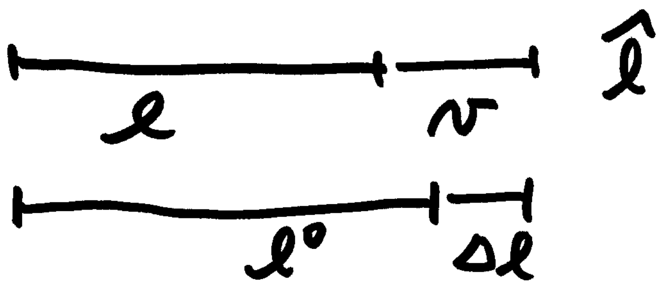
I/O nonlinear : <u>Derive</u> <u>linearized</u> <u>cond.</u> <u>eqn's.</u>

$$\hat{\ell} = G(x)$$

$$\hat{\ell} - G(x) = 0, \quad F(\hat{\ell}, x) = \hat{\ell} - G(x) = 0$$

Taylor Series Approx : $F(\hat{\ell}, x) \approx \underbrace{F(\ell^0, x^0)}_{\ell^0 - G(x^0)} + \frac{\partial F}{\partial \ell} \underbrace{\Delta \ell}_{\ell - \ell^0 + v} + \frac{\partial F}{\partial x} \Delta x = 0$



$$\ell + v = \ell^0 + \Delta \ell$$

$$\Delta \ell = \underline{\ell - \ell^0 + v}$$

$$F(\hat{\ell}, x) \approx \ell^0 - G(x^0) + I_n (\ell - \ell^0 + v) + \underbrace{\frac{\partial F}{\partial x}}_{B} \Delta x = 0$$

$$\underbrace{(l^0 - l^0)}_{0} + \underbrace{l - G(x^0)}_{F(l,\,x^0)} + v + B\Delta = 0$$

$$v + B\Delta = -F(l, x^0) \leftarrow \text{mis-closure}$$

$$\underset{(n,1)}{v} \quad \underset{(n,u)(u,1)}{B\Delta} = \underset{(n,1)}{-f}$$

$$v + B\Delta = f$$

$$\left.\begin{aligned} F_1(\hat{l}_1, x) &= \hat{l}_1 - G_1(x) = 0 \\ &\vdots \\ F_n(\hat{l}_n, x) &= \hat{l}_n - G_n(x) = 0 \end{aligned}\right\} \; n \text{ cond. eqn.}$$

$$\Delta = (B^T W B)^{-1} B^T W f$$

$$X_{i+1} = X_i + \Delta \quad , \quad \text{check } \Delta \text{ is small}$$

Obs only: $F(\hat{l}) = 0$

Taylor Series: $F(\hat{l}) \approx F(l^0) + \frac{\partial F}{\partial l} \cdot \Delta l = 0$

$$F(l^0) + \underbrace{\frac{\partial F}{\partial l}}_{= A}((l - l^0) + v) = 0$$

$$F(l^0) + A(l - l^0) + Av = 0$$

$$Av = -F(l^0) - A(l - l^0)$$

$$Av = f$$

$$Av = -F(\ell^0) - A\cdot(\ell-\ell^0)$$

$$(r,n)(n,1) = r,1 \quad - (r,n) \quad (n,1)$$

$$\boxed{Av = f}$$

ℓ: original obs.

ℓ^0: current (refined) value of obs.

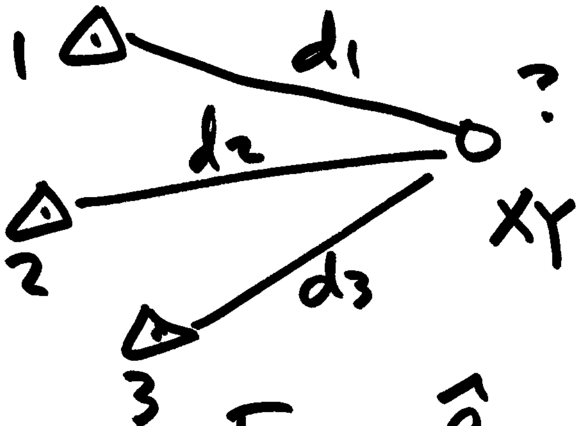
@ each iteration

$$k = W_e f$$

$$v = QA^T k$$

compute $\underline{\underline{\ell^0}} = \ell + v$

OK, we have derived ~~the~~ linearized form of cond. eqn's for both I/O & O/O, now finish prior example:



