Global Test

$H_0: \sigma^2 = \sigma_0^2$

$H_1: \sigma^2 > \sigma_0^2$

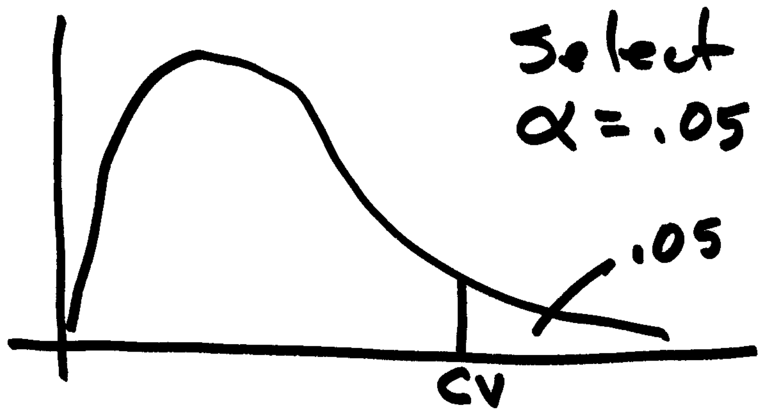
$\chi^* \quad \dfrac{v^T W v}{\sigma_0^2} \sim \chi^2_r$

r: redundancy

r: degrees of freedom

Select significance level .05

$\alpha = .05$



one sided

Probability of (Type I error) = .05

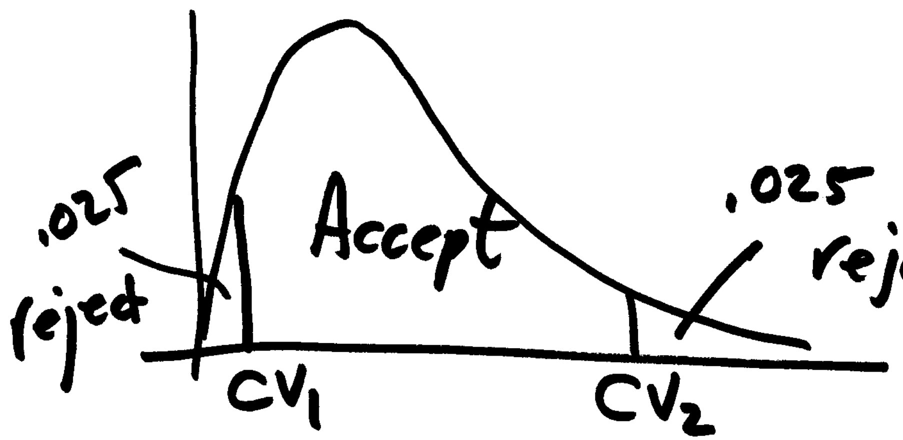
Prob (reject Ho when it's true)

two-sided test $H_0: \sigma = \sigma_0$

vs.

$H_1: \sigma \neq \sigma_0$

$$\chi^* = \frac{v^T W v}{\sigma_0^2}$$ ← "prior" value



if $CV_1 < \chi^* < CV_2$

accept H_0

otherwise

reject

$CV_1 = \text{icdf}('\text{chi2}', .025, r)$

$CV_2 = \text{icdf}('\text{chi2}', \underset{1-\alpha/2}{1-.025}, r)$

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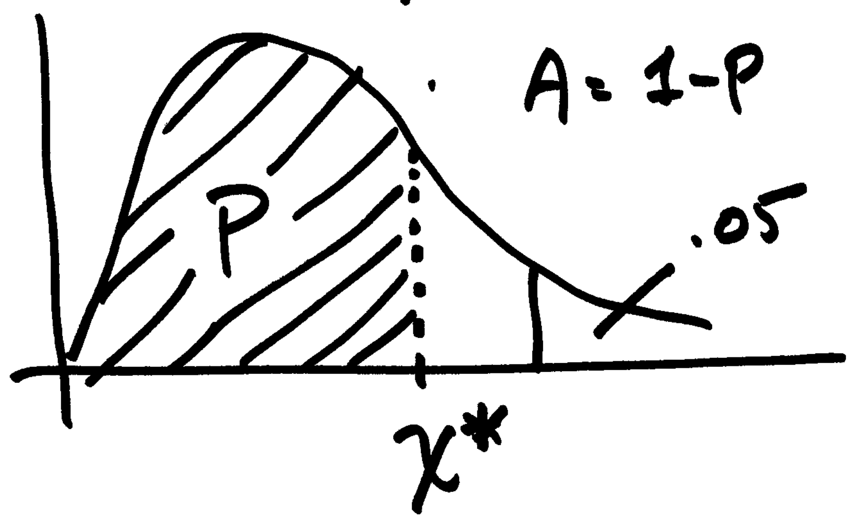
alternate way for decision: probability domain



reject if $\underline{\underline{A < \alpha}}$

$P = \text{cdf}('\text{chi2}', \chi^*, r)$

if accept H_0 in global test (F-test)

