prior derivation :

$$y = (x-\mu_x)^T \Sigma^{-1}(x-\mu_x) \sim \chi^2_n$$

only true if <u>pass global test</u>

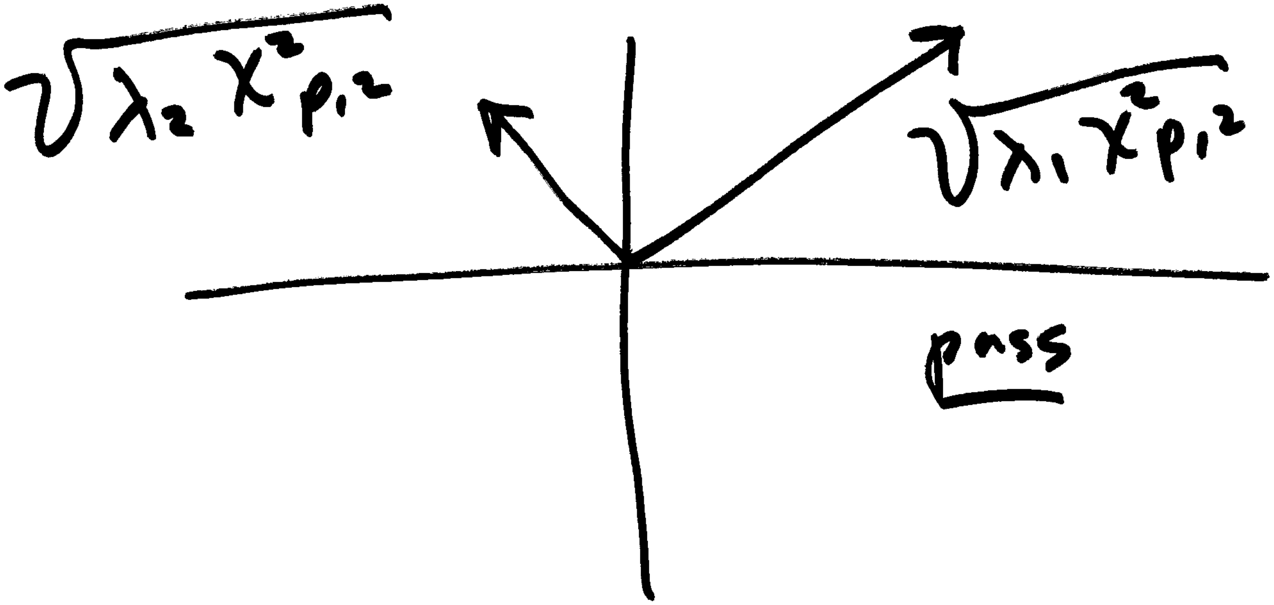
$$\Sigma = \sigma_0^2 Q$$

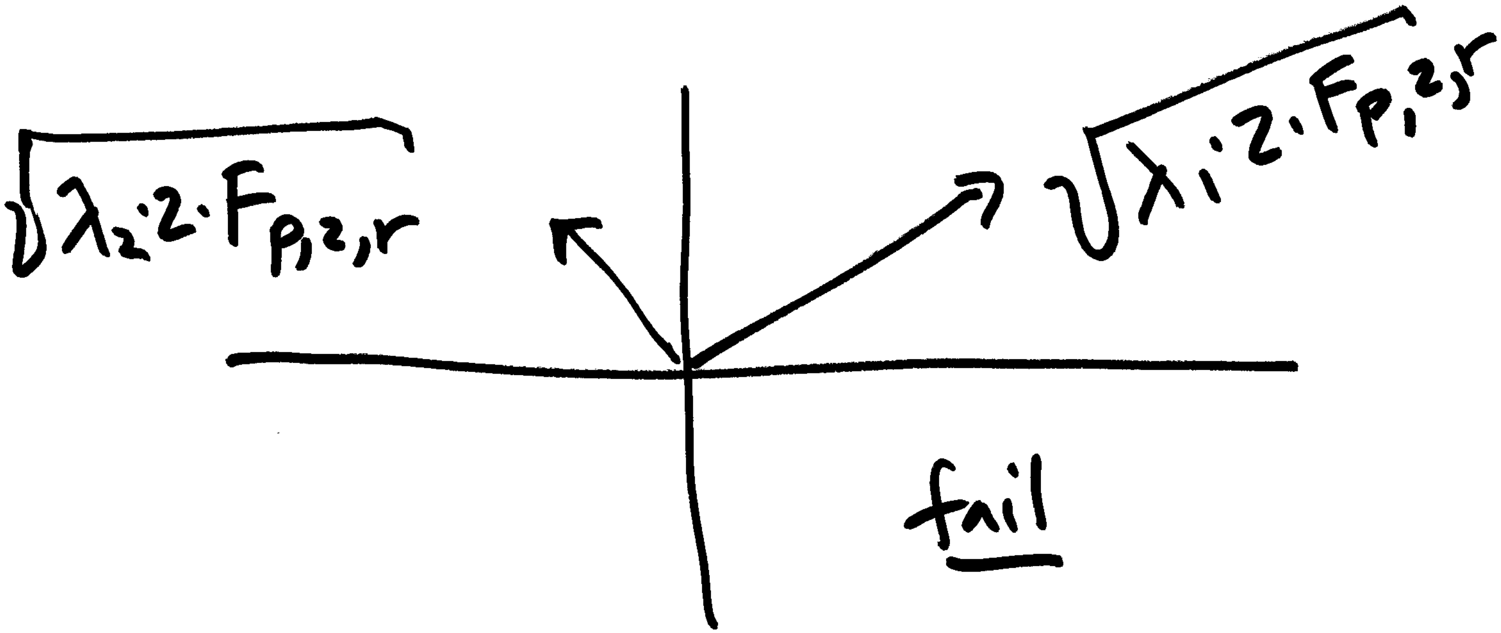
if <u>failed global test</u> (or did not do it)

$$y = (x-\mu_x)^T \Sigma^{-1}(x-\mu_x) \sim n \cdot F_{n,r}$$

$$\Sigma = \underline{\underline{\hat{\sigma}_0^2}}\, Q$$

$$\hat{\sigma}_0^2 = \frac{v^T W v}{r}$$


$\sqrt{\lambda_2 \chi^2_{p,2}}$
$\sqrt{\lambda_1 \chi^2_{p,2}}$
pass


$\sqrt{\lambda_2 \cdot 2 \cdot F_{p,2,r}}$
$\sqrt{\lambda_1 \cdot 2 \cdot F_{p,2,r}}$
fail

$S x = \lambda x$

(Σ)

eigenvector

eigenvalue

Matlab: Σ

$[V, D] = \text{eig}(S)$

columns are eigenvectors

diagonal matrix (eigenvalues)

