chapters 5, 6, 8 text book

probability,

statistics

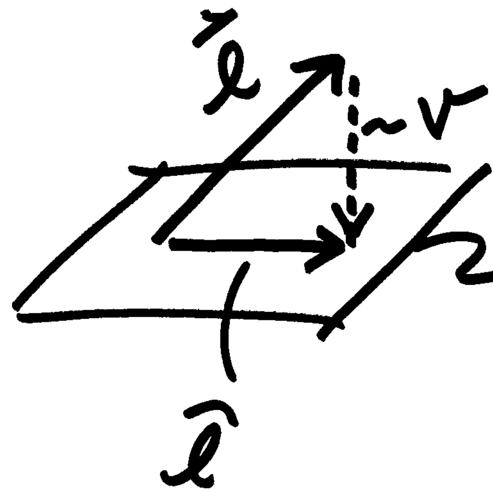
error propagation

Random variable < discrete {coin toss, rolling die} / continuous

we use only continuous R.V.

probability density function

But first, graphical view of Least Squares + I/O - O/O relationship:



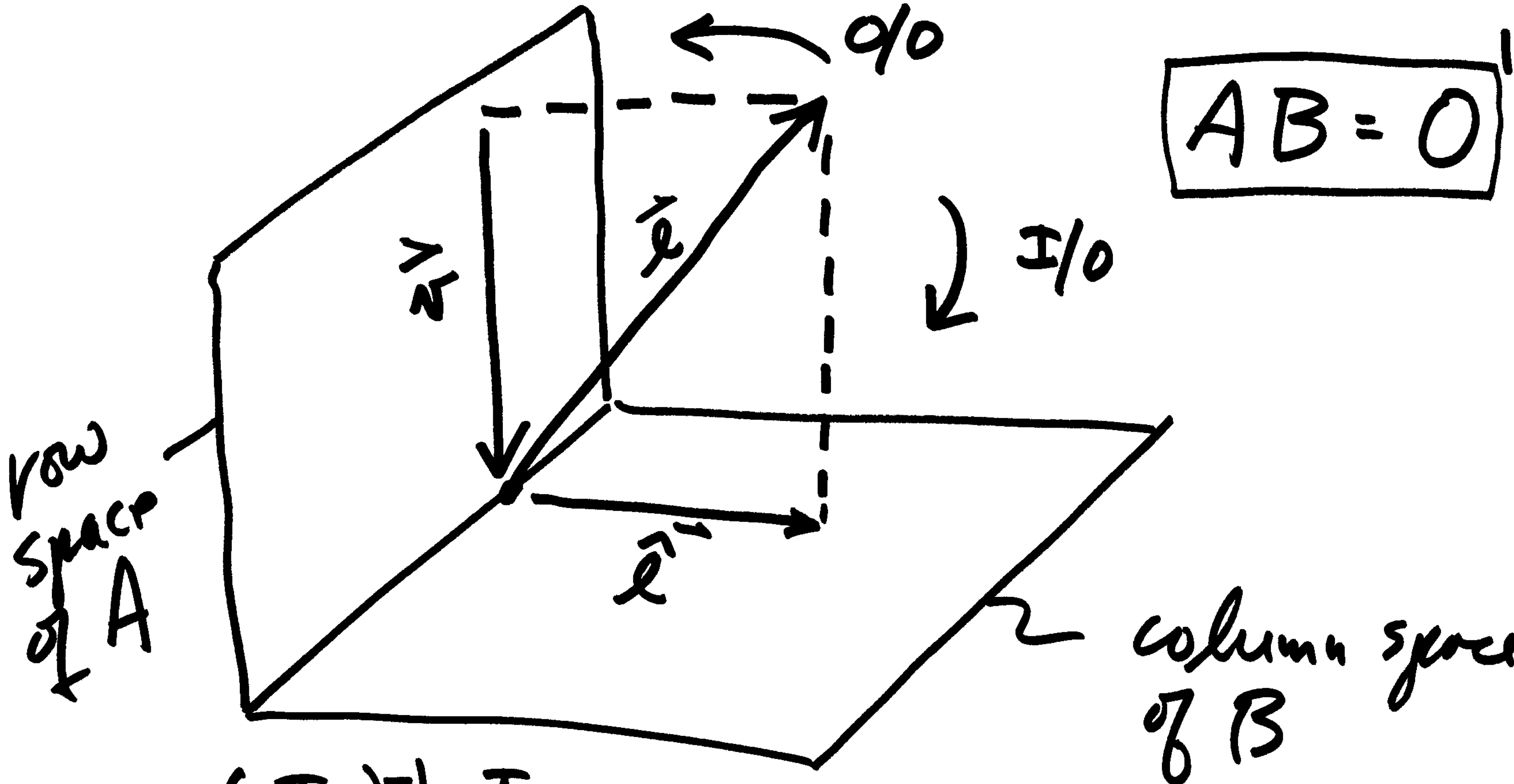
n-order

column space of B

<u>projection</u>: project $\vec{X}$ onto column space of matrix M:

$$P = M(M^TM)^{-1}M^TX$$

$$\boxed{AB = O}$$



$$P = M(M^TM)^{-1}M^TX \quad , \quad \text{assume } W = I$$

