$$y = f(x)$$

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix} = \begin{pmatrix} f_1(x_1, x_2 \cdots x_n) \\ f_2(x_1, x_2, \cdots, x_n) \\ \vdots \\ f_m(x_1, x_2 \cdots x_n) \end{pmatrix},$$

need Jacobian

$$J_{yx}$$

$$\frac{\partial y}{\partial x}$$

$$\frac{\partial y}{\partial x} = \begin{bmatrix} \partial y_1/\partial x_1 & \partial y_1/\partial x_2 & \cdots & \partial y_1/\partial x_n \\ \partial y_2/\partial x_1 & \partial y_2/\partial x_2 & \cdots & \partial y_2/\partial x_n \\ \vdots & & & \\ \partial y_m/\partial x_1 & \partial y_m/\partial x_2 & \cdots & \partial y_m/\partial x_n \end{bmatrix} \quad m, n$$

differentiation with respect to vector

$$a = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \qquad x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \qquad u = a^T x$$

$$\frac{\partial}{\partial x}(a^T x) = a^T, \qquad \frac{\partial}{\partial x}(Ax) = A$$

(vector x)

$$\frac{d}{dx}(u^T v) \qquad u = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}, \quad v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$$

$$\underline{\frac{\partial}{\partial x}(u^T v) = u^T \frac{\partial v}{\partial x} + v^T \frac{\partial u}{\partial x}}$$

$$\frac{\partial}{\partial x}(x^T A y) = y^T A^T$$

A: matrix of constants

$y^T A^T x$

(bilinear forms)

$$\frac{\partial}{\partial y}(x^T A y) = x^T A$$

$$\frac{\partial}{\partial x}(x^T A x) = 2x^T A$$

quadratic form

A: symmetric

$$\phi = v_1^2 + v_2^2 + \cdots + v_n^2 = \sum_{i=1}^{n} v_i^2$$

$$\phi = w_1 v_1^2 + w_2 v_2^2 + \cdots + w_n v_n^2 = \sum w_i v_i^2$$

$$W = \begin{bmatrix} w_1 & & & \phi \\ & w_2 & & \\ & & \ddots & \\ \phi & & & w_n \end{bmatrix}$$

W : symmetric even when full

$$\phi = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \begin{bmatrix} w_1 & & & \\ & w_2 & & \\ & & \ddots & \\ & & & w_n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

$$\boxed{\phi = v^T W v}$$

$w_i \sim \frac{1}{\sigma_i^2}$, $w_i = \frac{K}{\sigma_i^2}$

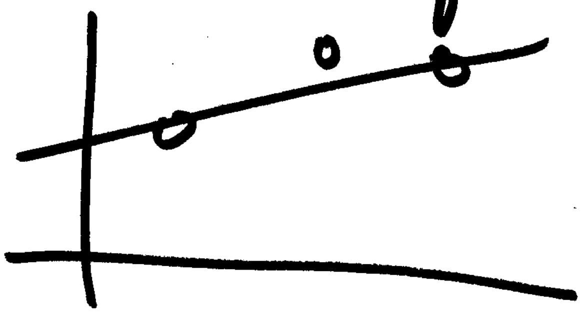
$$w_i = \frac{\sigma_0^2}{\sigma_i^2}$$

σ_0^2 you choose :

$\hat{\sigma}_0$ = allways = 1

$\sigma_0^2 = \sigma_i^2$ of common observation indirect observation:

condition equations in matrix form



$$y_i + v_i = m x_i + b$$

$$v_i - m x_i - b = -y_i$$

$$v_1 - x_1 m - b = -y_1$$

$$v_2 - x_2 m - b = -y_2$$

$$v_3 - x_3 m - b = -y_3$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} -x_1 & -1 \\ -x_2 & -1 \\ -x_3 & -1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} -y_1 \\ -y_2 \\ -y_3 \end{bmatrix}$$

$$V + B \Delta = f$$

$u = n_0 = \#$ params

$$\underline{\underline{V + B\Delta = f}}$$

$$\underset{n,1}{V} + \underset{n,u}{B}\ \underset{u,1}{\Delta} = \underset{n,1}{f}$$

matrix derivation, indirect obs. method

$\phi = V^T W V$ obj. function

$V + B\Delta = f$ cond. eqn's.

$V = \underline{f - B\Delta}$ substitute into ϕ

$$\phi = (f - B\Delta)^T W (f - B\Delta)$$

$$= (f^T - \Delta^T B^T) W (f - B\Delta)$$

$$= (f^T W - \Delta^T B^T W)(f - B\Delta)$$

$$\phi = f^T W f - f^T W B\Delta - \underline{\Delta^T B^T W f} + \Delta^T B^T W B \Delta$$

transp: $f^T W B \Delta$

$$\frac{\partial}{\partial x}(x^T A x) = 2x^T A$$

$$\underset{1,1}{\phi} = \underbrace{\underset{1,n}{V^T}\ \underset{n,n}{W}\ \underset{n,1}{V}}$$

$$\phi = f^T W f \underbrace{- f^T W B \Delta - f^T W B \Delta} + \Delta^T B^T W B \Delta$$

$$\phi = f^T W f - 2 f^T W B \Delta + \underbrace{\Delta^T \underline{B^T W B} \Delta}_{\text{quadratic form}}$$

minimize w.r.t. Δ

$$\frac{\partial \phi}{\partial \Delta} = -\cancel{2}\underset{1,n}{f^T}\ \underset{n,n}{W}\ \underset{n,n}{B} + \cancel{2}\Delta^T B^T W B = \underset{1,n}{0}$$

$$-B^T W f + B^T W B \Delta = \underset{n,1}{0}$$

$$\underbrace{B^T W B}_{N} \Delta = \underbrace{B^T W f}_{t}\ ,$$ normal equations

$N\Delta = t$, $\boxed{\Delta = N^{-1} t}$

parameter vector $\Delta = (B^T W B)^{-1} B^T W f$

residual vector $V = f - B\Delta$

adj. obs. vector $\hat{l} = l + V$

Steps: analyze problem

form n cond. eqn $V + \underline{\underline{B}}\Delta = \underline{\underline{f}}$

get weights W

$\Delta = (B^T W B)^{-1} B^T W f$

$V = f - B\Delta$

$\hat{l} = l + V$

Observation only – matrix derivation

$c = r$

$$\underset{c,n}{A}\ \underset{n,1}{v} = \underset{c,1}{f}$$

$$\begin{pmatrix} \lambda : LM \\ K : LM \end{pmatrix}$$

$$\Phi = v^T W v$$

$$\Phi' = v^T W v - 2K^T(Av - f)$$

2 or $\underline{1}$

+/-

$$\Phi' = v^T W v - 2K^T A v + 2K^T f$$

$$\to -2v^T A^T K + 2f^T K$$

diff w.r.t v, K

$$\frac{\partial \phi'}{\partial v} = \cancel{2}v^T W - \cancel{2}k^T A = 0.$$

$$\frac{\partial \phi'}{\partial k} = -\cancel{2}v^T A^T + \cancel{2}f^T = 0$$

$$Wv - A^T k = 0$$

$$-Av + f = 0, \quad -Av = -f$$

$$\begin{bmatrix} W & -A^T \\ -A & 0 \end{bmatrix}\begin{bmatrix} v \\ k \end{bmatrix} = \begin{bmatrix} 0 \\ -f \end{bmatrix}$$

full normal equations
Obs. only method.