CE506 – Homework #8 Solution – 11 Nov 02

based on problem/solution of homework #5

$\sigma_1 = 0.5$
$\sigma_2 = 0.2$
$\sigma_3 = 0.2$

$\sigma_0^2 = (0.5)^2 = 0.25$, $W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 6.25 & 0 \\ 0 & 0 & 6.25 \end{bmatrix}$

$n = 3$
$n_0 = 2$
$r = 1$

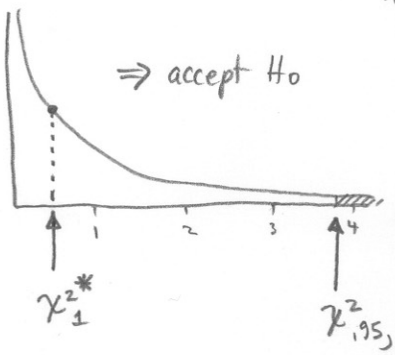
final parameter result

$\begin{bmatrix} \hat{X} \\ \hat{Y} \end{bmatrix} = \begin{bmatrix} 140.0660 \\ 90.1739 \end{bmatrix}$, residuals $V = \begin{bmatrix} 0.2745 \\ -0.0580 \\ 0.0197 \end{bmatrix}$

Note: they are small compared to the a priori σ's. So we know in advance we will pass global test at .05

global test @ .05 significance level

test statistic $\chi^{2*} = \frac{r \cdot \hat{\sigma}_0^2}{\sigma_0^2} = \frac{v^T W v}{\sigma_0^2} = \frac{0.0988}{0.25} = 0.3951$ $(r=1)$



$H_0: \sigma^2 = \sigma_0^2$
$H_1: \sigma^2 > \sigma_0^2$

so: (1) scale Q_{00} by σ_0^2
(2) use z for intervals
(3) use χ^2 for regions

$Q_{00} = \begin{bmatrix} .0852 & -.0294 \\ -.0294 & .6441 \end{bmatrix}$

$\Sigma_{00} = 0.25 \times Q_{00} = \begin{bmatrix} .0213 & -.0073 \\ -.0073 & .1610 \end{bmatrix}$

99% confidence interval for μ_X:

$\hat{X} \pm z_{.995} \cdot \sigma_{\hat{x}}$, $\sigma_{\hat{x}} = \sqrt{.0213} = .1460$

$z_{.995} = 2.5758$

$\hat{x} \pm .3760$, $140.0660 \pm .3760$

$139.6901 \rightarrow 140.4420$

99% confidence interval for μ_Y:

$\hat{y} \pm z_{.995} \cdot \sigma_{\hat{y}}$, $\sigma_{\hat{y}} = \sqrt{.1610} = .4013$

$z_{.995} = 2.5758$

$\hat{y} \pm 1.0336$, 90.1739 ± 1.0336

$89.1403 \rightarrow 91.2075$

Now for the 99% confidence region:

$$\sigma_{x'}^2 = \lambda_1 = \frac{\sigma_x^2+\sigma_y^2}{2} + \left[\frac{(\sigma_x^2-\sigma_y^2)^2}{4} + \sigma_{xy}^2\right]^{1/2} = .1614$$

$$\sigma_{y'}^2 = \lambda_2 = \frac{\sigma_x^2+\sigma_y^2}{2} - \left[\frac{(\sigma_x^2-\sigma_y^2)^2}{4} + \sigma_{xy}^2\right]^{1/2} = .0209$$

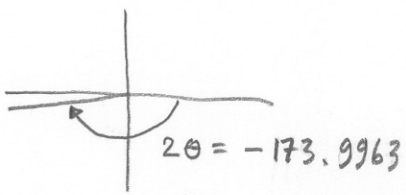
$(\sigma_x = .1460,\ \sigma_y = .4013,\ \sigma_{xy} = -.0073)$

$$\tan 2\theta = \frac{2\sigma_{xy}}{\sigma_x^2-\sigma_y^2} = \frac{-.0147}{-.1397} = \frac{2\cdot(-.0073)}{.0213-.1610}$$

quadrant III

II | I
III | IV

$\Delta y/\Delta x$



$2\theta = -173.9963$

$\theta = -86.9982$

$$a = \sqrt{\lambda_1 \cdot \chi^2_{.99,2}}, \quad b = \sqrt{\lambda_2 \cdot \chi^2_{.99,2}}$$

$\lambda_1 = .1614$

$\lambda_2 = .0209$

$\chi^2_{.99,2} = 9.2103$

$a = 1.2192$

$b = 0.4389$