$$\hat{\ell} = B(B^TB)^{-1}B^T\ell$$

$$\hat{\ell} = -B\underbrace{\underbrace{(B^TB)^{-1}}_{N^{-1}}\;\underbrace{B^T(-\ell)}_{t^{f}}}_{x,\,\Delta}$$

$$v = -A(AA^T)^{-1}A\,l$$

$$v = A^T \underbrace{\underbrace{\underbrace{(AA^T)}_{Q_e}{}^{-1}}_{w_e} \underbrace{(-Al)}_{f}}_{k}$$

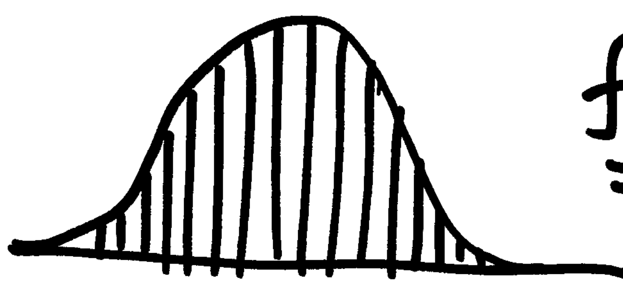
(v is negated from strict projection,
see figure)

construct. PDF for a RV) (X) :

1. sample X many times
2. make a histogram of occurrences of X

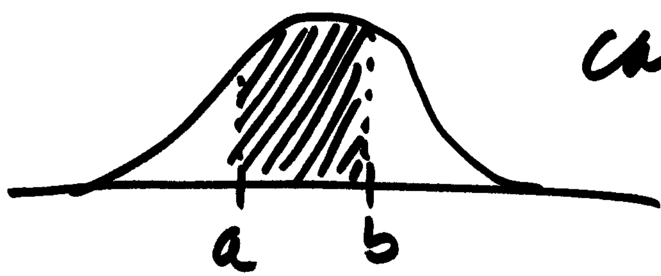
3. imagine (a) take ∞ number of samples
   (b) reduce the bin width

4.



$f(x)$

5. normalize so that area under curve $= 1$

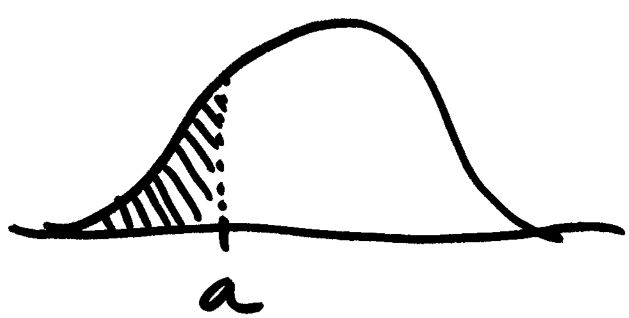


can interpret <u>areas</u> as <u>probabilities</u>

$P(a < X < b)$ = area under curve between a, b

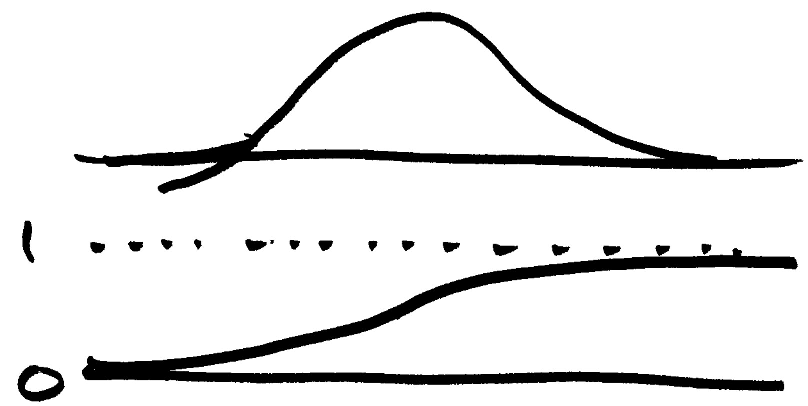
$$P(a < x < b) = \int_a^b f(x)\,dx$$

$$P(x < a) = \int_{-\infty}^a f(x)\,dx$$



$F(a) = P(X < a)$

density function $f(x)$



distribution function $F(x)$

accumulation (integral)

rate of accumulation (derivative)

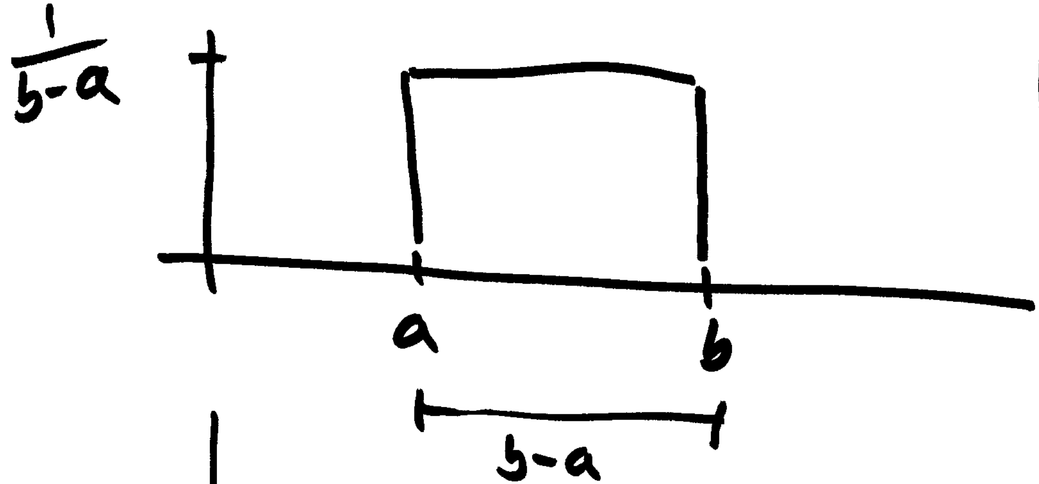
$$F(a) = \int_{-\infty}^{a} f(x)\, dx$$

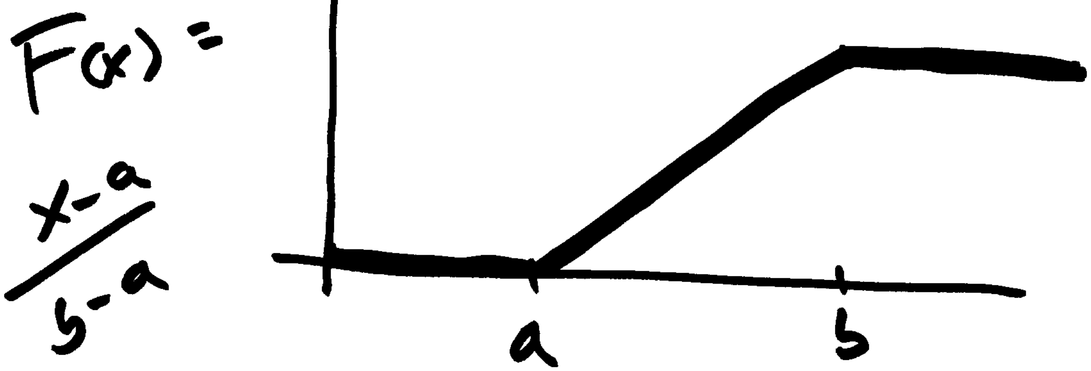
$$f(a) = \frac{d}{dx} F(x) = F'(x)$$

F.T.O.C. $\int_a^b \underline{f}(x)\,dx = \underline{F}(b) - F(a)$

(Fundamental Theorem of Calculus)

