# Linearization of Coplanarity Equation

$$\vec{a}_1 = M_1^T \begin{bmatrix} x_1 - x_0 \\ y_1 - y_0 \\ -f \end{bmatrix} \qquad \vec{a}_2 = M_2^T \begin{bmatrix} x_2 - x_0 \\ y_2 - y_0 \\ -f \end{bmatrix} \qquad F = \begin{vmatrix} b_x & b_y & b_z \\ a_{1x} & a_{1y} & a_{1z} \\ a_{2x} & a_{2y} & a_{2z} \end{vmatrix}$$

unknowns are $\omega, \varphi, k$ of $M_2$, $b_y$ & $b_z$. observations are $x_1, y_1, x_2, y_2$

$$\frac{\partial F}{\partial x_1} = \begin{vmatrix} b_x & b_y & b_z \\ \frac{\partial}{\partial x_1}[a_{1x} & a_{1y} & a_{1z}] \\ a_{2x} & a_{2y} & a_{2z} \end{vmatrix}$$

$\frac{\partial}{\partial x_1} \vec{a}_1 =$ 1st column of $M_1^T$

$\frac{\partial}{\partial x_1} (\vec{a}_1)^T =$ 1st row of $M_1$

$$\frac{\partial F}{\partial x_1} = \begin{vmatrix} b_x & b_y & b_z \\ m_{1_{11}} & m_{1_{12}} & m_{1_{13}} \\ a_{2x} & a_{2y} & a_{2z} \end{vmatrix}$$

$$\frac{\partial F}{\partial y_1} = \begin{vmatrix} b_x & b_y & b_z \\ \frac{\partial}{\partial y_1}[a_{1x} & a_{1y} & a_{1z}] \\ a_{2x} & a_{2y} & a_{2z} \end{vmatrix}$$

$\frac{\partial}{\partial y_1} \vec{a}_1 =$ 2nd column of $M_1^T$

$\frac{\partial}{\partial y_1} (\vec{a}_1)^T =$ 2nd row of $M_1$

$$\frac{\partial F}{\partial y_1} = \begin{vmatrix} b_x & b_y & b_z \\ m_{1_{21}} & m_{1_{22}} & m_{1_{23}} \\ a_{2x} & a_{2y} & a_{2z} \end{vmatrix}$$

$$\frac{\partial F}{\partial x_2} = \begin{vmatrix} b_x & b_y & b_z \\ a_{1x} & a_{1y} & a_{1z} \\ \frac{\partial}{\partial x_2}[a_{2x} & a_{2y} & a_{2z}] \end{vmatrix}$$

$\frac{\partial}{\partial x_2} (\vec{a}_2) =$ 1st column of $M_2^T$

$\frac{\partial}{\partial x_2} (\vec{a}_2)^T =$ 1st row of $M_2$

$$\frac{\partial F}{\partial x_2} = \begin{vmatrix} b_x & b_y & b_z \\ a_{1x} & a_{1y} & a_{1z} \\ m_{2_{11}} & m_{2_{12}} & m_{2_{13}} \end{vmatrix}$$

$$\frac{\partial F}{\partial y_2} = \begin{vmatrix} b_x & b_y & b_z \\ a_{1x} & a_{1y} & a_{1z} \\ \frac{\partial}{\partial y_2}[a_{2x} & a_{2y} & a_{2z}] \end{vmatrix}$$

$\frac{\partial}{\partial y_2} \vec{a}_2 =$ 2nd column of $M_2^T$

$\frac{\partial}{\partial y_2} (\vec{a}_2)^T =$ 2nd row of $M_2$

$$\frac{\partial F}{\partial y_2} = \begin{vmatrix} b_x & b_y & b_z \\ a_{1x} & a_{1y} & a_{1z} \\ m_{2_{21}} & m_{2_{22}} & m_{2_{23}} \end{vmatrix}$$

$$\frac{\partial F}{\partial \omega} = \begin{vmatrix} b_x & b_y & b_z \\ a_{1x} & a_{1y} & a_{1z} \\ \frac{\partial}{\partial \omega}[a_{2x} & a_{2y} & a_{2z}] \end{vmatrix}$$

$$\frac{\partial}{\partial \omega} \vec{a}_2 = \left(\frac{\partial M_2}{\partial \omega}\right)^T \begin{bmatrix} x_2 - x_0 \\ y_2 - y_0 \\ -f \end{bmatrix}$$

