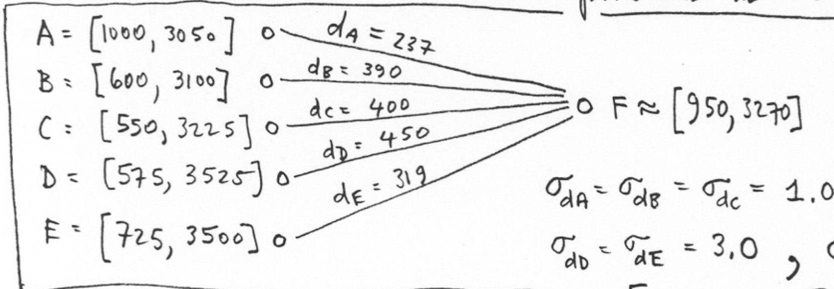
CE 506 Post Adjustment Statistical Analysis, Part 1

note: some matlab functions have changed since this was written.

Use the homework trilateration problem as the data set;



$n = 5$, $n_0 = 2$, $r = 3$

$\sigma_{dA} = \sigma_{dB} = \sigma_{dC} = 1.0$

$\sigma_{dD} = \sigma_{dE} = 3.0$, $\sigma_0^2 = 1.0$

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/9 & 0 \\ 0 & 0 & 0 & 0 & 1/9 \end{bmatrix}$$

After adjustment we get

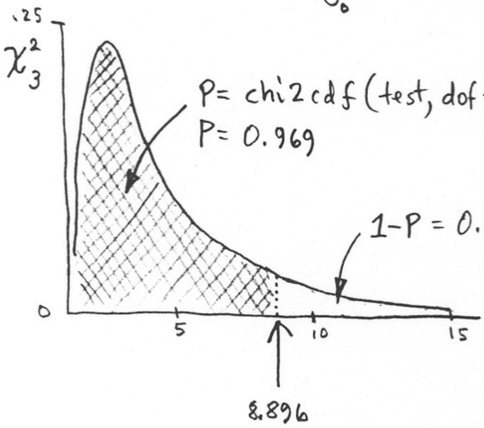
$[\hat{X}_F, \hat{Y}_F] = [946.574, 3279.786]$

$$\hat{\ell} = \begin{bmatrix} \hat{d}_A \\ \hat{d}_B \\ \hat{d}_C \\ \hat{d}_D \\ \hat{d}_E \end{bmatrix} = \begin{bmatrix} 235.915 \\ 390.431 \\ 400.341 \\ 445.194 \\ 312.393 \end{bmatrix}, \quad v = \begin{bmatrix} -1.085 \\ 0.431 \\ 0.341 \\ -4.806 \\ -6.607 \end{bmatrix}$$

$$\Sigma_{\Delta\Delta} = \sigma_0^2 Q_{\Delta\Delta} = \begin{bmatrix} .522 & -.089 \\ -.089 & .804 \end{bmatrix}, \quad \hat{\sigma}_0^2 = \frac{v^T W v}{r} = 2.965$$

(1). Global Test;

Test statistic $= \dfrac{v^T W v}{\sigma_0^2} = 8.896$



$H_0 : \sigma^2 = \sigma_0^2$ (Null Hypothesis)

true variance of unit weight

assumed variance of unit weight

$H_1 : \sigma^2 > \sigma_0^2$ (Alternate Hypothesis)

@ .05 level of significance we reject Ho

@ .01 level of significance we accept Ho

one-sided global test shown here, test done by probabilities, not critical values

You must choose a level of significance (or alpha, probability of Type I error) then either accept or reject the Null Hypothesis.

(2) Construct a confidence interval for a parameter; 90% conf. interval:

If Accept Ho

90% conf. int.: $\hat{x} \pm z \cdot \sigma_{\hat{x}}$

$P\{\hat{x} - z\sigma_{\hat{x}} > \mu_x > \hat{x} + z\sigma_{\hat{x}}\} = 0.90 = 2\Phi(z) - 1$

