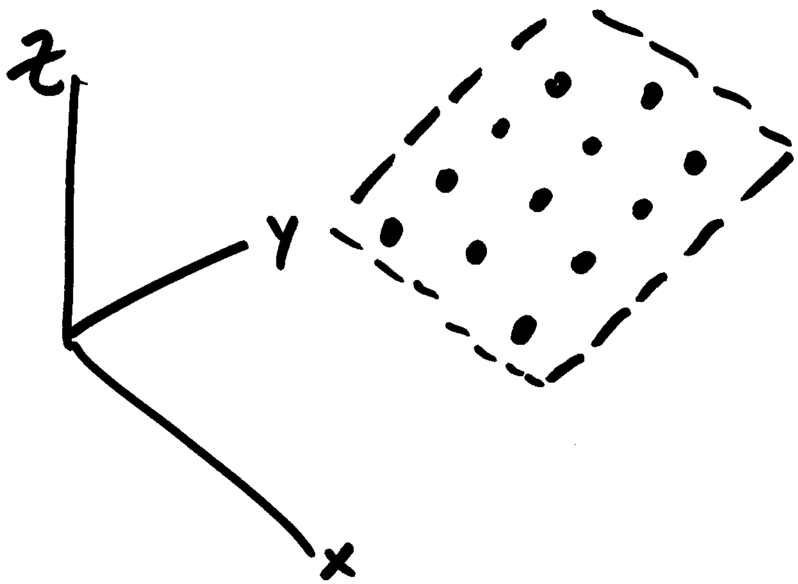
I/O Cond. Eqn $v + \underline{B}\Delta = \underline{f}$, $\underline{W}$

$$\Delta = (B^T W B)^{-1} B^T W f$$

O/O Cond. Eqn $\underline{A} v = \underline{f}$, $\underline{W}$

$$k = W_e f$$
$$v = Q A^T k$$



fit plane to set of points

x, y constant (errorless)

z observation

$$z = a_0 + a_1 x + a_2 y$$

$$z_i + v_i - a_0 - a_1 x_i - a_2 y_i = 0$$

$$\boxed{v_i - \underline{a_0} - \underline{a_1} x_i - \underline{a_2} y_i = -z_i}$$

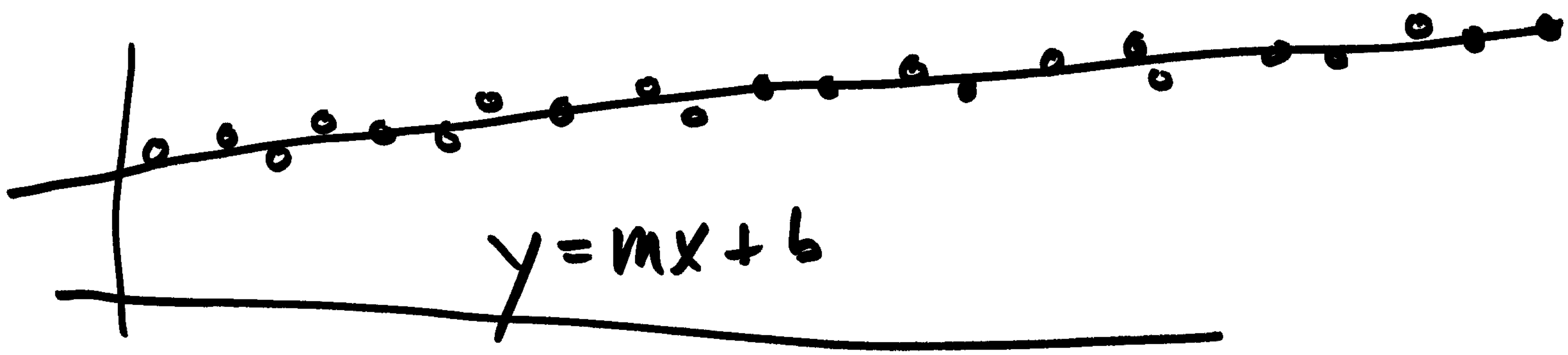
$$v + B\Delta = f$$

$$z = a_0 + a_1 x + a_2 x^2 + a_3 y + a_4 y^2 + a_5 xy$$

general 2^{nd} order surface

x, y : errorless constants
z : observation

sequential formation of normal equations



$$y_i + v_i = m x_i + b, \qquad v_i - m x_i - b = -y_i$$

$$v_1 + \begin{bmatrix} -x_1 & -1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = -y_1$$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \begin{pmatrix} -x_1 & -1 \\ -x_2 & -1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} y_1 \\ -y_2 \end{pmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{pmatrix} -x_1 & -1 \\ -x_2 & -1 \\ -x_3 & -1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} -y_1 \\ -y_2 \\ -y_3 \end{pmatrix}$$

n points:

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} + \begin{bmatrix} -x_1 & -1 \\ -x_2 & -1 \\ \vdots & \vdots \\ -x_n & -1 \end{bmatrix} \begin{pmatrix} m \\ b \end{pmatrix} = \begin{bmatrix} -y_1 \\ -y_2 \\ \vdots \\ -y_n \end{bmatrix}$$

$$\underset{(n,1)}{V} + \underset{(n,u)}{B}\ \underset{(u,1)}{\Delta} = \underset{(n,1)}{f}$$

suppose 1000 points,

$$\underset{(1000,2)}{B} \quad \underset{(1000,1)}{f} \quad \underset{(1000,1000)}{W}$$

assume W is <u>diagonal</u>

$n=3$, B^TWB

$$\begin{bmatrix} -x_1 & -x_2 & -x_3 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} w_1 & & \\ & w_2 & \\ & & w_3 \end{bmatrix} \begin{bmatrix} -x_1 & -1 \\ -x_2 & -1 \\ -x_3 & -1 \end{bmatrix} = N$$

$$\begin{bmatrix} -w_1x_1 & -w_2x_2 & -w_3x_3 \\ -w_1 & -w_2 & -w_3 \end{bmatrix} \begin{bmatrix} -x_1 & -1 \\ -x_2 & -1 \\ -x_3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} w_1x_1^2 + w_2x_2^2 + w_3x_3^2 & w_1x_1 + w_2x_2 + w_3x_3 \\ w_1x_1 + w_2x_2 + w_3x_3 & w_1 + w_2 + w_3 \end{bmatrix}$$

$$B^TWf \quad \begin{pmatrix} -x_1 & -x_2 & -x_3 \\ -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} w_1 & & \\ & w_2 & \\ & & w_3 \end{pmatrix} \begin{pmatrix} -y_1 \\ -y_2 \\ -y_3 \end{pmatrix}$$

$$\begin{bmatrix} -w_1x_1 & -w_2x_2 & -w_3x_3 \\ -w_1 & -w_2 & -w_3 \end{pmatrix} \begin{pmatrix} -y_1 \\ -y_2 \\ -y_3 \end{pmatrix} = t$$

$$\begin{pmatrix} \underline{w_1 x_1 y_1} + \underline{\underline{w_2 x_2 y_2}} + \underline{\underline{w_3 x_3 y_3}} \\ \underline{w_1 y_1} + \underline{\underline{w_2 y_2}} + \underline{\underline{w_3 y_3}} \end{pmatrix}$$

for N & t form directly by Accumulation

b_i is i^{th} row of B

f_i is i^{th} element of f

w_i is i^{th} element of W (w_{ii})

$$N = \sum_{i=1}^{n} b_i^T w_i b_i$$

$$t = \sum_{i=1}^{n} b_i^T w_i f_i$$

when done $N\Delta = t$, $\Delta = N^{-1} t$

$v_i = f_i - b_i \Delta$

an equation is linear in a variable if that variable appears with, at most, a scalar multiplier.
(constant)

$$y = 5a + 3x, \quad y - 5a - 3x = 0$$

linear in y, a, x

$y = ax$ if both a & x are variables
not linear a, or x
if a constant, then linear in X
if X constant, then linear in a

$y = a \cdot \cos\theta$ a, θ both variable, then nonlinear in a & θ

if θ is constant, linear in a

if a constant, nonlinear in θ

$y = ax^2$ if a, x both variable, not linear in either

if x constant, x^2 const, linear in a

if a constant, not linear in x

$y = a_0 + a_1\underline{\underline{x}} + a_2\underline{\underline{x}}^2$, if x constant then linear in a_0, a_1, a_2

