tion 3.221 is ill-conditioned when the only available data are control and tie points. In this case, not all the elements of the exterior orientation of the image are simultaneously recoverable in a block adjustment solution.

For many applications, Equation 3.221 has inherently more parameters than needed and only a subset of the correction parameters need to be recovered. In cases where the parameters are not known, a two-pass solution may be necessary. In the first pass, estimates of the camera unknown parameters are computed. In the second pass, the correction parameters are solved for while holding the other parameters constant.

3.4 SYSTEMATIC ERROR CORRECTIONS

3.4.1 Atmospheric Refraction

The refractive index of air decreases with decreasing density. As a result, the light rays bend as the refractive index of the air decreases with altitude. Photogrammetric equations assume that light rays travel in straight paths, so corrections are applied to photo coordinates to compensate for refraction,.

If we assume that the atmosphere is composed of discrete layers and apply Snell's law for refraction between any two successive layers we obtain:

$$(n_i + dn)\sin\alpha_i = n_i \sin(\alpha_i + \Delta\alpha) \tag{3.240}$$

where n_i is the refractive index, and α is the angle of incidence. Since the angle $d\alpha$ is very small, this gives:

$$\Delta\alpha = \frac{dn}{n}\tan\alpha \tag{3.241}$$

In its simplest form, Equation 3.241 can be expressed as:

$$\Delta\alpha = K\tan\alpha \tag{3.242}$$

where $\Delta\alpha$ is angle of displacement, α is the angle the ray makes with vertical, and K is a constant related to atmospheric conditions. The constant K can be thought of as the amount of refraction for a ray at an angle of 45 degrees, and evaluated by assuming a density profile for the atmosphere.

In an unpublished article, Bertram (Thompson, 1966) derived the following expression for K using the Air Force ARDC (Air Research and Development Command) 1959 model for the atmosphere.

$$K = \left[\frac{2410H}{H^2 - 6H + 250} - \frac{2410h}{h^2 - 6h + 250}\left(\frac{h}{H}\right)\right]\cdot 10^{-6} \tag{3.243}$$

where H, the flying height above mean sea level and h the terrain elevation, are both expressed in kilometers, and K is in radians (Table 3.12).

Saastamoinen (1974), using the I.C.A.N. atmosphere, expressed K as:

$$K = \left[\frac{2335}{H-h}\left[(1-0.02257h)^{5.256} - (1-0.02257H)^{5.256}\right] - 277.0(1-0.02257H)^{4.256}\right]\cdot 10^{-6} \tag{3.244}$$

for flying heights of up to 11 km, and for flying heights over 11 km:

$$K = \left[\frac{2335}{H-h}(1-0.02257h)^{5.256} - 0.8540^{H-11}(82.2 - \frac{521}{H-h})\right]\cdot 10^{-6} \tag{3.245}$$

Using the U.S. Standard Atmosphere, 1962, Saastamoinen (1974), offers the following simplified equation for K which is valid up to flying heights of 9 km:

$$K = [13(H - h)[1 - 0.02(2H + h)]] \cdot 10^{-6} \quad (3.246)$$

This formula gives the standard values of photogrammetric refraction correctly to within ±0.5 μrad. Equation 3.246 gives corrections comparable to those proposed by Schut (1969) and Bertram (1969).

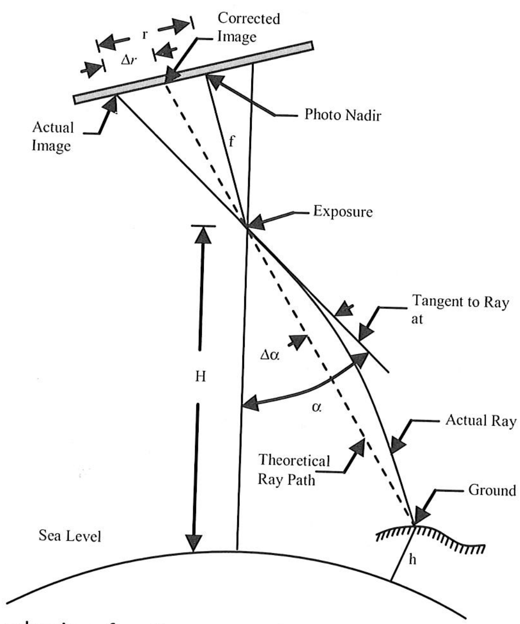

Figure 3.52 Atmospheric refraction correction.

From Figure 3.52, the radial distance r' from the principal point to the corrected image location can then be computed by substituting Equation 3.242 into 3.247 and simplifying to the following general atmospheric refraction for near vertical photograph:

$$r' = f\tan(\alpha - \Delta\alpha) \quad (3.247)$$

$$\Delta r = -K\left(r + \frac{r^3}{f^2}\right) \quad (3.248)$$

$$\Delta x = \frac{x}{r}\Delta r$$

$$\Delta y = \frac{y}{r}\Delta r$$

The second term in the parentheses of Equation 3.248 is directly proportional to the cube of the radial distance and inversely proportional to the square of the focal length. Atmospheric refraction is therefore larger in wide-angle and super-wide-angle photography than in normal-angle photography, and the distortion will be more pronounced at the edges of the format.

If the camera is tilted the image displacement due to refraction is different from the vertical case. Given the orientation matrix M of the tilted photograph, angle α in Equation 3.242 can be computed using as: