

1 hour exam

1 page of notes

definition, vocabulary

short answer

math. modelling:  $n, n_0, r$

linear LS - I/O, O/O - longhand

I/O, O/O - matrix

max:  $2 \times 2$  system of equations

non linear LS I/O, O/O

weights

error propagation  $Y = AX + b, \Sigma_{xx} \rightarrow \Sigma_{yy}$

confidence intervals

Chapters 1, 3, 4, 5, 6, 8

adjustment

Prob-

Error Prop.

Post Adj. Stat

HW2 - general comments for HW  
 Solution of LS problem

Summary of solution:

method

$n, n_0, r$

parameters selected

condition equations

nonlinear problems

1st iteration  $B, W, f$  AfW

show evidence of convergence

statement on convergence

$\Delta, X$  v: primary means for QA

$\hat{e}$

= global test  
 = any other error prop

15-4

$$Av = f$$

$$\underline{Av} = d - \underline{A\Delta}$$

$$z_i + v_i = a_0 + a_1 x + a_2 y + a_3 (x^2 + y^2)$$

$$v_i - a_0 - a_1 x - a_2 y - a_3 (x^2 + y^2) = -z_i$$

$$(v_i) + \begin{pmatrix} -1 & -x & -y & -(x^2 + y^2) \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = (-z_i)$$

$$v + B \Delta = f$$

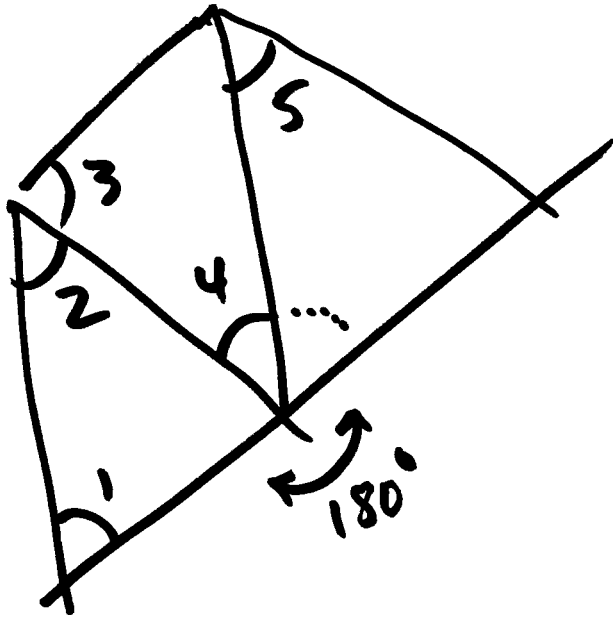
$$v = f - B\Delta$$

$$* v = -(f - B\Delta)$$

$$\omega_i = \frac{\sigma_0^2}{\sigma_i^2} \neq \frac{\sigma_0}{\sigma_i}$$

$$= \left( \frac{\sigma_0}{\sigma_i} \right)^2$$

15-5



15-6

if  $X \sim N(0, \Sigma)$  then

$X^T A X \sim \chi_r^2$  if  $A \Sigma$  is

idempotent + rank  $r$

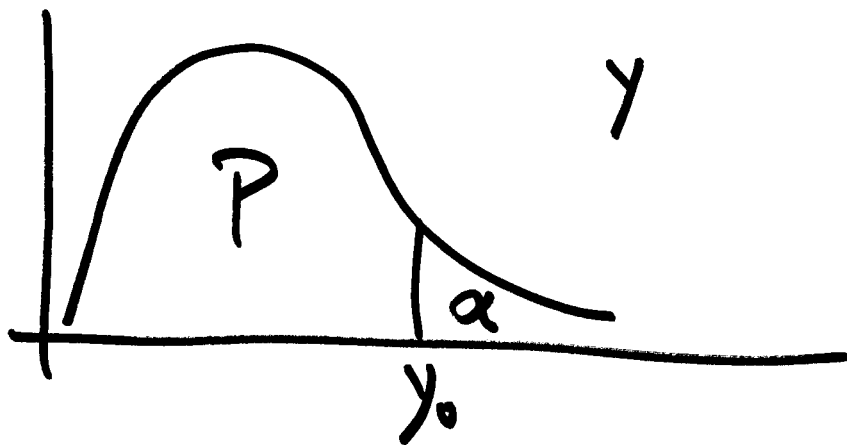
$$\uparrow$$

$$(A^2 = A)$$

$$\boxed{\frac{V^T W U}{\sigma_0^2} \sim \chi_r^2}$$

$$Y = (X - \mu_x)^T \Sigma^{-1} (X - \mu_x) \sim \chi_n^2$$

15-7



ask question: what region of  $X$  corresponds to area "P"?

