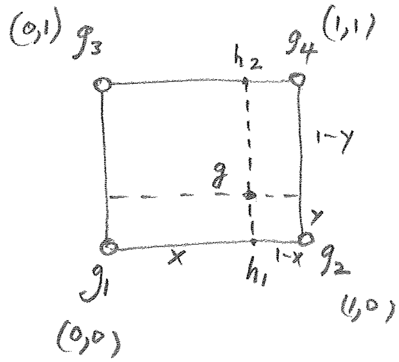
Bilinear interpolation: 3 ways, all yield same result



(1)

1. linear interp. (2x) along x-dir. to produce h_1, h_2
2. linear interp. along y-dir to produce g

$$h_1 = x \cdot g_2 + (1-x) \cdot g_1$$

$$h_2 = x \cdot g_4 + (1-x) \cdot g_3$$

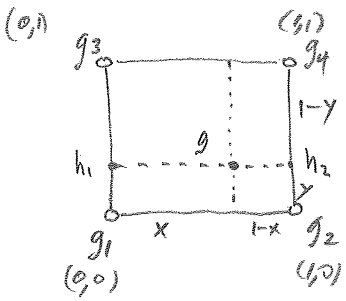
$$g = y \cdot h_2 + (1-y) \cdot h_1 \quad \text{(subs. from above)}$$

$$g = y \cdot \left[x \cdot g_4 + (1-x) \cdot g_3 \right] + (1-y) \cdot \left[x \cdot g_2 + (1-x) \cdot g_1 \right]$$

$$g = xy \cdot g_4 + y \cdot g_3 - xy \cdot g_3 + x \cdot g_2 + g_1 - x \cdot g_1 - xy \cdot g_2 - y \cdot g_1 + xy \cdot g_1$$

$$\boxed{g = (1 - x - y + xy) g_1 + (x - xy) g_2 + (y - xy) g_3 + (xy) g_4}$$

(2)



1. linear interp. (2x) along y-dir. to produce h_1, h_2
2. linear interp along x-dir to produce g

$$h_1 = y \cdot g_3 + (1-y)\, g_1$$

$$h_2 = y \cdot g_4 + (1-y)\, g_2$$

$$g = x \cdot h_2 + (1-x) \cdot h_1 \quad \text{(subs. from above)}$$

$$g = x \cdot \left[y \cdot g_4 + (1-y) \cdot g_2 \right] + (1-x) \cdot \left[y \cdot g_3 + (1-y) \cdot g_1 \right]$$

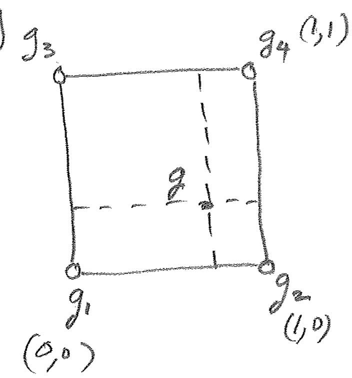
$$g = xy \cdot g_4 + x \cdot g_2 - xy \cdot g_2 + y \cdot g_3 + g_1 - y \cdot g_1 - xy \cdot g_3 - x \cdot g_1 + xy \cdot g_1$$

$$\boxed{g = (1 - x - y + xy) g_1 + (x - xy) g_2 + (y - xy) g_3 + (xy) g_4}$$

Same as above

(3) $g = a_0 + a_1 x + a_2 y + a_3 xy$

evaluate this "condition equation" at each of the 4 corners:



$g_1 = a_0$

$g_2 = a_0 + a_1$

$g_3 = a_0 + a_2$

$g_4 = a_0 + a_1 + a_2 + a_3$

$$\Rightarrow \quad \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Since matrix is triangular, we can solve by "back substitution"

$a_0 = g_1$

$a_1 = g_2 - g_1$

$a_2 = g_3 - g_1$

$a_3 = g_4 - g_1 - g_2 + g_1 - g_3 + g_1, \quad a_3 = g_1 - g_2 - g_3 + g_4$

now substitute solved values for $a_0 \to a_3$ into the original equation where, now, x, y can be anything.

$$g = g_1 + (g_2 - g_1)x + (g_3 - g_1)y + (g_1 - g_2 - g_3 + g_4)xy$$

$$\boxed{g = (1 - x - y + xy)g_1 + (x - xy)g_2 + (y - xy)g_3 + (xy)g_4}$$

Same as prior