$v_1 = .025$ $\quad \hat{\ell}_1 = 100 + .025 = 100.025$

$v_2 = -.010$ $\quad \hat{\ell}_2 = 100 - .010 = 99.99$

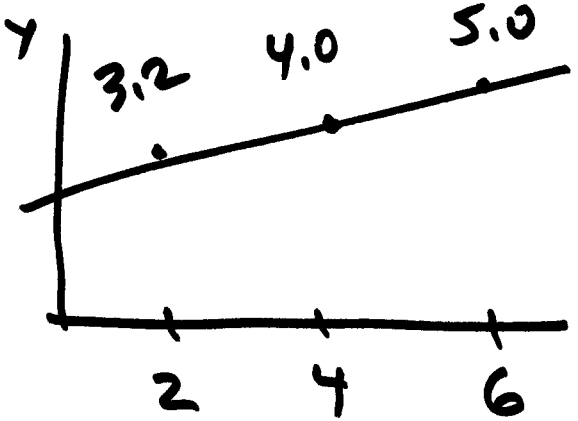
$v_3 = -.035$ $\quad \hat{\ell}_3 = 100.08 - .035 = 100.045$

$v_4 = -.025$ $\quad \hat{\ell}_4 = 200.04 - .025 = 200.015$

$v_5 = .035$ $\quad \hat{\ell}_5 = 200 + .035 = 200.035$

$\hat{\ell}_i = \ell_i + v_i$ $\quad$ ← adjusted obs.

Ex 3-5



fixed

y: obs, x: fixed

$n = 3$

$n_0 = 2$

$r = 1$

indirect observation method

parameters (n_0) : slope, intercept m, b

$Y_1 + V_1 = m X_1 + b$ $\quad 3.2 + V_1 = m \cdot 2 + b$

$Y_2 + V_2 = m X_2 + b$ $\quad 4.0 + V_2 = m \cdot 4 + b$

$Y_3 + V_3 = m X_3 + b$ $\quad 5.0 + V_3 = m \cdot 6 + b$

$$V_1 = \underline{m}\cdot 2 + \underline{b} - 3.2$$
$$V_2 = \underline{m}\cdot 4 + \underline{b} - 4.0$$
$$V_3 = \underline{m}\cdot 6 + \underline{b} - 5.0$$

$$\Phi = V_1^2 + V_2^2 + V_3^2$$

LS objective function

~~$\Phi = (2m+b-3.2)^2$~~

$$\Phi = (2m+b-3.2)^2 + (4m+b-4.0)^2 + (6m+b-5.0)^2$$

$$\partial\Phi/\partial m = 2(2m+b-3.2)2 + 2(4m+b-4.0)4 + 2(6m+b-5.0)6 = 0$$

$$\partial\Phi/\partial b = 2(2m+b-3.2) + 2(4m+b-4.0) + 2(6m+b-5.0) = 0$$

$$56m + 12b = 52.4$$
$$12m + 3b = 12.2$$

$$\begin{bmatrix} 56 & 12 \\ 12 & 3 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 52.4 \\ 12.2 \end{bmatrix}$$

$$\begin{pmatrix} m \\ b \end{pmatrix} = \begin{bmatrix} .45 \\ 2.2667 \end{bmatrix}$$

