$$\begin{bmatrix} W & -A^T \\ -A & 0 \end{bmatrix} \begin{bmatrix} v \\ k \end{bmatrix} = \begin{bmatrix} 0 \\ -f \end{bmatrix}$$

full $\begin{bmatrix} v \\ k \end{bmatrix}$

Full

$$Wv - A^T k = 0 \longrightarrow \quad Wv = A^T k$$

$$-Av = -f \qquad v = W^{-1} A^T k$$

$$Q = W^{-1}$$

substitute into 2nd equ

$$Av = f$$

$$\underbrace{AQA^T}_{Qe} k = f$$

partition

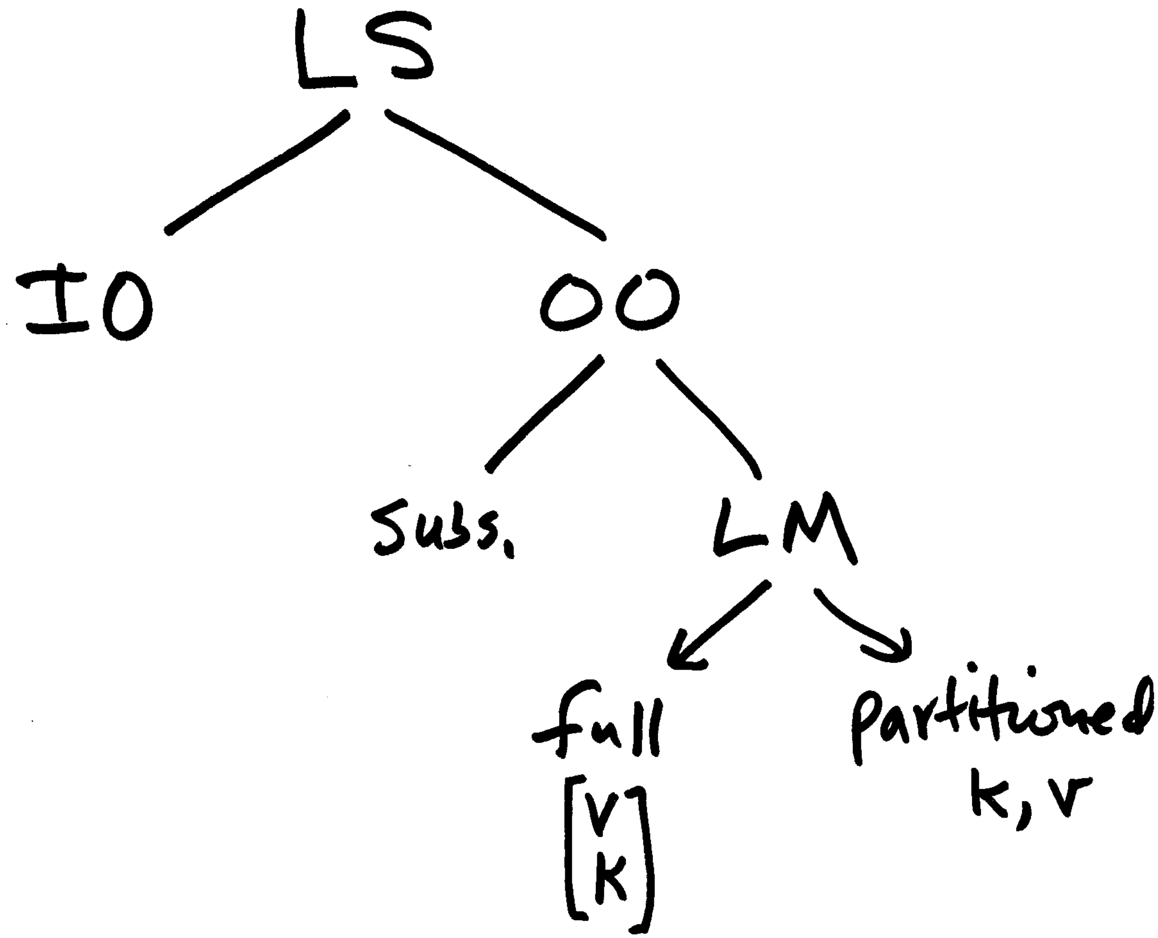
$$\boxed{v = QA^T k}$$

$$\boxed{k = We\, f}$$

$$Qe\, k = f$$

$$k = Qe^{-1} f, \quad We = Qe^{-1}$$

full Normal equations vs. partition solution by block gauss elim.



