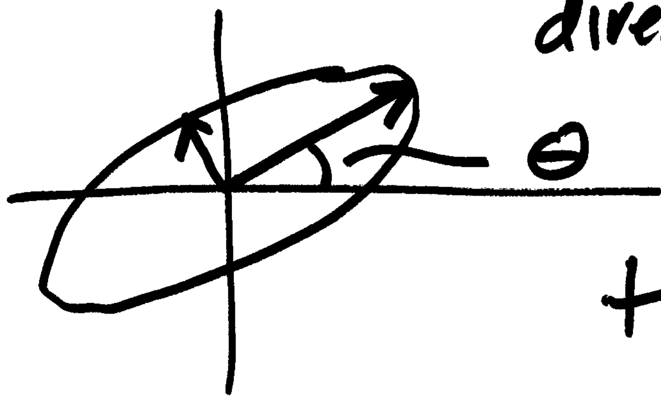


directions of axes : eigenvectors

$$\tan 2\theta = \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2}$$

<u>lengths</u> of <u>axes</u> for probability P

1.) pass global test

$$\left.\begin{array}{l} \sqrt{\lambda_1 \chi^2_{P,2}} \\ \\ \sqrt{\lambda_2 \chi^2_{P,2}} \end{array}\right\} \text{axes lengths}$$

2.) do not pass global test

$$\left.\begin{array}{l} \sqrt{\lambda_1 2 \cdot F_{p,2,r}} \\ \\ \sqrt{\lambda_2 2 \cdot F_{p,2,r}} \end{array}\right\}$$ axes lengths

r: redundancy in adj

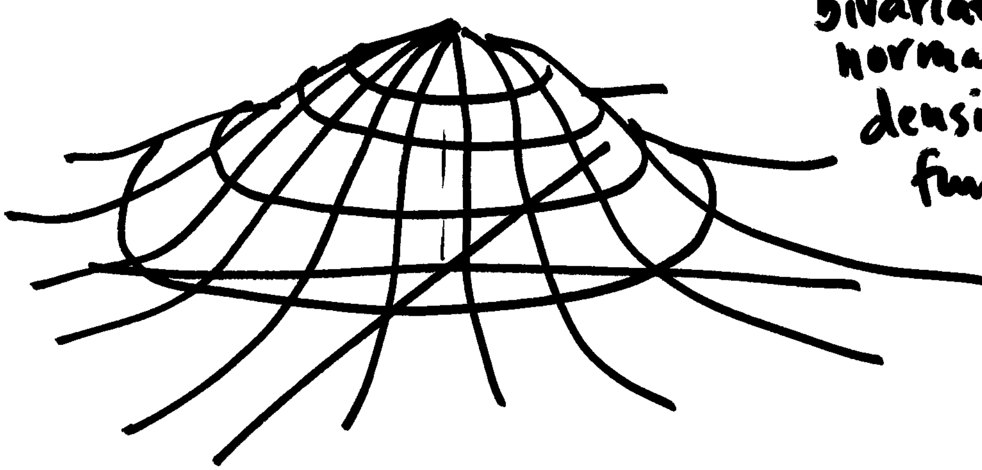
$$\underbrace{\sqrt{\lambda}}_{\sigma_{x'}} \cdot \underbrace{\sqrt{2 F_{p,2,r}}}_{C}$$ equivalent notation

Multivariate Normal

$$f(x_1, x_2, \dots x_n) = \frac{1}{(2\pi)^{n/2}\sqrt{|\Sigma|}} \cdot \exp\left(-\tfrac{1}{2} \cdot (x-\mu_x)^T \Sigma^{-1} (x-\mu_x)\right)$$

$n=2$, bivariate normal

$$f(x,y) = \frac{1}{2\pi\sqrt{|\Sigma|}} \exp\left(-\tfrac{1}{2}\left[\begin{matrix} x-\mu_x & y-\mu_y \end{matrix}\right] \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix}^{-1} \begin{pmatrix} x-\mu_x \\ y-\mu_y \end{pmatrix}\right]$$