make sure you can solve 2x2 linear equations by hand

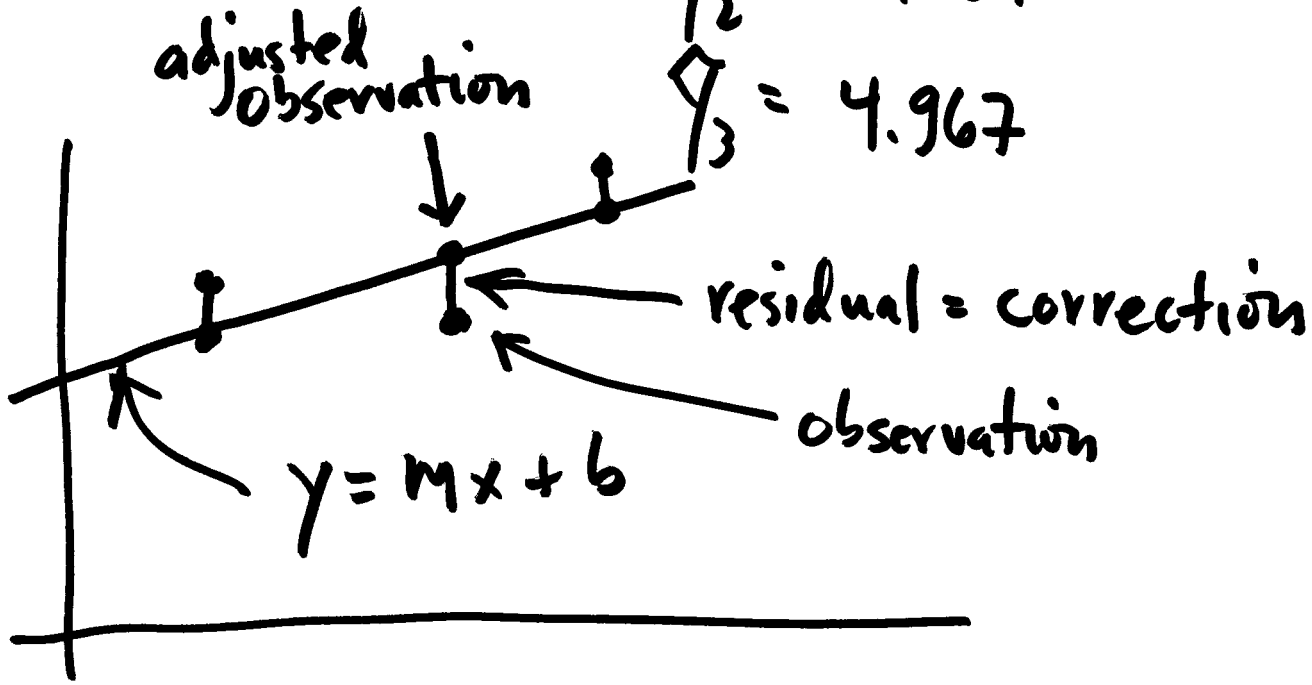
solve 2x2 :
- gauss elimination
- cramer's rule
- inverse of 2x2

