$n = 8$ what if $n_0$ too small, $r$ too large?

$n_0 = 4$

$r = 4$

$n_0 = 3$

$r = 5$

0/0

too many rows

$5 \left\{ \boxed{\quad A \quad} \right.$ $\Rightarrow$ $AQA^T$ probably be rank deficient singular, near singular

$\operatorname{rank}(A) = \#\,\text{rows}(A)$

I/0

3

too few parameters

$B^T W B$

full rank

invertible

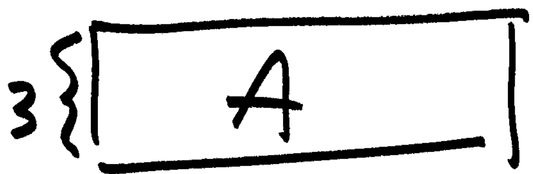
wrong answer

Consequences of incorrect counting of LS problem model elements.

$n_0$ too large $\Rightarrow$ $r$ too small

$$\frac{n_0 = 5}{r = 3}$$

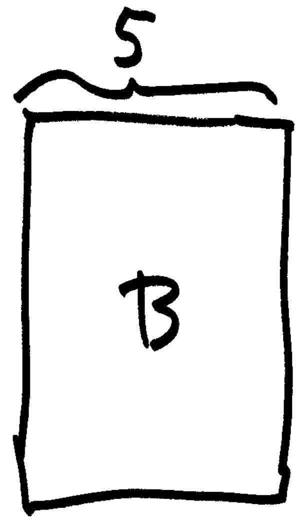
O/O




too few eqn's

$AQA^T =$
full rank
invertible
$\Rightarrow$ wrong answer

I/O




too many parameters

$B^TWB$ likely rank deficient

singular
or
nearly singular

$$A = \begin{bmatrix} 2 & 4 & 5 \\ 4 & 8 & 10 \end{bmatrix}$$ rank of A = 1

$AQA^T$

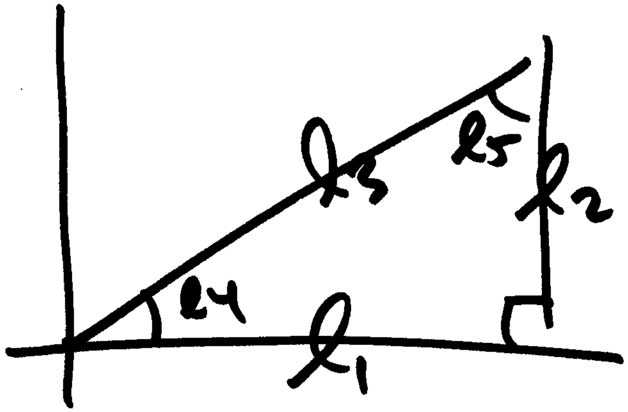
rank (A) = 1

$$A = \begin{bmatrix} 2 & 4 & 5 \\ 4 & 8 & 10.00001 \end{bmatrix}$$ rank (A) = 2

$$\left.\begin{aligned} Q_e &= AQA^T \\ N &= B^T W B \end{aligned}\right\}$$ cond (N)

avoid large

cond > $\underline{\underline{10^{12}}}$




$n = 5$
$n_0 = 2$
$r = 3$

$l_1 = 10.1$
$l_2 = 7.4$
$l_3 = 12.5$
$\Big\}\ \sigma = 0.1$

$l_4 = 36.22^\circ$
$l_5 = 53.78^\circ$
$\Big\}\ \sigma = .005R$

$\hat{l}_1^2 + \hat{l}_2^2 - \hat{l}_3^2 = 0 \qquad F_1$

$\hat{l}_4 + \hat{l}_5 - \pi/2 = 0 \qquad F_2$

$\hat{l}_4 - \tan^{-1}\left(l_2 / l_1\right) = 0 \qquad F_3$

observation only
non linear
<u>example</u>

$A = \dfrac{\partial F}{\partial l}$

$$A = \begin{bmatrix} 2l_1 & 2l_2 & -2l_3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ \frac{l_2}{l_1^2+l_2^2} & \frac{-l_1}{l_1^2+l_2^2} & 0 & 1 & 0 \end{bmatrix}$$

$(l^0)$

$l$ original observations

$l^0$ current estimate

$\underline{\underline{W}}$

$$f = \left[ \begin{bmatrix} -(l_1^2+l_2^2-l_3^2) \\ -(l_4+l_5-\pi/2) \\ -(l_4-\tan^{-1}(l_2/l_1)) \end{bmatrix}_{(l^0)} - A * (l-l^0) \right]$$

1. analytically (derivatives)
2. symbolic processing in matlab
3. numerically

3 ways to obtain derivatives

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\frac{df}{dx} \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$\Delta x$ : small number

$$\frac{\partial f}{\partial x_i} = \lim_{\Delta x_i \to 0} \frac{f(x_1, x_2, \ldots, x_i + \Delta x_i, \ldots x_n) - f(x_1, x_2, \ldots x_n)}{\Delta x_i}$$

$$\frac{\partial f}{\partial x_i} \approx \frac{f(x_1, x_2, \ldots x_i + \Delta x_i, \ldots x_n) - f(x_1, x_2, \ldots x_n)}{\Delta x_i}$$

(partial derivative approximation)