$1.90 = 2\Phi(z)$, $0.95 = \Phi(z)$

table lookup $z = 1.645$, or

$z = \text{norminv}(.95, 0, 1)$

$z = 1.645$

$\Sigma_{\Delta\Delta} = \sigma_0^2 Q_{\Delta\Delta}$

$\sigma_{\hat{x}} = \sqrt{.522} = .72$

confidence interval $\hat{x} \pm z \cdot \sigma_{\hat{x}}$

$946.574 \pm (1.645)(.72)$

$\boxed{946.574 \pm 1.184}$

If Reject Ho

90% conf. int.: $\hat{x} \pm t_{r=3} \cdot \hat{\sigma}_{\hat{x}}$

$P\{\hat{x} - t\hat{\sigma}_{\hat{x}} > \mu_x > \hat{x} + t\hat{\sigma}_{\hat{x}}\} = 0.90 = 2F(t) - 1$

$1.90 = 2F(t)$, $0.95 = F(t)$

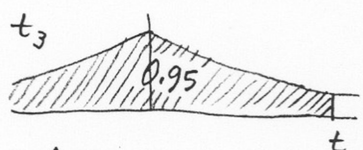


table lookup $t = 2.353$, or

$t = \text{tinv}(0.95, 3)$

$t = 2.353$

$\hat{\Sigma}_{\Delta\Delta} = \hat{\sigma}_0^2 Q_{\Delta\Delta}$

$\hat{\sigma}_{\hat{x}} = \sqrt{1.547} = 1.243$

confidence interval $\hat{x} \pm t \cdot \hat{\sigma}_{\hat{x}}$

$946.574 \pm (2.353)(1.243)$

$\boxed{946.574 \pm 2.925}$

CE 506 Post Adjustment Statistical Analysis
Part 2 : Confidence Regions

After adjustment, make Global Test on $\frac{v^T W v}{\sigma_0^2}$ as in previous handout.

if accept H_0, then $\Sigma_{\Delta\Delta} = \sigma_0^2 Q_{\Delta\Delta}$, σ_0^2 apriori value, $Q_{\Delta\Delta} = (B^T W B)^{-1}$ indi. obs.

if reject H_0, then $\hat{\Sigma}_{\Delta\Delta} = \hat{\sigma}_0^2 Q_{\Delta\Delta}$, $\hat{\sigma}_0^2 = \frac{v^T W v}{r}$ a posteriori value, $Q_{\Delta\Delta}$ same

Extract 2x2 submatrix from $\Sigma_{\Delta\Delta}$ or $\hat{\Sigma}_{\Delta\Delta}$ as appropriate

$$\Sigma_{\Delta\Delta} = \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1 y_1} & \sigma_{x_1 x_2} & \sigma_{x_1 y_2} & \sigma_{x_1 x_3} & \sigma_{x_1 y_3} \\ & \sigma_{y_1}^2 & \sigma_{y_1 x_2} & \sigma_{y_1 y_2} & \sigma_{y_1 x_3} & \sigma_{y_1 y_3} \\ & & \sigma_{x_2}^2 & \sigma_{x_2 y_2} & \sigma_{x_2 x_3} & \sigma_{x_2 y_3} \\ & & & \sigma_{y_2}^2 & \sigma_{y_2 x_3} & \sigma_{y_2 y_3} \\ & \text{Sym.} & & & \sigma_{x_3}^2 & \sigma_{x_3 y_3} \\ & & & & & \sigma_{y_3}^2 \end{bmatrix} \qquad [\ ] = S = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}$$

From prior homework trilateration problem,

$$Q_{\Delta\Delta} = \begin{bmatrix} .522 & -.089 \\ -.089 & .804 \end{bmatrix}, \quad \sigma_0^2 = 1, \quad \hat{\sigma}_0^2 = \frac{v^T W v}{r} = 2.965, \quad r = 3$$

now construct a $P = 95\%$ confidence region for the point, centered @ estimate $[\hat{x}, \hat{y}]$

If accept H_0

$$\Sigma_{\Delta\Delta} = \sigma_0^2 Q_{\Delta\Delta} = 1 \times \begin{bmatrix} .522 & -.089 \\ -.089 & .804 \end{bmatrix} = \begin{bmatrix} .522 & -.089 \\ -.089 & .804 \end{bmatrix} = S$$

compute eigenvalues and { eigenvectors, or orientation angle, θ

matlab: [eigvec, eigval] = eig (S)

$$\text{eigvec} = \begin{bmatrix} -.961 & -.278 \\ -.278 & .961 \end{bmatrix}, \quad \text{eigval} = \begin{bmatrix} .496 & 0 \\ 0 & .830 \end{bmatrix}$$

hand calc.: $\lambda_1 = \sigma_{x'}^2 = \frac{\sigma_x^2 + \sigma_y^2}{2} + \left[\frac{[\sigma_x^2 - \sigma_y^2]^2}{4} + \sigma_{xy}^2\right]^{1/2}$