$SV = VD$

$S = VDV^T$

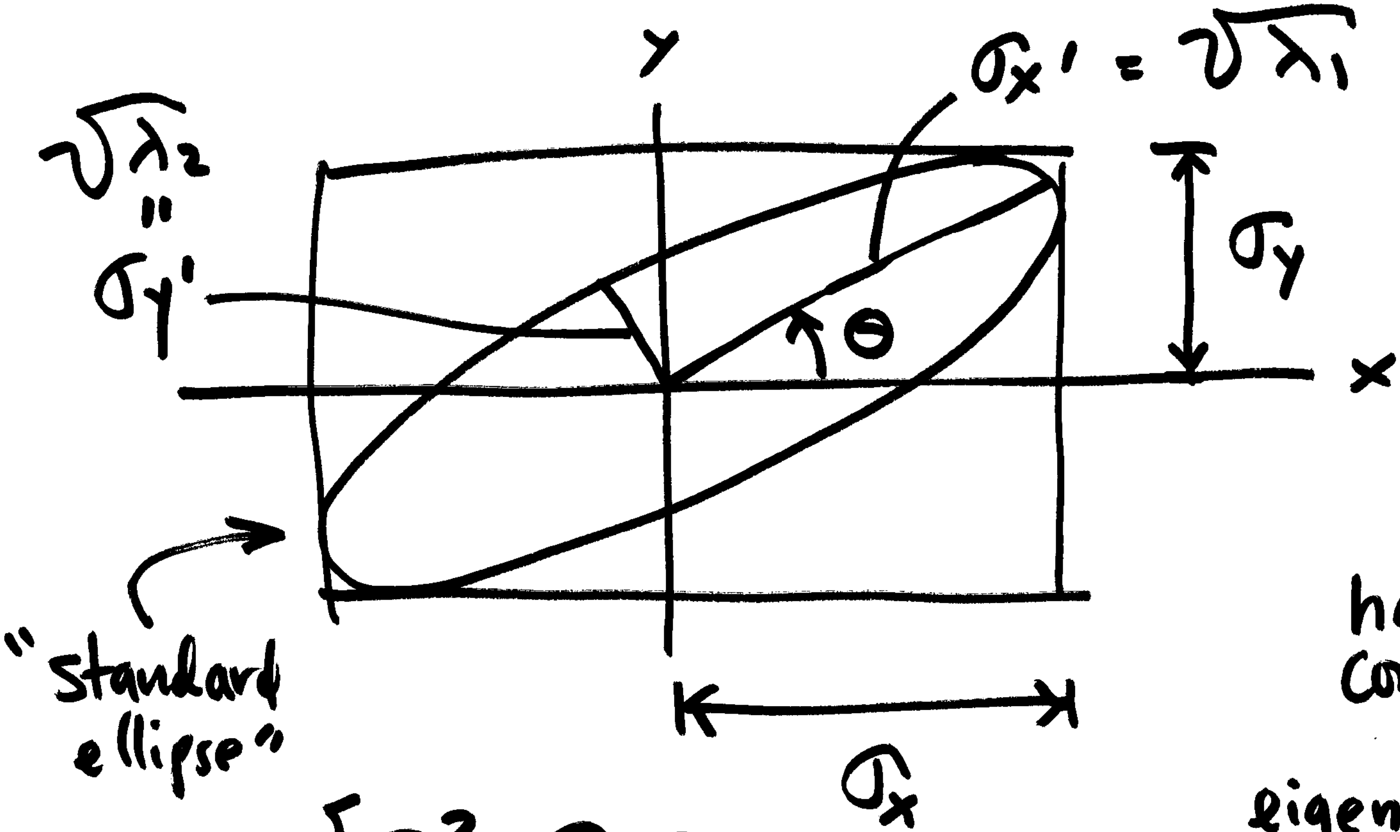
$V^T S V = D \;/\; R \Sigma R^T = D$

<u>eigenvectors</u>

col's of V

rows of V^T

rows of R



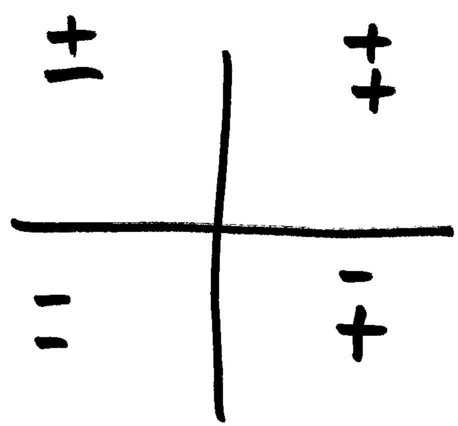
hand computation of eigenvalues

$$\Sigma_{2,2} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}$$

$$\lambda_{1,2} = \sigma_{x'}^2, \sigma_{y'}^2 = \frac{\sigma_x^2 + \sigma_y^2}{2} \pm \left[\frac{(\sigma_x^2 - \sigma_y^2)^2}{4} + \sigma_{xy}^2 \right]^{1/2}$$

$$\tan 2\theta = \frac{2\,\sigma_{xy}}{\sigma_x^2 - \sigma_y^2}$$

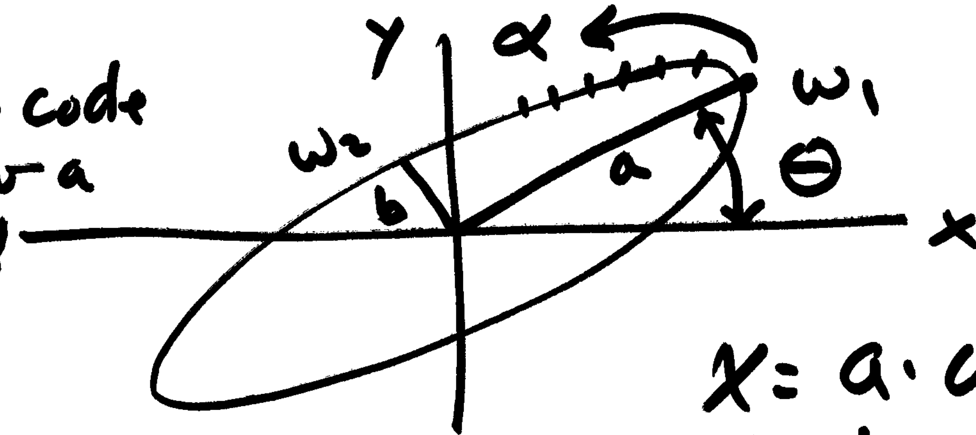
caution: obtain $\tan 2\theta$ with signs of both numerator + denominator



$$2\theta = \text{atan2}(\text{num}, \text{den})$$

this function returns value in the correct quadrant.