use σ_0^2 to scale Q's

if reject H_0 in global test

then use $\hat{\sigma}_0^2 = \frac{V^T W V}{r}$ to scale Q's

$$\Sigma_{xx} = \sigma_0^2 Q_{xx}$$

$$\updownarrow$$

$$\hat{\sigma}_0^2$$

$l_1 \quad \sigma_1 = .5$

$l_2 \quad \sigma_2 = 1.5$

$l_3 \quad \sigma_3 = 1.0$

$\sigma_0 = .5, \ l_1 : w = 1$

$w_1 = (.5)^2/(.5)^2 = \underline{1}$

$w_2 = (.5)^2/(1.5)^2 = 0.111$

$w_3 = (.5)^2/1^2 = .25$

$\sigma_0 = 1.5, \ w_2 = 1$

$w_1 = (1.5)^2/(.5)^2 = 9$

$w_2 = (1.5)^2/(1.5)^2 = 1$

$w_3 = (1.5)^2/1^2 = 2.25$

$\sigma_0 = 1.0, \ w_3 = 1$

$w_1 = 4$

$w_2 = 0.444$

$w_3 = 1$

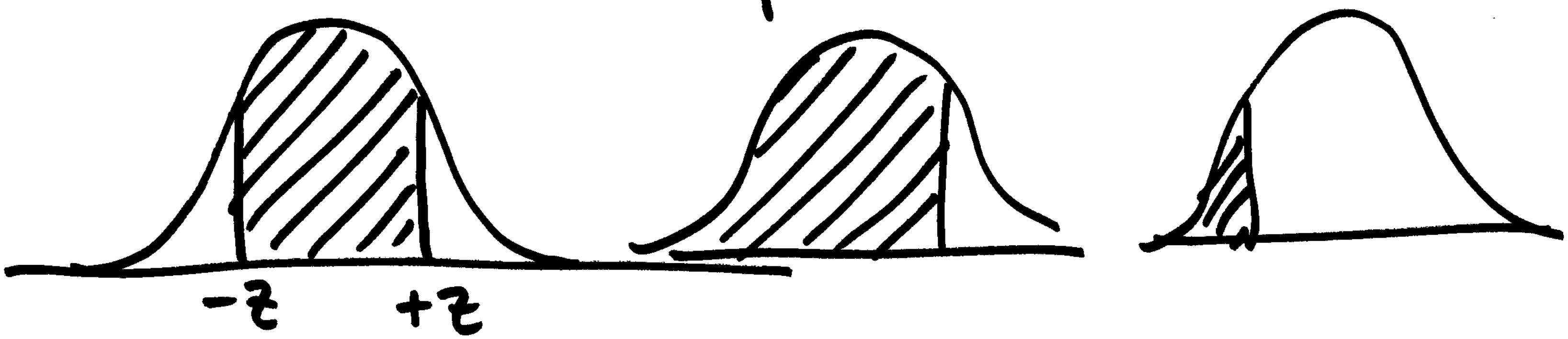
$$H_0 : \sigma = \sigma_0$$

$$\chi^* = \frac{V^T W V}{\sigma_0^2} , \quad \begin{bmatrix} \sigma_0^2/\sigma_1^2 & & \\ & \ddots & \\ & & \sigma_0^2/\sigma_n^2 \end{bmatrix} = \sigma_0^2 \cdot \Sigma^{-1} = W$$

$$\chi^* = \frac{V^T \cdot \cancel{\sigma_0^2} \Sigma^{-1} V}{\cancel{\sigma_0^2}} = V^T \Sigma^{-1} V$$