$\lambda_2 = \sigma_{y'}^2 = \frac{\sigma_x^2 + \sigma_y^2}{2} - \left[\frac{[\sigma_x^2 - \sigma_y^2]^2}{4} + \sigma_{xy}^2\right]^{1/2}$

$\lambda_1 = .830$, $\lambda_2 = .496$ ✓

θ: $\tan 2\theta = \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2} = \frac{-.178}{-.282}$,

II	I
III	IV

$\Rightarrow 2\theta$ in Q. III

$2\theta = \text{atan2}(-.178, -.282) = -2.578$ Radians $= -147.7$ Degrees

$\theta = -1.289$ Radians $= -73.8$ Degrees

Take Larger of 2 eigenvalues $\lambda_1 = .830$,

then $\sigma_{x'} = \sqrt{\lambda_1} = 0.911$

direction given by corresponding eigen vector $[-.278, .961]$ (Δx, Δy) or by angle θ

If reject H_0

$$\hat{\Sigma}_{\Delta\Delta} = \hat{\sigma}_0^2 Q_{\Delta\Delta} = 2.965 \times \begin{bmatrix} .522 & -.089 \\ -.089 & .522 \end{bmatrix} = \begin{bmatrix} 1.547 & -.264 \\ -.264 & 2.384 \end{bmatrix} = S$$

compute eigenvalues and { eigenvectors, or orientation angle, θ

matlab: [eigvec, eigval] = eig (S)

$$\text{eigvec} = \begin{bmatrix} -.961 & -.278 \\ -.278 & .961 \end{bmatrix}, \quad \text{eigval} = \begin{bmatrix} 1.471 & 0 \\ 0 & 2.460 \end{bmatrix}$$

hand calc. formula at left

$\lambda_1 = 2.460$, $\lambda_2 = 1.471$

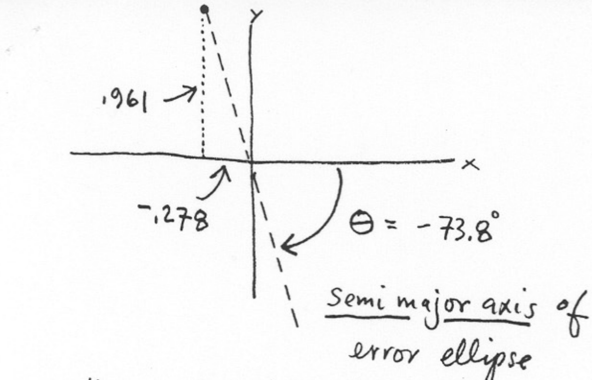
θ: $\tan 2\theta = \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2} = \frac{-.528}{-.836}$

θ: same as at left, $\theta = -73.8$ Degrees

Take Larger of 2 eigenvalues $\lambda_1 = 2.460$

then $\sigma_{x'} = \sqrt{\lambda_1} = 1.568$

direction given by corresponding eigenvector $[-.278, .961]$ (Δx, Δy) or by angle θ

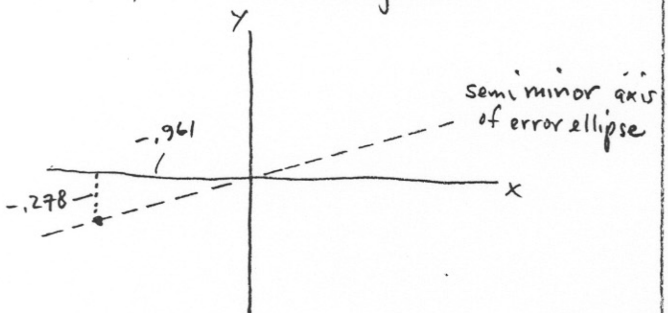


Take smaller of 2 eigenvalues, $\lambda_2 = .496$

then $\sigma_{y'} = \sqrt{\lambda_2} = .704$

direction given by corresponding eigenvector,

$\underset{\Delta x}{[-.961,}\ \underset{\Delta y}{-.278]}$, will always be 90° from semi major axis.