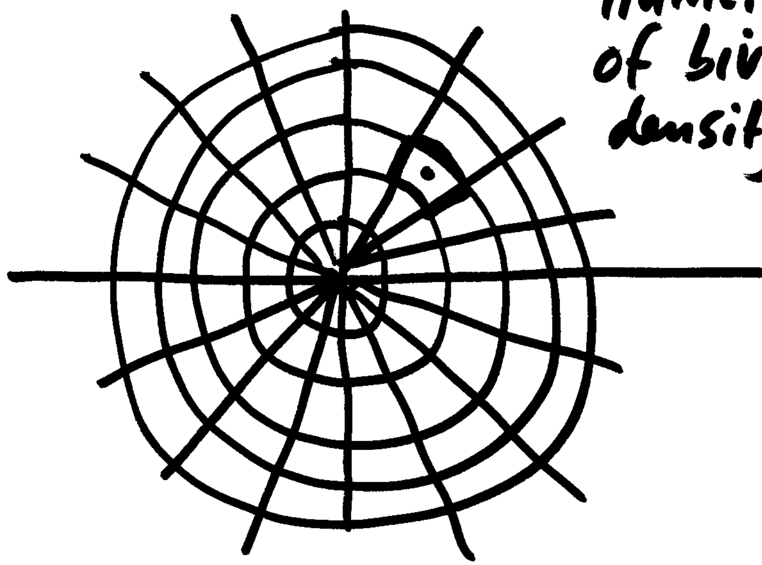


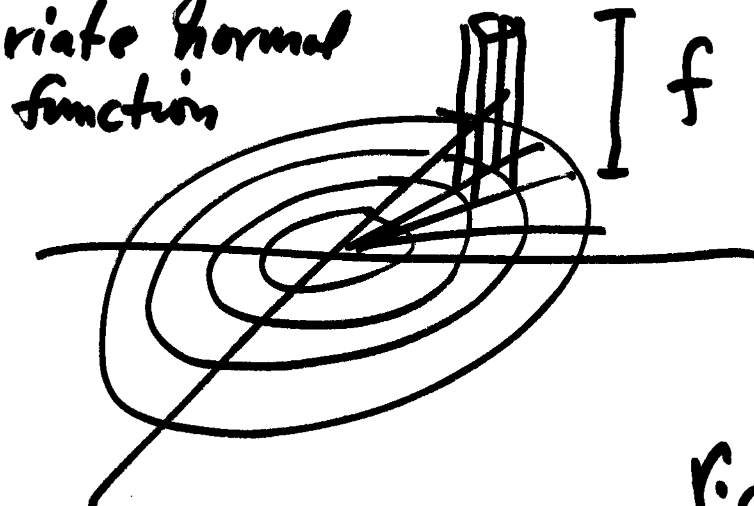
bivariate normal density function.

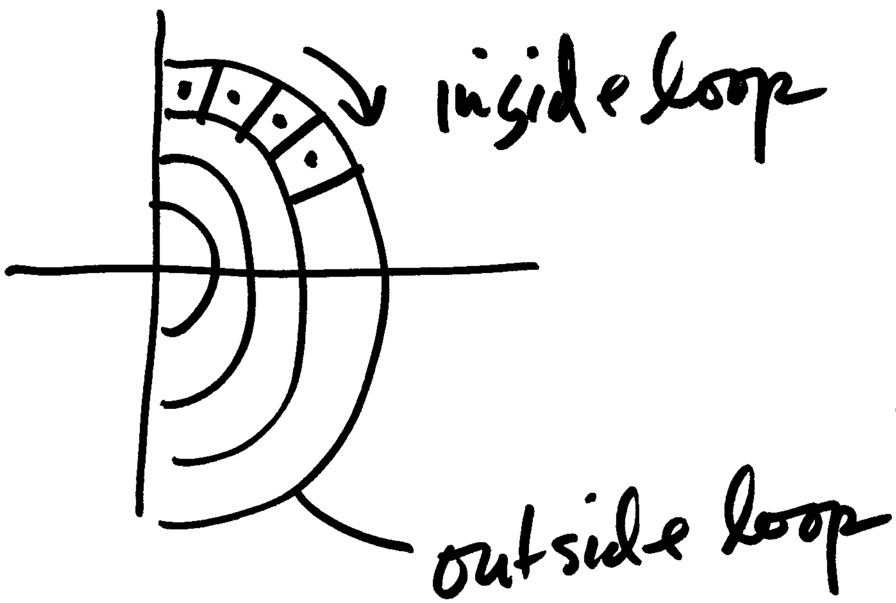
How to compute "Circular Error"?