matlab code to draw a rotated ellipse



$x = a \cdot \cos\alpha$
$y = b \cdot \sin\alpha$

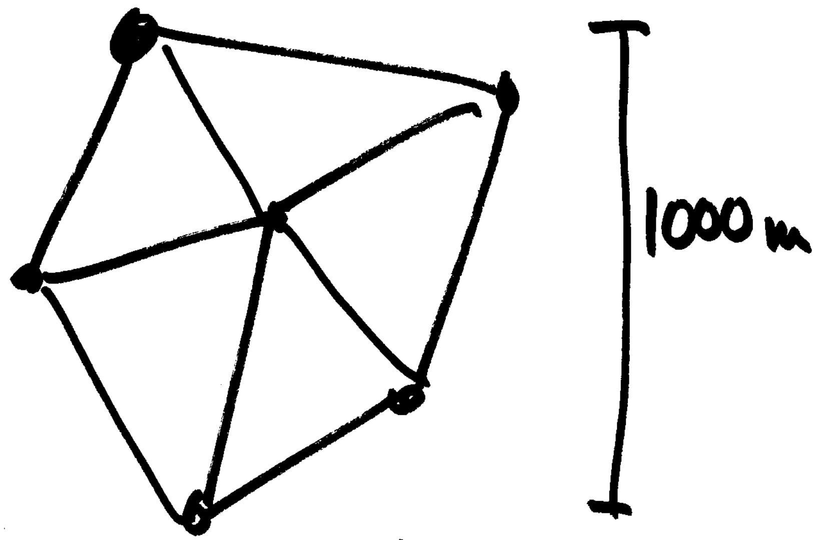
th = θ
a = semi-major
b = semi-minor
x_0 = a
y_0 = 0
nseg = 50

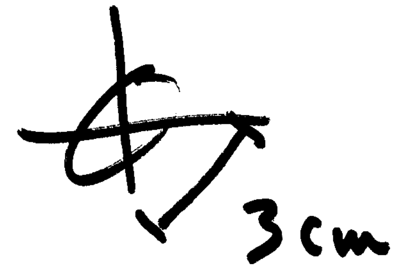
dalpha = 2*pi/nseg

```
for i=1:nseg
    alpha = i*dalpha
    x1 = a*cos(alpha)
    y1 = b*sin(alpha)
```

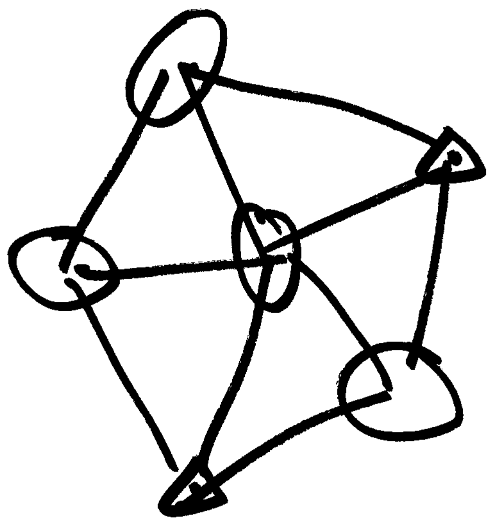
```
pX0 = cos(th) * x0 - sin(th) * y0
pY0 = sin(th) * X0 + cos(th) * y0
Px1 = cos(th) * x1 - sin(th) * y1
Py1 = sin(th) * x1 + cos(th) * y1
plot([px0 px1], [py0 Py1], '-r');
if (i == 1)
    hold on
    end
x0 = x1
y0 = y1
end
```

(it works!)





to see error ellipses : <u>exaggerate</u> <u>scale</u>



500 m — map

5 cm — error