$V_1 = -.033$
$V_2 = .067$
$V_3 = -.033$

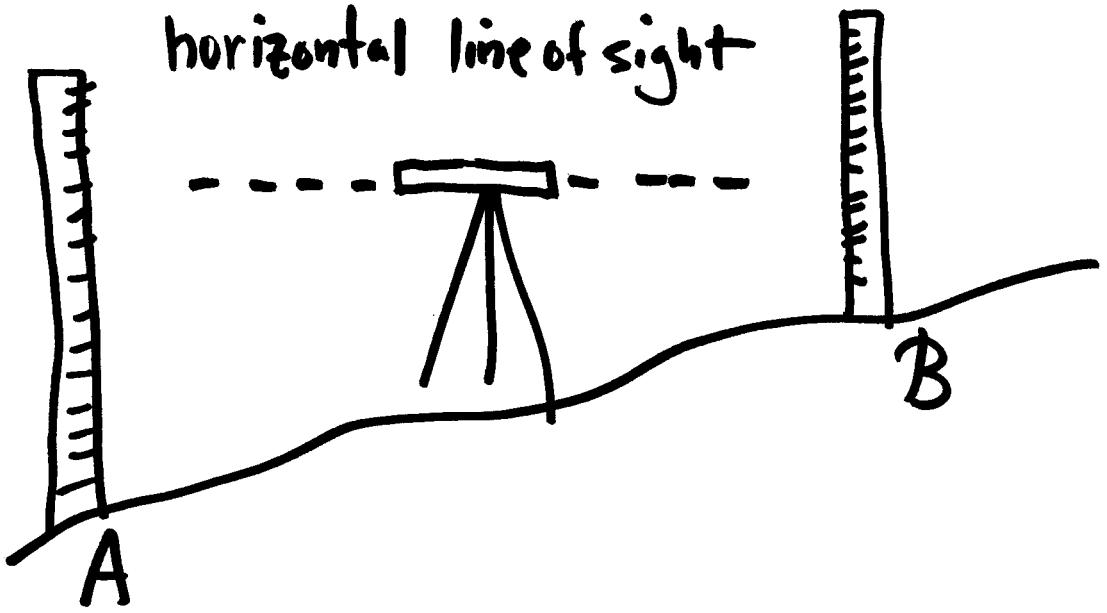
Compare V_i with σ_i

$\hat{Y}_i = Y_i + V_i$

$\hat{Y}_1 = 3.167$
$\hat{Y}_2 = 4.067$
$\hat{Y}_3 = 4.967$



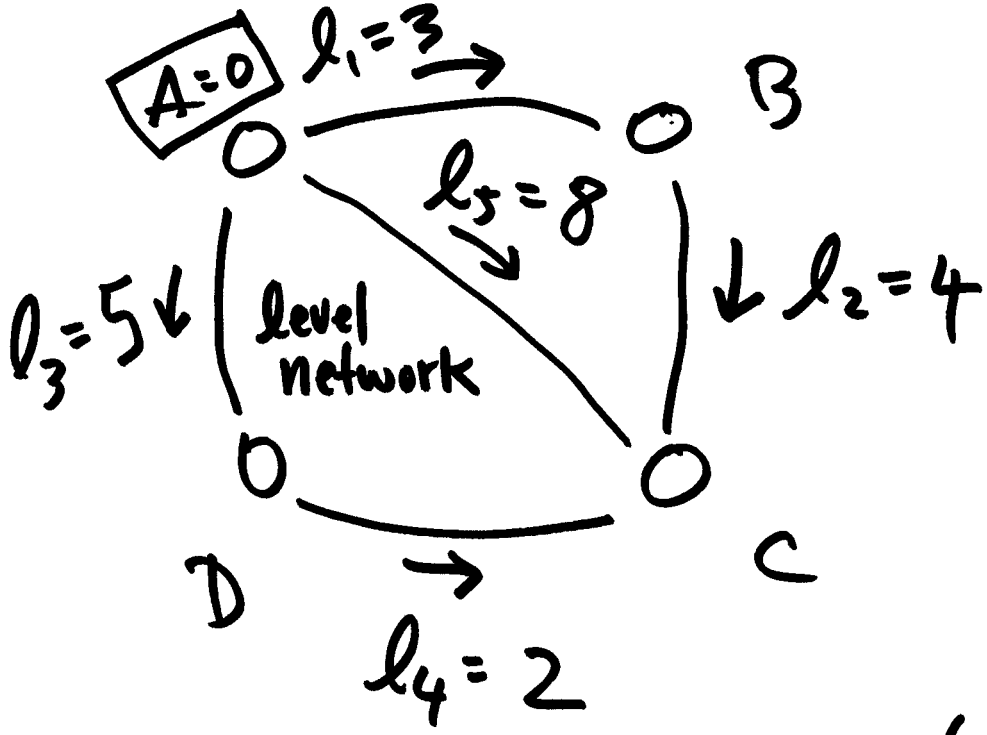
Leveling



difference in elevation between A & B =

rod reading @ A − rod reading @ B

note: "horizontal" means tangent to equipotential surface, not ellipsoid surface



→ = uphill

$n = 5$

$n_0 = 3$

$r = 2$

Choose $n_0 = 3$ parameters B, C, D

$l_1 + V_1 = B$ $\quad V_1 = B - 3$

$l_2 + V_2 = C - B$ $\quad V_2 = C - B - 4$

$l_3 + V_3 = D$ $\quad V_3 = D - 5$

$l_4 + V_4 = C - D$ $\quad V_4 = C - D - 2$

$l_5 + V_5 = C$ $\quad V_5 = C - 8$

$$\Phi = \sum V_i^2$$

$$\Phi = (B-3)^2 + (C-B-4)^2 + (D-5)^2 + (C-D-2)^2 + (C-8)^2$$

$$\frac{\partial \Phi}{\partial B} = \tfrac{1}{2}(B-3) + \tfrac{1}{2}(C-B-4)(-1) = 0$$

$$\frac{\partial \Phi}{\partial C} = \tfrac{1}{2}(C-B-4) + \tfrac{1}{2}(C-D-2) + \tfrac{1}{2}(C-8) = 0$$

$$\frac{\partial \Phi}{\partial D} = \tfrac{1}{2}(D-5) + \tfrac{1}{2}(C-D-2)(-1) = 0$$

$$2B - C = -1$$
$$-B + 3C - D = 14$$
$$-C + 2D = 3$$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} B \\ C \\ D \end{bmatrix} = \begin{bmatrix} -1 \\ 14 \\ 3 \end{bmatrix}$$