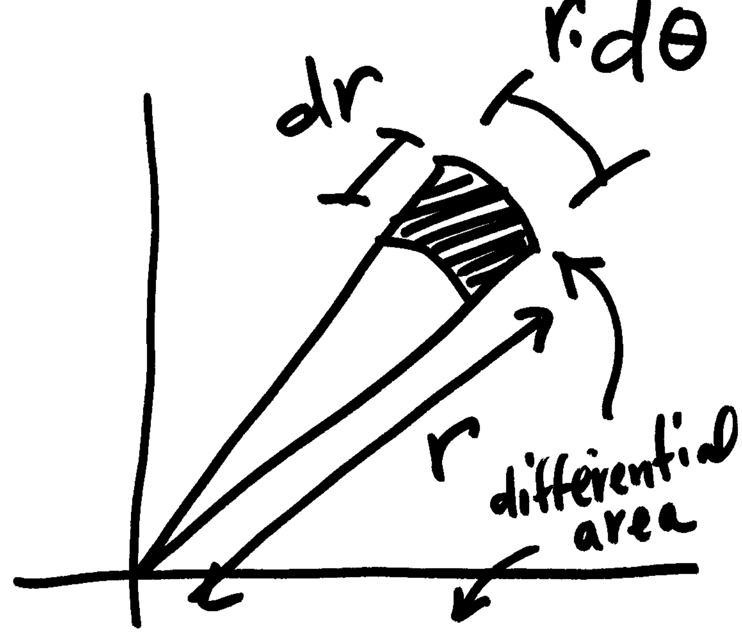
see following...

numerical integration of bivariate normal density function









area =

$$\underline{r \cdot dr \cdot d\theta}$$

change variables of integration: cartesian → polar

$$\iint_{\text{circle}} f(x,y)\, dx\, dy$$

$$\left.\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}\right\} \quad J = \begin{bmatrix} \partial x/\partial r & \partial x/\partial\theta \\ \partial y/\partial r & \partial y/\partial\theta \end{bmatrix}$$

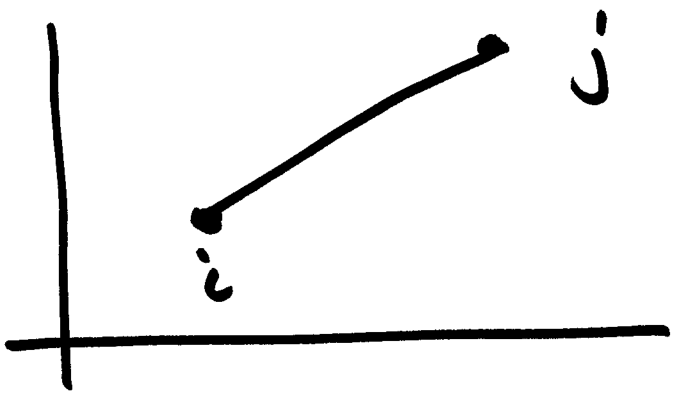
$$\int_0^{2\pi}\int_0^{R} f(r\cos\theta, r\sin\theta)\, \overset{r}{\overset{\downarrow}{|J|}}\, dr\, d\theta$$

$$J = \begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix}$$

$$\begin{aligned} |J| &= r\cos^2\theta + r\sin^2\theta \\ &= r(\underbrace{\sin^2\theta + \cos^2\theta}_{1}) \\ &= r \end{aligned}$$

Ch. 10 : develop generic LS solution to a class of problems

distance condition (2D)



$$d_{ij} = \left[(x_j - x_i)^2 + (y_j - y_i)^2\right]^{1/2}$$

$$F_d = d_{ij} - \left[(x_j - x_i)^2 + (y_j - y_i)^2\right]^{1/2} = 0$$

$$B = \left[\frac{\partial F}{\partial x_i} \;\; \frac{\partial F}{\partial y_i} \;\; \frac{\partial F}{\partial x_j} \;\; \frac{\partial F}{\partial y_j}\right]$$

$$f = -(F)$$

$$d_{ij} - \left[(x_j - x_i)^2 + (y_j - y_i)^2\right]^{1/2} = 0$$

$$\frac{\partial F}{\partial x_i} = \cancel{+}\,\frac{1}{\cancel{2}}(\cdot)^{-1/2} \cdot \cancel{2}\,(x_j - x_i)(\cancel{-}1)$$

$$= (x_j - x_i)/D_{ij}$$

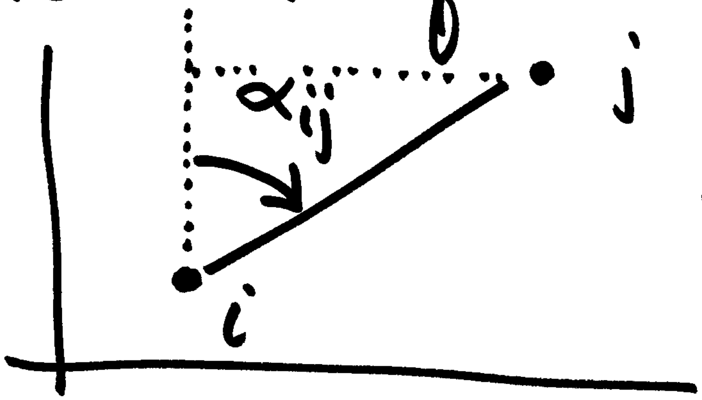
$$\frac{\partial F}{\partial y_i} = (y_j - y_i)/D_{ij}$$