examples : **uniform**

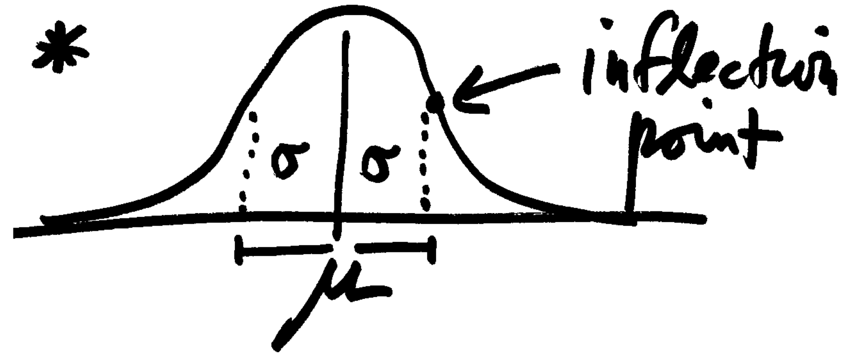




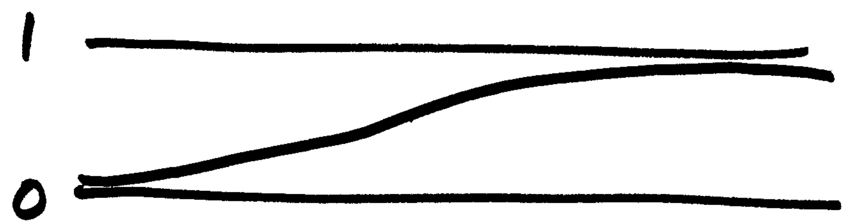
2. example : normal distribution

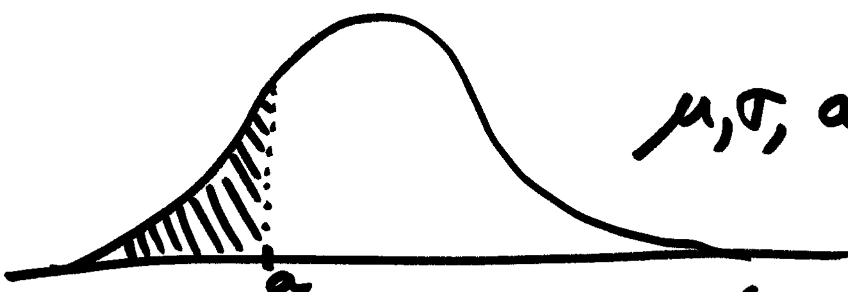
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

*



parameters : μ, σ





μ, σ, a

$$P(x < a) = \int_{-\infty}^{a} f(x)\, dx$$

$$= F(a)$$

* (note: f(x) above was incorrect in lecture!)

obtain numerical values of F(a)

by: table look up ✓

numerical approximation

- calculator ✓
- matlab ✓
- ...

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matlab

$$Y = \text{pdf}(\text{'norm'}, x, \mu, \sigma)$$

$$P = \text{cdf}(\text{'norm'}, x, \mu, \sigma)$$

$$x = \text{icdf}(\text{'norm'}, P, \mu, \sigma)$$

pdf : graph of function
numerical integration

cdf : hypothesis tests

icdf : critical values, confidence interval

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tables : back of textbook

normal : $\dfrac{X - \mu_x}{\sigma_x} = Z \sim N(0,1)$

$N(0,1)$

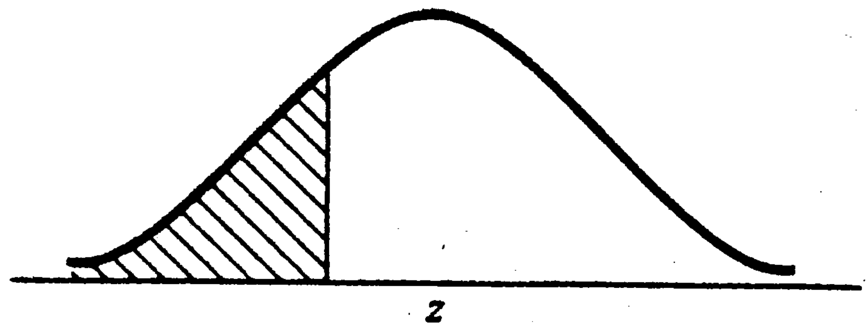
Table I. Values of the Standard Normal Distribution