GCP's = error free

$n = 3$

$\underline{n_0 = 2}$

$r = 1$

$$F_1 = \hat{l}_1 - \left[(x-x_1)^2 + (y-y_1)^2\right]^{1/2} = 0$$

$$\vdots$$

$$v + B\Delta = f$$

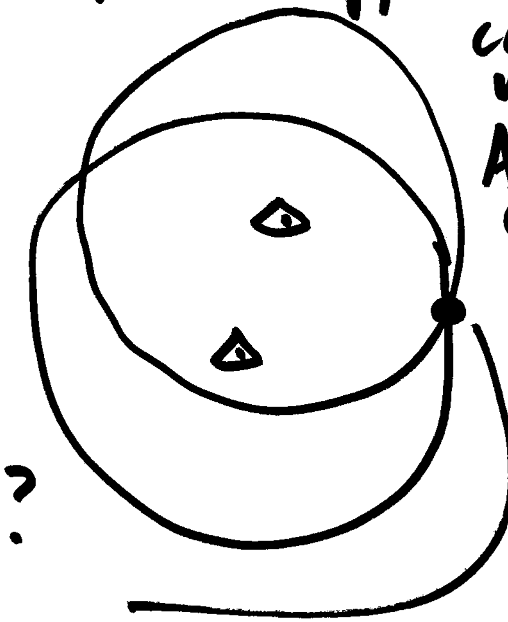
$$\frac{\partial F}{\partial x} = -\cancel{\tfrac{1}{2}}\left[(x-x_1)^2 + (y-y_1)^2\right]^{-1/2} \cdot \cancel{2}(x-x_1)$$

$$\frac{\partial F}{\partial y} = -\tfrac{1}{2}\underbrace{\left[\quad \cdot \quad\right]}_{D_1}^{-1/2} \cdot 2(y-y_1)$$

$$D_i = \left[(x-x_i)^2 + (y-y_i)^2\right]^{1/2}$$

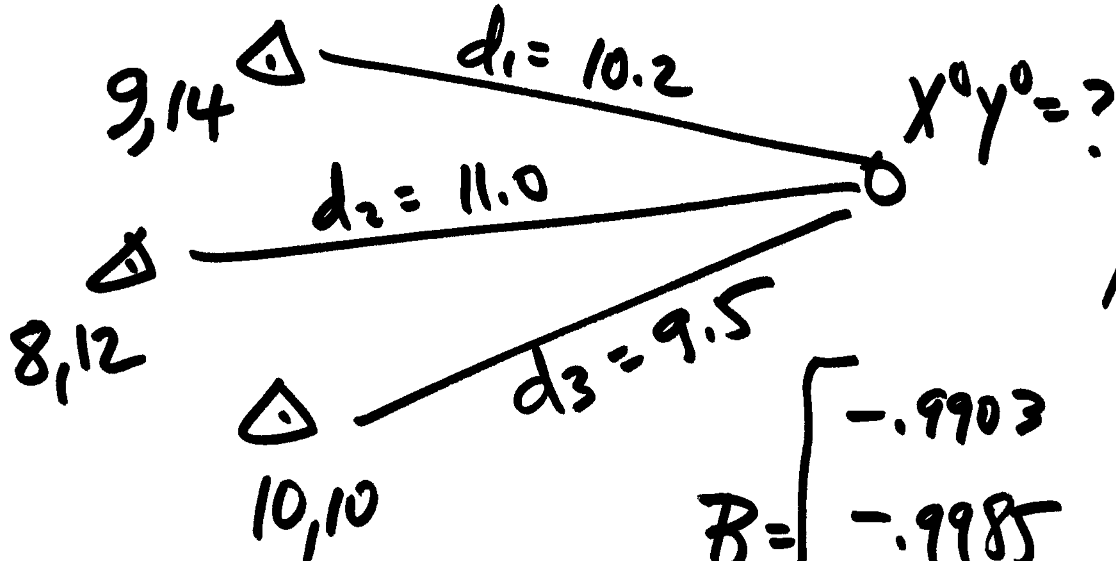
$$W = \begin{bmatrix} \sigma_0^2/\sigma_1^2 & & \\ & \sigma_0^2/\sigma_2^2 & \\ & & \sigma_0^2/\sigma_3^2 \end{bmatrix}$$

initial approximations



can use AutoCAD

19.0, 12.6



$$B = \begin{bmatrix} -.9903 & .1386 \\ -.9985 & -.0544 \\ -.9607 & -.2775 \end{bmatrix}, \quad f = \begin{pmatrix} -.1024 \\ .0163 \\ -.1319 \end{pmatrix}$$