Least Squares solution options

IO = indirect observations

OO = observations only

LM = lagrange multipliers

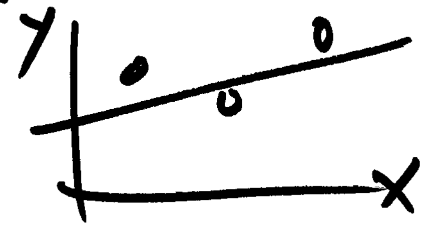
observation only – matrix approach – steps

1. analyze problem
2. form condition equations

$$A v = f$$

3. get weights, W matrix
4. A, f, W $\Rightarrow$ solve for v, k
5. $k = W_e f$, $v = Q A^T k$
6. $\hat{l} = l + v$
7. evaluate v's:
   are they reasonable?

example for IO, by matrix approach
data from Ex 3-5, 4-1, 4-6



x	y
2	3.2
4	4.0
6	5.0

$W = I_3$

$n = 3$
$n_0 = 2 \longrightarrow m, b$
$r = 1$ n equations

$y_i + v_i = m x_i + b$

$v + B\Delta = f$ — desired form for equations

$v_i - m x_i - b = -y_i$

$$\begin{bmatrix}\cancel{v_1}\\ \cancel{v_2}\\ \cancel{v_3}\end{bmatrix}$$

$v_1 - 2m - b = -3.2$
$v_2 - 4m - b = -4.0$
$v_3 - 6m - b = -5.0$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} -2 & -1 \\ -4 & -1 \\ -6 & -1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} -3.2 \\ -4.0 \\ -5.0 \end{bmatrix}$$

$$W = I_3$$

$$W = eye(3)$$

$$v + B \Delta = f$$

$$B, f, W$$

$$\underbrace{B^T W B}\Delta = \underbrace{B^T W f}$$

$$N \Delta = t$$

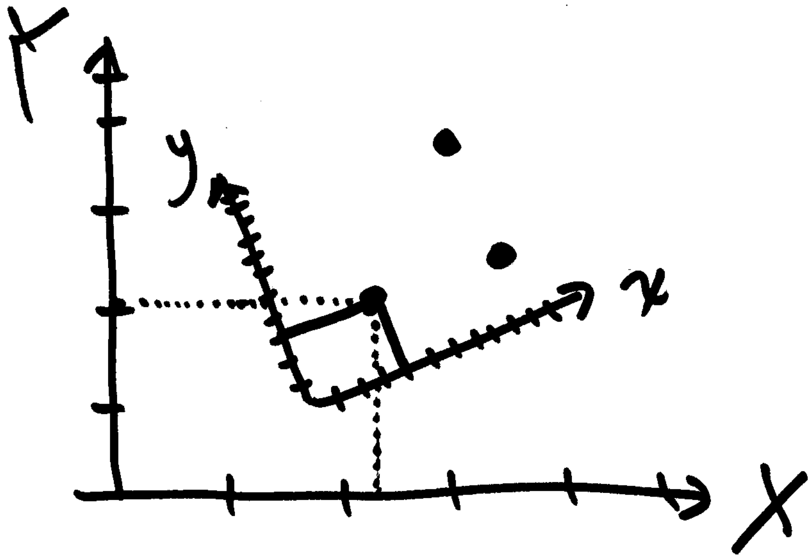
$$\Delta = N^{-1} t$$

$$\Delta = \begin{bmatrix} 0.45 \\ 2.667 \end{bmatrix}, \quad v = f - B\Delta = \begin{bmatrix} -.0333 \\ .0667 \\ -.0333 \end{bmatrix}$$

$$\hat{l} = l + v$$

4 parameter coordinate transformation

~~conf~~ 2D conformal coord. transf.