Symmetric

$$\begin{pmatrix} B \\ C \\ D \end{pmatrix} = \begin{bmatrix} 3.25 \\ 7.5 \\ 5.25 \end{bmatrix}$$

parameter values

$V_1 = .25$

$V_2 = .25$

$V_3 = .25$

$V_4 = .25$

$V_5 = -.5$

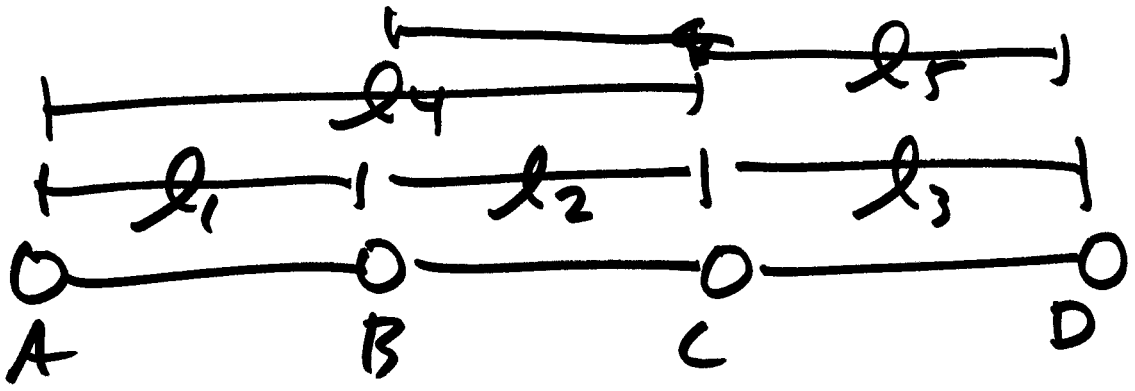
$\hat{\ell}_1 = 3.25$

$\hat{\ell}_2 = 4.25$

$\hat{\ell}_3 = 5.25$

$\hat{\ell}_4 = 2.25$

$\hat{\ell}_5 = 7.5$



$n = 5$

$n_0 = 3$

$r = 2$

Observations only : need r conditions eqns

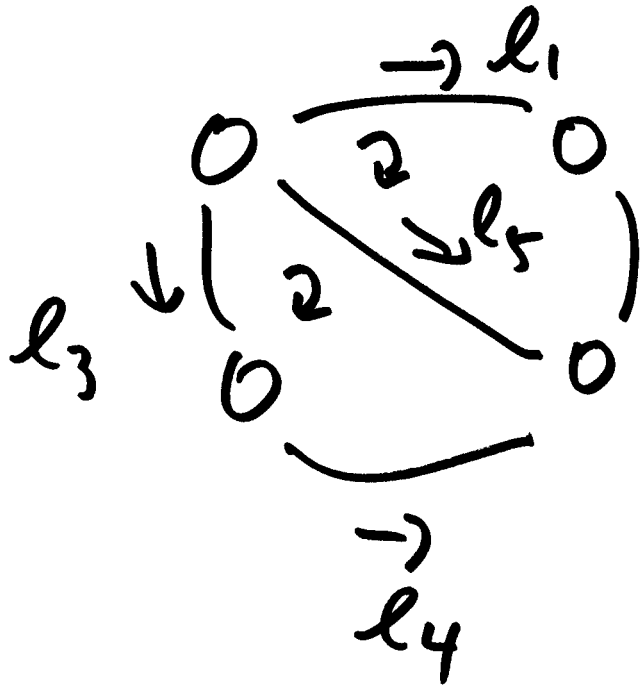
$\hat{\ell}_1 + \hat{\ell}_2 = \hat{\ell}_4$ ✓

$\hat{\ell}_2 + \hat{\ell}_3 = \hat{\ell}_5$

$\hat{\ell}_1 + \hat{\ell}_5 = \hat{\ell}_3 + \hat{\ell}_4$

$\hat{\ell}_2 + \hat{\ell}_1 = \hat{\ell}_4$ ✓

LS method #2



$n = 5$

$n_0 = 3$

$r = 2$ ←

$r = 2$ cond. eqn's for obs. only

$$\hat{\ell}_1 + \hat{\ell}_2 - \hat{\ell}_5 = 0$$
$$\hat{\ell}_5 - \hat{\ell}_4 - \hat{\ell}_3 = 0$$ } ✓

$$\hat{\ell}_1 + \hat{\ell}_2 - \hat{\ell}_4 - \hat{\ell}_3 = 0$$

only 2 equations can be independent