$$\frac{\partial}{\partial \omega} (\vec{a}_2)^T = \left[\left(\frac{\partial M_2}{\partial \omega}\right)^T \begin{bmatrix} x_2 - x_0 \\ y_2 - y_0 \\ -f \end{bmatrix}\right]^T$$

$$\frac{\partial F}{\partial \varphi} = \begin{vmatrix} b_x & b_y & b_z \\ a_{1x} & a_{1y} & a_{1z} \\ \frac{\partial}{\partial \varphi}[a_{2x} & a_{2y} & a_{2z}] \end{vmatrix} \qquad \frac{\partial}{\partial \varphi}\vec{a}_2 = \left(\frac{\partial M_2}{\partial \varphi}\right)^T \begin{bmatrix} x_2 - x_0 \\ y_2 - y_0 \\ -f \end{bmatrix}$$

$$\frac{\partial}{\partial \varphi}(\vec{a}_2)^T = \left[\left(\frac{\partial M_2}{\partial \varphi}\right)^T \begin{bmatrix} x_2 - x_0 \\ y_2 - y_0 \\ -f \end{bmatrix}\right]^T$$

$$\frac{\partial F}{\partial K} = \begin{vmatrix} b_x & b_y & b_z \\ a_{1x} & a_{1y} & a_{1z} \\ \frac{\partial}{\partial K}[a_{2x} & a_{2y} & a_{2z}] \end{vmatrix} \qquad \frac{\partial}{\partial K}\vec{a}_2 = \left(\frac{\partial M_2}{\partial K}\right)^T \begin{bmatrix} x_2 - x_0 \\ y_2 - y_0 \\ -f \end{bmatrix}$$

$$\frac{\partial}{\partial K}(\vec{a}_2)^T = \left[\left(\frac{\partial M_2}{\partial K}\right)^T \begin{bmatrix} x_2 - x_0 \\ y_2 - y_0 \\ -f \end{bmatrix}\right]^T$$

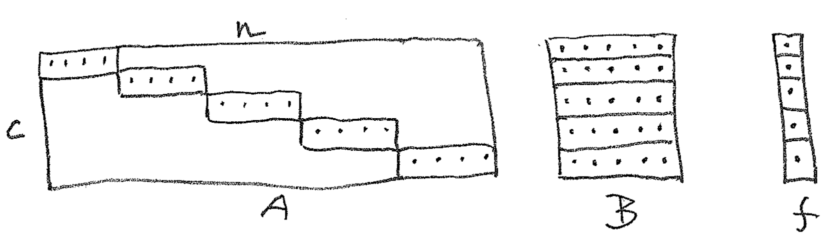
$$\frac{\partial F}{\partial b_y} = \begin{vmatrix} 0 & 1 & 0 \\ a_{1x} & a_{1y} & a_{1z} \\ a_{2x} & a_{2y} & a_{2z} \end{vmatrix} = -\left(a_{1x} a_{2z} - a_{2x} a_{1z}\right)$$

$$\frac{\partial F}{\partial b_z} = \begin{vmatrix} 0 & 0 & 1 \\ a_{1x} & a_{1y} & a_{1z} \\ a_{2x} & a_{2y} & a_{2z} \end{vmatrix} = a_{1x} a_{2y} - a_{2x} a_{1y}$$

Suggest $[a, b, F] = \text{coplan\_lin}(l^0, p^0, io)$

$$a = \left[\frac{\partial F}{\partial x_1} \; \frac{\partial F}{\partial y_1} \; \frac{\partial F}{\partial x_2} \; \frac{\partial F}{\partial y_2}\right] \qquad b = \left[\frac{\partial F}{\partial \omega} \; \frac{\partial F}{\partial \varphi} \; \frac{\partial F}{\partial K} \; \frac{\partial F}{\partial b_y} \; \frac{\partial F}{\partial b_z}\right] \qquad F = \begin{vmatrix} \vdots & \vdots & \vdots \end{vmatrix}$$

$$l^0 = \begin{bmatrix} x_1^0 \\ y_1^0 \\ x_2^0 \\ y_2^0 \end{bmatrix} \qquad p^0 = \begin{bmatrix} b_x \\ b_y^0 \\ b_z^0 \\ \omega^0 \\ \varphi^0 \\ K^0 \end{bmatrix} \qquad io = \begin{bmatrix} x_0 \\ y_0 \\ f \end{bmatrix}$$




$$f = -F - A(l - l^0)$$