$$\frac{\partial F}{\partial x_j} = -\frac{1}{\cancel{2}}(\cdot)^{-1/2} \cdot \cancel{2}\,(x_j - x_i)$$

$$= -(x_j - x_i)/D_{ij}$$

$$\frac{\partial F}{\partial y_j} = -(y_j - y_i)/D_{ij}$$

azimuth equation



$$\alpha_{ij} = \tan^{-1}\left(\frac{x_j - x_i}{y_j - y_i}\right)$$

$$F_\alpha = \alpha_{ij} - \tan^{-1}\left(\frac{x_j - x_i}{y_j - y_i}\right) = 0$$

$$B = \begin{bmatrix} \frac{\partial F_\alpha}{\partial x_i} & \frac{\partial F}{\partial y_i} & \frac{\partial F}{\partial x_j} & \frac{\partial F}{\partial y_j} \end{bmatrix} \qquad f = (-F)$$

$$\frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

$$\frac{dF_\alpha}{\partial x_i} = -\frac{1}{1+\left(\frac{\Delta x}{\Delta y}\right)^2} \cdot \frac{-1}{\Delta y} \left(\frac{\Delta y}{\Delta y}\right)$$

$$= \frac{\Delta y}{\Delta x^2 + \Delta y^2} = \frac{Y_j - Y_i}{D_{ij}^2}$$

$$\frac{\partial F_\alpha}{\partial y_i} = -\frac{1}{1+\left(\frac{\Delta x}{\Delta y}\right)^2} \cdot \frac{v\,du - u\,dv}{v^2}$$

$$\frac{\Delta y \cdot 0 - \Delta x(-1)}{\Delta y^2}$$

$$-\frac{\Delta x}{\Delta x^2 + \Delta y^2} = -\frac{(X_j - X_i)}{D_{ij}^2}$$

$$\partial F_\alpha / \partial x_j = -(Y_j - Y_i) / D_{ij}^2$$

$$\partial F_\alpha / \partial y_j = \frac{X_j - X_i}{D_{ij}^2}$$

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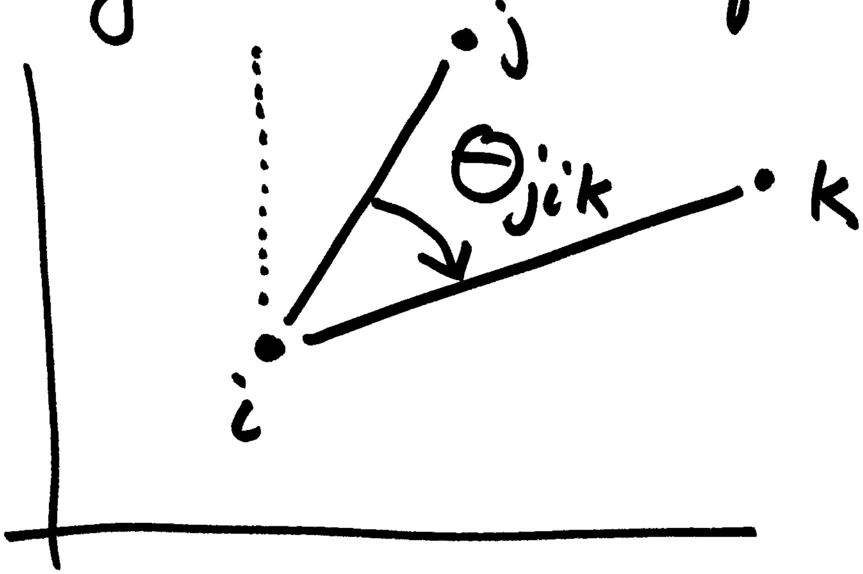
angle condition equations



$$\Theta_{jik} = \alpha_{ik} - \alpha_{ij}$$

$$\theta_{jik} = \tan^{-1}\left(\frac{x_k - x_i}{y_k - y_i}\right) - \tan^{-1}\left(\frac{x_j - x_i}{y_j - y_i}\right)$$

$$F_\theta = \theta_{jik} - \tan^{-1}\left(\frac{x_k - x_i}{y_k - y_i}\right) + \tan^{-1}\left(\frac{x_j - x_i}{y_j - y_i}\right) = 0$$

$B = \ldots$ (to be continued)