11-12

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du = P[Z \leqslant z]$$

p. 326

F(z)



z	0	1	2	3	4	5	6	7
−3.	.0013	.0010	.0007	.0005	.0003	.0002	.0002	.0001
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089

z	0	1	2	3	4	5	6	7	8
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306
1.5	.9332	.9345	.9357	.9370	.9382	[illegible]	[illegible]	[illegible]	[illegible]

can also use pdf, cdf, icdf

'norm' = normal

'unif' = uniform

'f' = F

'chi2' = chi-squared

't' = t

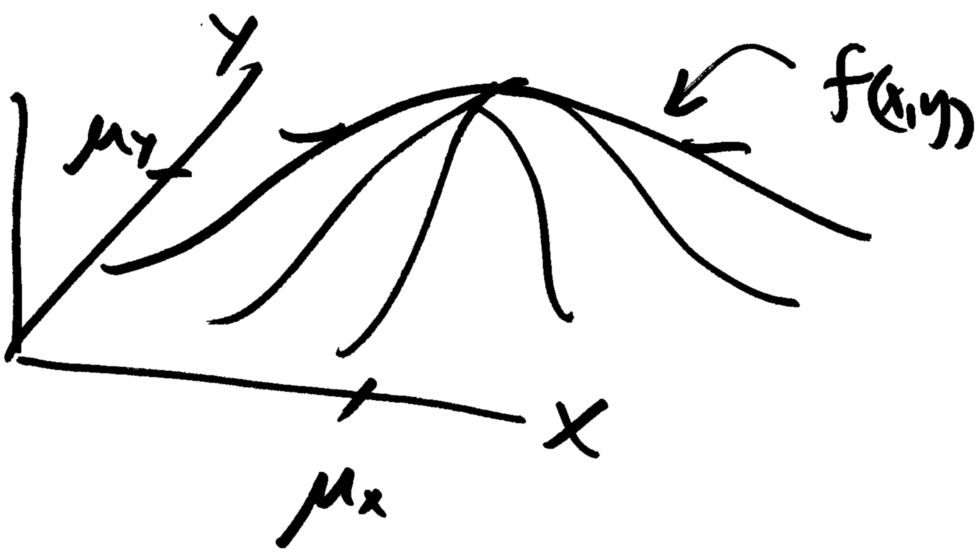
'ncf' = non-central F

'ncx2' = non-central chi-squared

11-15

MVN $\vec{X}: \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{pmatrix}$

$$f(\vec{X}) = \frac{1}{(2\pi)^{n/2}\sqrt{|\Sigma|}} \exp\left\{-\frac{1}{2} \underset{(1,n)}{(\vec{X}-\vec{\mu})^T} \underset{(n,n)}{\Sigma^{-1}} \underset{(n,1)}{(\vec{X}-\vec{\mu})}\right\}$$



2D r: correlation coefficient 11-16

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-r^2}} \cdot \exp$$

$$\left\{\frac{-1}{2(1-r^2)}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2r\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]\right\}$$

$$\Sigma = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix} = \begin{pmatrix} \sigma_x^2 & r\cdot\sigma_x\sigma_y \\ r\cdot\sigma_x\sigma_y & \sigma_y^2 \end{pmatrix}$$

$$r = \frac{\sigma_{xy}}{\sigma_x\sigma_y}$$

$$\sigma_{xy} = r\sigma_x\sigma_y$$

expectation

expected value of X, $\quad E\{x\}$

discrete RV : $E\{X\} = \sum_{i=1}^{n} x_i \, p(x_i) =$

$$x_1 \cdot p(x_1) + x_2 \cdot p(x_2) + \cdots + x_n p(x_n)$$

$$E(x) = \mu_x$$

continuous $E(x) = \int_{-\infty}^{\infty} x \cdot \underline{f(x)\, dx}$

$$E\{(X-\mu_x)^2\} = \sigma^2 \quad \text{variance} \quad \sigma_{xx}$$

$$E\{(X-\mu_x)(Y-\mu_y)\} = \sigma_{xy} \quad \text{covariance}$$