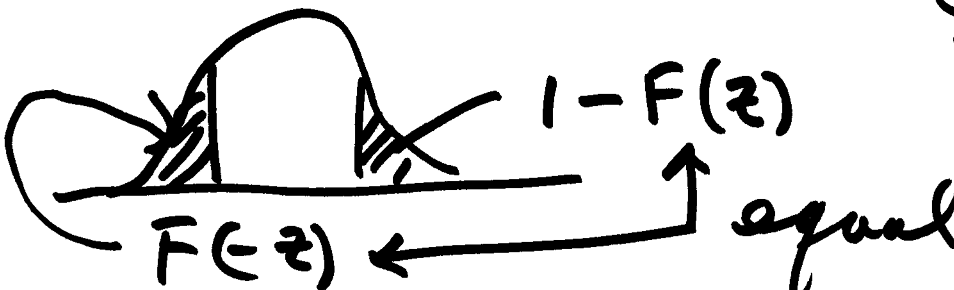
Confidence intervals

assume normality for observations



$$P(-z < RV < +z) = F(z) - F(-z)$$

$$\Phi(z) - \Phi(-z)$$



$$F(z) - (1 - F(z))$$

$$2F(z) - 1$$

result of adjustment $\hat{x}$

standardize $\hat{x}$: $z = \dfrac{\hat{x} - \mu_x}{\sigma_x}$

||

mean 0, $\sigma = 1$

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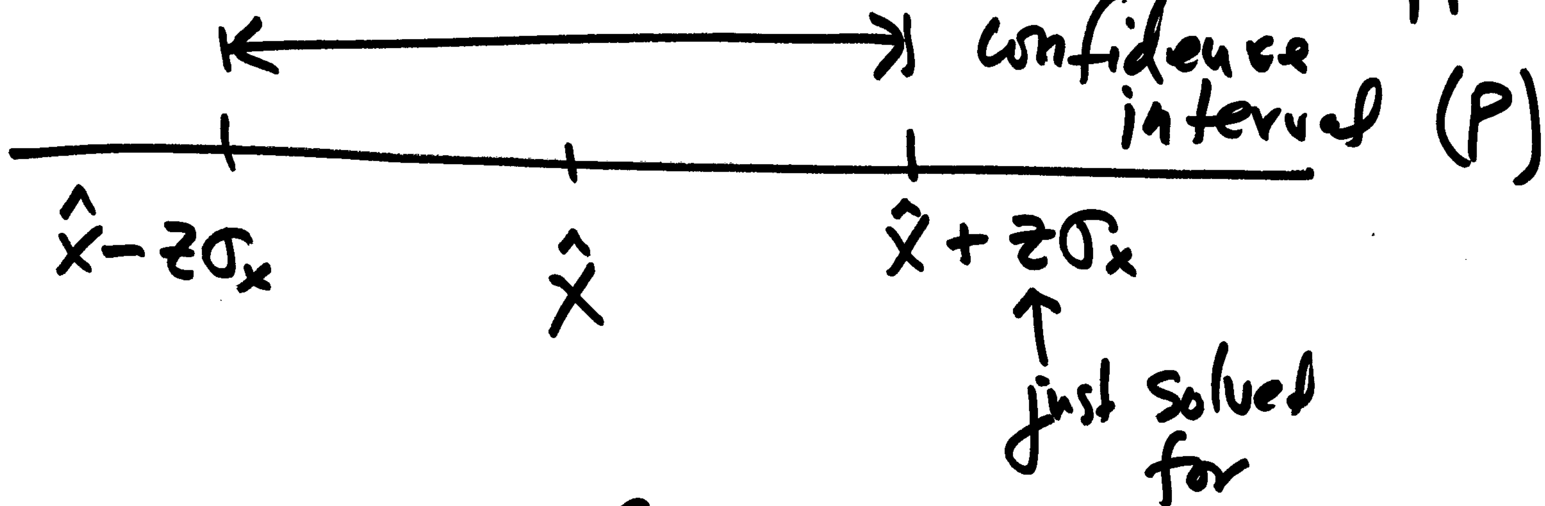
prior σ_0^2 passed test

$$P\left(-z < \frac{\hat{x} - \mu_x}{\sigma_x} < +z\right) = \underline{2F(z) - 1}$$

$$P\left(-z\sigma_x < \hat{x} - \mu_x < +z\sigma_x\right)$$

$$P\left(-\hat{x} - z\sigma_x < -\mu_x < -\hat{x} + z\sigma_x\right)$$

$$P\left(\hat{x} + z\sigma_x > \mu_x > \hat{x} - z\sigma_x\right)$$



this derivation ok for use of <u>prior σ_0^2</u>
passed global test

$$\frac{\hat{x}-\mu_x}{\hat{\sigma}_x} \sim t_r$$

$$\Sigma = \hat{\sigma}_0^2 Q_{xx}$$

$$\hat{\sigma}_0^2 = \frac{v^T W v}{r}$$

$$P\left(-t < \frac{\hat{x}-\mu_x}{\hat{\sigma}_x} < +t\right) = \underbrace{F(t) - F(-t)}_{2F(t)-1}$$

$$P\left(-t\hat{\sigma}_x < \hat{x}-\mu_x < t\hat{\sigma}_x\right) =$$

$$P\left(-\hat{x}-t\hat{\sigma}_x < -\mu_x < -\hat{x}+t\hat{\sigma}_x\right)$$

$$P\left(\hat{x}+t\hat{\sigma}_x > \mu_x > \hat{x}-t\hat{\sigma}_x\right)$$

$$= 2F(t)-1$$

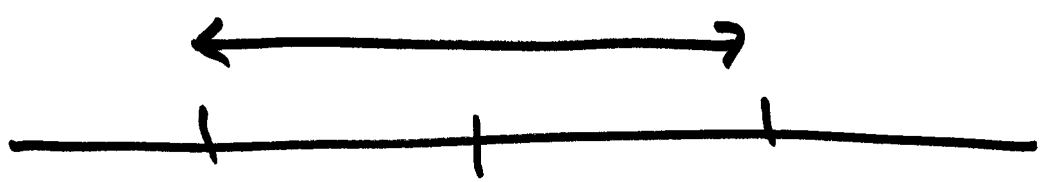
(use this one if fail to pass global test) select P

solve $P = 2F(t) - 1$, for $F(t)$

$$F(t) = \frac{P+1}{2}$$

$$t = \text{icdf}('t', (P+1)/2, r)$$

$r \rightarrow$ degrees of freedom



$\hat{x} - t\hat{\sigma}_x$ $\quad$ $\hat{x}$ $\quad$ $\hat{x} + t\hat{\sigma}_x$

recall

$\hat{x} - z\sigma_x$ $\quad$ $\hat{x} + z\sigma_x$