4 parameters

shift X, Y

rotation

scale

$$\begin{pmatrix} x \\ y \end{pmatrix} = S \cdot \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

S

θ

t_x, t_y

problem $S \cdot \cos\theta$ is <u>nonlinear</u>

linear : <u>addition</u> + <u>scalar multiplication</u>

$$\left.\begin{aligned} s\cdot\cos\theta &= a \\ s\cdot\sin\theta &= b \\ t_x &= c \\ t_y &= d \end{aligned}\right\}\quad \underset{\substack{\uparrow \\ \text{observations}}}{\begin{pmatrix} x \\ y \end{pmatrix}} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \underset{\substack{\uparrow \\ \text{constants}}}{\begin{pmatrix} X \\ Y \end{pmatrix}} + \begin{pmatrix} c \\ d \end{pmatrix}$$

analysis of problem :

3 points $\Rightarrow$ 6 equations, 6 observations

$$\begin{aligned} n &= 6 \\ n_0 &= 4 \quad \text{4 parameter } a, b, c, d \\ \hline r &= 2 \end{aligned}$$

a, b, c, d = linear parameters

s, θ, t_x, t_y = non linear parameters

#	x	y	X	Y	$\sigma_{obs.}$
1.	2.35	3.00	1.0	1.0	.03
2.	3.60	2.70	2.0	1.0	.06
3.	3.80	3.90	2.0	2.0	.03

---

form: $V + B\Delta = f$

$$x_i = aX_i + bY_i + c$$
$$y_i = -bX_i + aY_i + d$$

$$x_i + V_{x_i} - aX_i - bY_i - c = 0$$
$$y_i + V_{y_i} + bX_i - aY_i - d = 0$$

$$V_{x_i} - aX_i - bY_i - c = -x_i$$
$$V_{y_i} + bX_i - aY_i - d = -y_i$$

$$\begin{bmatrix} V_{x_i} \\ V_{y_i} \end{bmatrix} + \begin{bmatrix} -X_i & -Y_i & -1 & 0 \\ -Y_i & X_i & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -x_i \\ -y_i \end{bmatrix}$$

$$\begin{bmatrix} V_{x_1} \\ V_{y_1} \\ V_{x_2} \\ V_{y_2} \\ V_{x_3} \\ V_{y_3} \end{bmatrix} + \begin{bmatrix} -1 & -1 & -1 & 0 \\ -1 & 1 & 0 & -1 \\ -2 & -1 & -1 & 0 \\ -1 & 2 & 0 & -1 \\ -2 & -2 & -1 & 0 \\ -2 & 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -2.35 \\ -3.00 \\ -3.60 \\ -2.70 \\ -3.8 \\ -3.9 \end{bmatrix}$$

$$\underset{(n,1)}{V} + \underset{(n,u)}{B} \; \underset{(u,1)}{\Delta} = \underset{(n,1)}{f}$$

Weights & weight matrix

$\sigma_{x_1}, \sigma_{y_1}, \sigma_{x_3}, \sigma_{y_3} = .03$, $\sigma^2 = .0009$

$\sigma_{x_2}, \sigma_{y_2} = .06$, $\sigma^2 = .0036$

$w_i = \dfrac{\sigma_0^2}{\sigma_i^2}$, $\sigma_0^2 = .0009$ choose

choice is arbitrary

$$w_{x_1 y_1 x_3 y_3} = \frac{.0009}{.0009} = 1$$

$$w_{x_2, y_2} = \frac{.0009}{.0036} = 0.25$$

$$W = \begin{bmatrix} 1 & & & & & \\ & 1 & & & \phi & \\ & & .125 & & & \\ & & & .125 & & \\ & \phi & & & 1 & \\ & & & & & 1 \end{bmatrix}$$

have $B, f, W \Rightarrow$ numerical solution

$$\Delta = (B^T W B)^{-1} B^T W f = \begin{bmatrix} 1.7850 \\ .2700 \\ .9000 \\ 2.0750 \end{bmatrix}, \quad v = f - B\Delta = \begin{bmatrix} .005 \\ -.010 \\ -.060 \\ .020 \\ .010 \\ .005 \end{bmatrix}$$