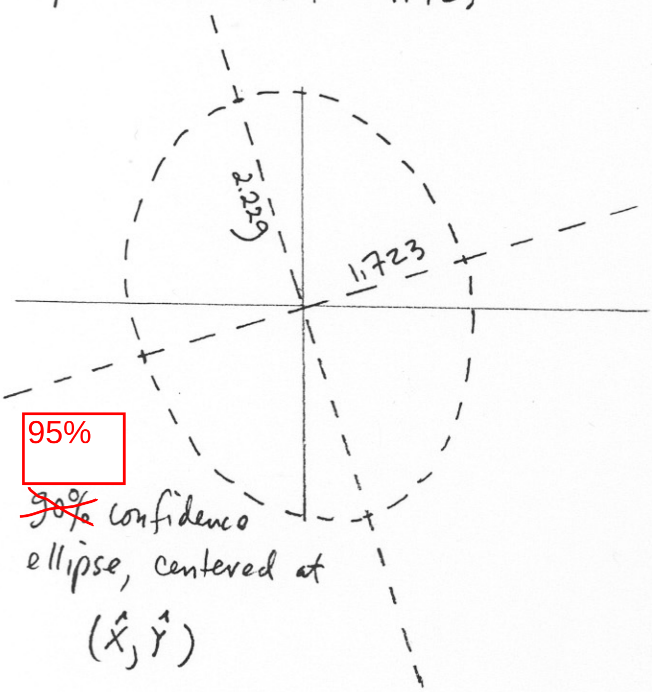
for P confidence level, scale $\sigma_{x'}, \sigma_{y'}$ by C where $C = \sqrt{\chi^2_{2,P}}$ for $P = 0.95$

$C = \sqrt{\chi^2_{2,.95}} = \sqrt{5.991} = 2.447$

C = sqrt(chi2inv(.95,2))

$C \cdot \sigma_{x'} = 2.447 \cdot 0.911 = 2.229$

$C \cdot \sigma_{y'} = 2.447 \cdot 0.704 = 1.723$



95%

~~90%~~ confidence ellipse, centered at $(\hat{x}, \hat{y})$

same as at left

Take smaller of 2 eigenvalues $\lambda_2 = 1.471$

then $\sigma_{y'} = \sqrt{\lambda_2} = 1.213$

direction given by corresponding eigenvector

Same as left

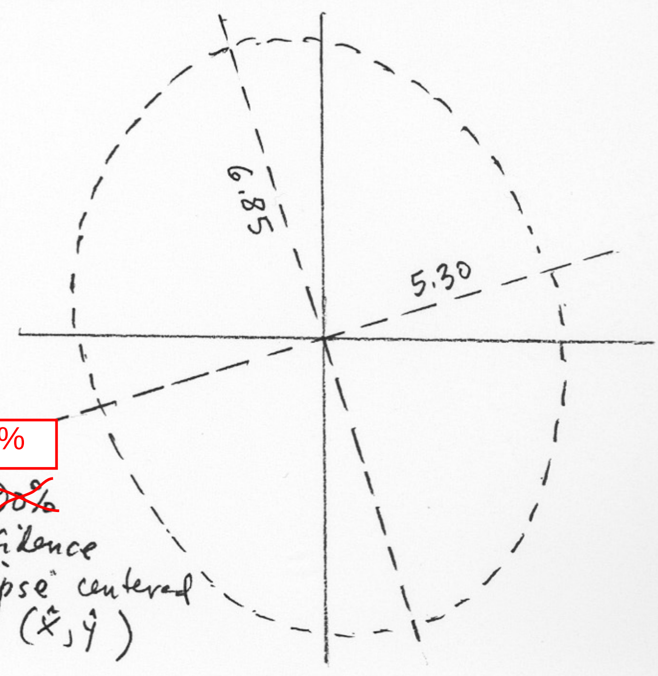
for P confidence level, scale $\sigma_{x'}, \sigma_{y'}$ by C where $C = \sqrt{2F_{2,r,P}}$ for $P = 0.95$

$C = \sqrt{2F_{2,3,.95}} = \sqrt{19.104} = 4.37$

C = sqrt(2*finv(.95,2,3))

$C \cdot \sigma_{x'} = 4.37 \cdot 1.568 = 6.85$

$C \cdot \sigma_{y'} = 4.37 \cdot 1.213 = 5.30$



95%

~~90%~~

confidence ellipse centered at $(\hat{x}, \hat{y})$