$W = I$

$$\partial F/\partial x = -(x-x_1)/D_1 \,, \quad \partial F/\partial y = -(y-y_1)/D_1$$

$$B = \begin{bmatrix} -(x-x_1)/D_1 & -(y-y_1)/D_1 \\ -(x-x_2)/D_2 & -(y-y_2)/D_2 \\ -(x-x_3)/D_3 & -(y-y_3)/D_3 \end{bmatrix}$$

wherever you see x, y, use $\underline{\underline{x^0, y^0}}$ current approximation

$$f = \begin{bmatrix} -(d_1 - D_1) \\ -(d_2 - D_2) \\ -(d_3 - D_3) \end{bmatrix}$$

O - C observed - computed

$$\Delta = (B^T W B)^{-1} B^T W f = \begin{bmatrix} .0672 \\ .0925 \end{bmatrix}, \quad \begin{pmatrix} x^0 \\ y^0 \end{pmatrix} = \begin{matrix} 19.0672 \\ 12.6925 \end{matrix}$$

---

iteration #2

$$B = \begin{bmatrix} -.9916 & .1287 \\ -.9980 & -.0624 \\ -.9586 & -.2846 \end{bmatrix}, \quad f = \begin{pmatrix} -.0482 \\ .0888 \\ -.0414 \end{pmatrix}$$

$W = I_3$

$$\Delta = \begin{matrix} -.0004 \\ .0013 \end{matrix} \qquad \begin{pmatrix} x^0 \\ y^0 \end{pmatrix} = \begin{matrix} 19.0667 \\ 12.6939 \end{matrix}$$

---

iteration #3

$$\Delta = \begin{matrix} 1.16 \times 10^{-6} \\ 1.34 \times 10^{-5} \end{matrix}, \qquad \begin{matrix} x = 19.0667 \\ y = 12.6939 \end{matrix}$$

all(v) : true if all elements of vector
are non-zero

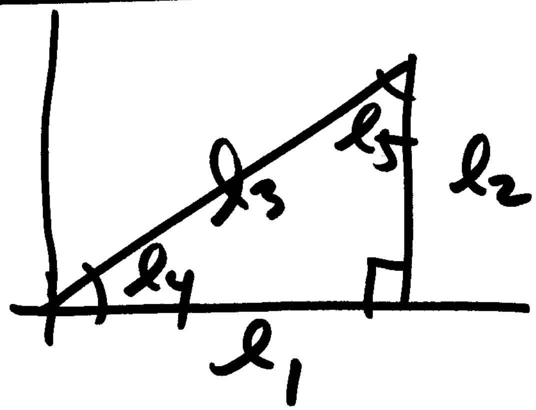
(1 = true
0 = false)

if all(abs(del) < 1e-05)

```
n_iter = 0
keep_going = 1
while (keep_going == 1)
    ...
    if all(abs(del) < 1e-05)
        keep_going = 0
    end
    n_iter = n_iter + 1
end
```

$\boxed{\text{matlab tips}}$

while((keep_going == 1) & (n_iter ≤ 10))



$n = 5$
$n_0 = 2$
$r = 3$

observations only example

$$\hat{l}_1^2 + \hat{l}_2^2 = \hat{l}_3^2$$

$$\hat{l}_4 + \hat{l}_5 = \pi/2 \; R$$

$$\hat{l}_4 = \tan^{-1}\left(\hat{l}_2 / \hat{l}_1\right)$$

for nonlinear problems always <u>radian</u> <u>units</u> for angles

$$F_1 = \hat{l}_1^2 + \hat{l}_2^2 - \hat{l}_3^2 = 0$$

$$F_2 = \hat{l}_4 + \hat{l}_5 - \pi/2 = 0$$

$$F_3 = \hat{l}_4 - \tan^{-1}(\hat{l}_2 / \hat{l}_1) = 0$$

$$Av = f$$

$$\downarrow$$

$$A = \frac{\partial F}{\partial l}$$

$$\frac{\partial F_1}{\partial l_1} = 2l_1 \Big|_{l_1^0},$$

$$f = \left[-F(l^0) - A(l - l^0)\right]$$

$$\boxed{\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\cdot\frac{du}{dx}}$$

learn it !

$$\partial F_1/\partial l_2 = 2l_2, \quad \partial F_1/\partial l_3 = -2l_3$$

$$\partial F_2/\partial l_4 = 1, \quad \partial F_2/\partial l_5 = 1$$

$$\partial F_3/\partial l_1 = -\left(\frac{1}{1+(l_2^2/l_1^2)}\right)\cdot -\frac{l_2}{l_1^2} = \frac{l_2}{l_1^2 + l_2^2}$$

$$\frac{\partial F_3}{\partial l_2} = -\frac{1}{1+\left(\frac{l_2^2}{l_1^2}\right)} \cdot \frac{1}{l_1} = \frac{-l_1}{l_1^2+l_2^2}$$

$$\frac{\partial F_3}{\partial l_4} = \underline{1}$$