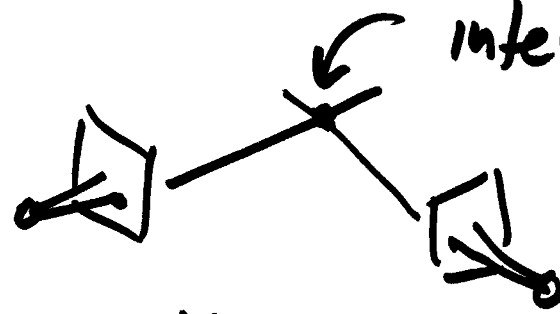
$$\underbrace{M^T \begin{pmatrix} x - x_0 \\ y - y_0 \\ -f \end{pmatrix}} = \lambda \begin{pmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \lambda \begin{pmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{pmatrix}$$

$$\frac{u}{w} = \frac{X - X_L}{Z - Z_L}, \quad \frac{v}{w} = \frac{Y - Y_L}{Z - Z_L}$$

intersection coords

XYZ



$X_L Y_L Z_L$

$\omega \varphi \kappa$

XYZ

unknown

photogrammetry example

$$uZ - uZ_L = wX - wX_L$$
$$vZ - vZ_L = wY - wY_L$$

$$-wX + uZ = uZ_L - wX_L$$
$$-wY + vZ = vZ_L - wY_L$$

$$\begin{bmatrix} -w & 0 & u \\ 0 & -w & v \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} uZ_L - wX_L \\ vZ_L - wY_L \end{bmatrix}$$

"linear" if we consider u, v, w to be <u>constants</u>.
They actually have observations embedded in them.

2 images
2 rays
in
space

$$\begin{bmatrix} -w_1 & 0 & u_1 \\ 0 & -w_1 & v_1 \\ -w_2 & 0 & u_2 \\ 0 & -w_2 & v_2 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} u_1 Z_{L_1} - w_1 X_{L_1} \\ v_1 Z_{L_1} - w_1 Y_{L_1} \\ u_2 Z_{L_2} - w_2 X_{L_2} \\ v_2 Z_{L_2} - w_2 Y_{L_2} \end{pmatrix}$$

4 eq. 3 unknowns

linear solution

- no approximations
- no iterations

computer vision

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

7 parameters

$\lambda, \omega, \phi, \kappa, t_x, t_y, t_z$

conformal 3D coord. transf.

assume same scale, $\lambda = 1$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

unknowns: m_{ij} ; t_x, t_y, t_z

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} X & Y & Z & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & X & Y & Z & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & X & Y & Z & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{31} \\ m_{32} \\ m_{32} \\ t\text{'s} \end{bmatrix}$$

solve <u>linear</u> problem for m_{ij} & t_x, t_y, t_z

<u>overparameterized</u>

$c \cdot c = 1$, $r \cdot r = 1$ etc.

$c_1 \cdot c_2 = 0$, $r_1 \cdot r_2 = 0$

1. estimate m_{ij} in linear mode
2. combine with 6 nonlinear constraints
   solve NL/LS
3. extract ω, φ, κ

RPC : rapid positioning capability
rational polynomial coefficients

$$x = \frac{a_0 + a_1 X + a_2 Y + a_3 Z + a_4 X^2 + a_5 Y^2 + a_6 Z^2 + a_7 XY + a_8 XZ + a_9 YZ + a_{10} X^3 + a_{11} Y^3 + a_{12} Z^3 + a_{13} X^2 Y + a_{14} X^2 Z + a_{15} Y^2 X + a_{16} Y^2 Z + a_{17} Z^2 X + a_{18} Z^2 Y + a_{19} XYZ}{1 + b_1 X + b_2 Y + b_3 Z + \cdots}$$

$$y = \frac{c_0 + c_1 X + c_2 Y + c_3 Z + \cdots}{1 + d_1 X + d_2 Y + d_3 Z + \cdots}$$

$$\left.\begin{array}{ll} a_0 - a_{19} & 20 \\ 1 \cdots b_{19} & 19 \end{array}\right) \; 39$$

$$\left.\begin{array}{ll} c_0 - c_{19} & 20 \\ 1 - d_{19} & 19 \end{array}\right) \; 39$$

$\Big\}$ <u>78 unknowns</u>

$$x = \frac{P_a(X, Y, Z)}{P_b(X, Y, Z)} \qquad y = \frac{P_c(X, Y, Z)}{P_d(X, Y, Z)}$$