$$P\left(y = (x - \mu_x)^T \Sigma^{-1} (x - \mu_x) < y_0\right) = 1 - \alpha$$

$$R \Sigma R^T = D$$

$$M = \underline{R^* \Sigma^{-1} R^T} = D^{-1}$$

$D$ : diagonal  
eigenvalues of  $\Sigma$

$$D = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}$$

$$x = \begin{pmatrix} x \\ y \end{pmatrix} \quad 15-8$$

$$D^{-1} = M = \begin{pmatrix} 1/d_1 & 0 \\ 0 & 1/d_2 \end{pmatrix}$$

$R^T R$   
insert this @  $\uparrow$

$$w = R(x - \mu)$$

$$P\left((x - \mu)^T \Sigma^{-1} (x - \mu) < \chi^2_{p,n}\right) = P$$

$$\rightarrow P\left(\underbrace{(x - \mu)^T}_{w^T} \underbrace{R^T R \Sigma^{-1} R^T R}_M \underbrace{(x - \mu)}_w < \chi^2_{p,n}\right) = P$$



$$P(\omega^T M \omega < \chi^2_{P,n}) = P \quad 15-9$$

$$P\left((w_1, w_2) \begin{pmatrix} 1/d_1 & 0 \\ 0 & 1/d_2 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} < \chi^2_{P,n}\right) = P$$

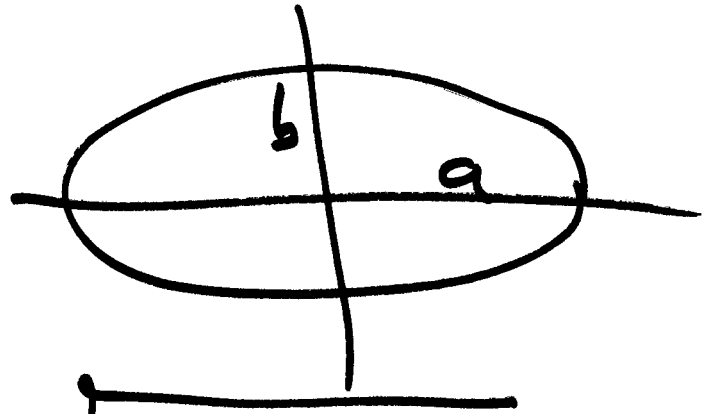
$$P\left(\frac{w_1^2}{d_1} + \frac{w_2^2}{d_2} < \chi^2_{P,n}\right) = P$$

$$P\left(\frac{w_1^2}{d_1 \chi^2_{P,n}} + \frac{w_2^2}{d_2 \chi^2_{P,n}} < 1\right) = P$$

— this inequality represents interior of ellipse —

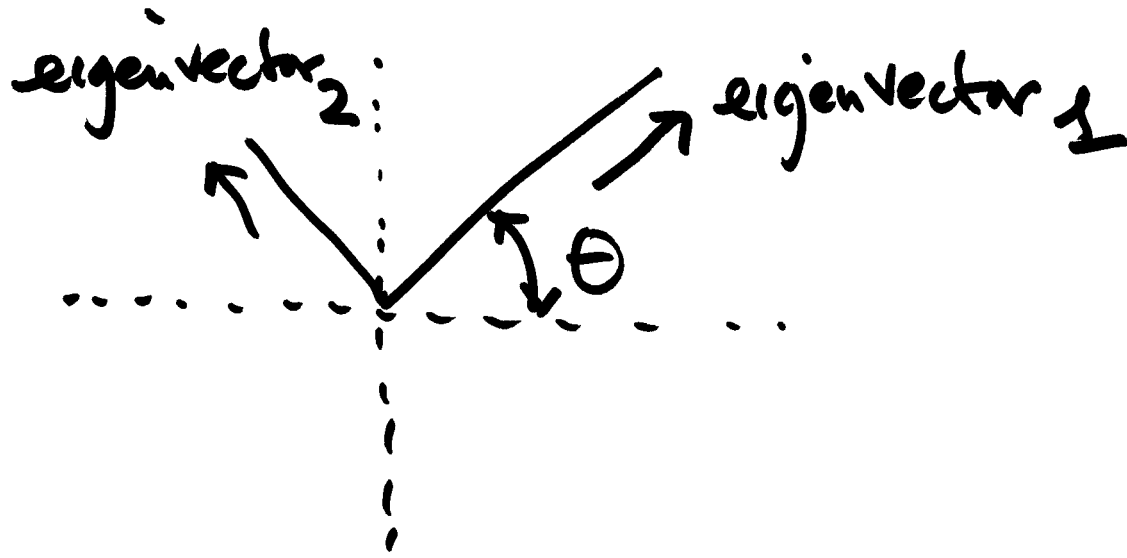
15-10

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Semi major axis =  $\sqrt{d_1 \chi^2_{p,n}}$

Semi-minor axis =  $\sqrt{d_2 \chi^2_{p,n}}$



$$d_1 = \lambda_1$$

$$d_2 = \lambda_2$$