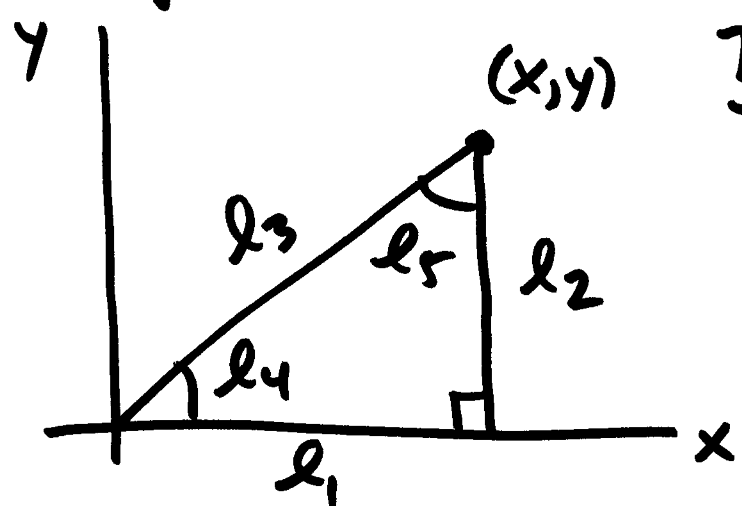
variables: observations

parameters

$$d_{12} = \left[(x_1 - x_2)^2 + (x_1 - y_2)^2\right]^{1/2}$$

$$d_{12} - \left[(x_1 - x_2)^2 + (y_1 - y_2)^2\right]^{1/2} = 0$$

<u>example</u>



$$n = 5$$
$$\underline{n_0 = 2}$$
$$r = 3$$

0/0

$c = 3$

1. $\hat{l}_1^{\,2} + \hat{l}_2^{\,2} = \hat{l}_3^{\,2}$, $\hat{l}_3 = \left[\hat{l}_1^{\,2} + \hat{l}_2^{\,2}\right]^{1/2}$
2. $\hat{l}_4 + \hat{l}_5 = 90^\circ$ or $\pi/2\ R$
3. $\hat{l}_4 = \tan^{-1}(\hat{l}_2 / \hat{l}_1)$

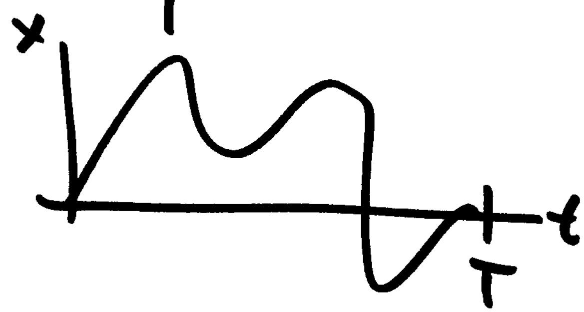
I/O $c = n = 5$

x, y params.

$$\hat{\ell}_1 = x$$
$$\hat{\ell}_2 = y$$
$$\hat{\ell}_3 = [x^2 + y^2]^{1/2}$$
$$\hat{\ell}_4 = \tan^{-1}(y/x)$$
$$\hat{\ell}_5 = \tan^{-1}(x/y)$$

linear problems

- level networks
- curve fit, surface fit (Regression)
- ANOVA $\quad Y_i = \beta_0 + \beta_1 \underline{X_{i1}} + \beta_2 \underline{X_{i2}} + \beta_3 \underline{X_{i3}} + \epsilon_i$

  X: indicator var's $\quad 0, 1$
- coordinate transform $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix}$
- simple angle figures
- simple linear networks
- 

$$x(t) = a_0 + a_1 \cos \frac{2\pi}{T} t + b_1 \sin \frac{2\pi}{T} t + \dots$$

most others are <u>non linear</u>

photogrammetry : resection,
intersection
BBA

plane surveying : triang., trilat, traverse

GPS : pseudo ranges, phase

LIDAR : point cloud merging
fitting geom. primitives