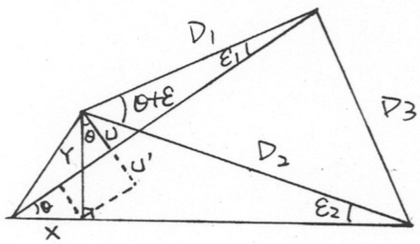
CE506 Homework # 7 31-Oct.-02

6-14)

$$D_3^2 = D_1^2 + D_2^2 - 2D_1D_2\cos(\theta+\varepsilon),\quad \varepsilon \approx 0 \Rightarrow D_3^2 = D_1^2+D_2^2-2D_1D_2\cos\theta$$



$$U = Y\cos\theta \quad U' = \underline{Y\cos\theta - X\sin\theta}$$

$$\varepsilon = \varepsilon_2 - \varepsilon_1 = \frac{Y}{D_2} - \frac{U}{D_1} = \frac{Y}{D_2} - \frac{Y\cos\theta - X\sin\theta}{D_1}$$

$$= \frac{\sin\theta}{D_1}X + \frac{D_1 - D_2\cos\theta}{D_1D_2}Y$$

$$\sigma_\varepsilon^2 = \left(\frac{\partial\varepsilon}{\partial X}\right)^2\sigma_x^2 + \left(\frac{\partial\varepsilon}{\partial Y}\right)^2\sigma_y^2 \qquad \sigma_x = \sigma_y = \sigma.$$

$$= \left(\frac{\sin\theta}{D_1}\right)^2\sigma^2 + \left(\frac{D_1 - D_2\cos\theta}{D_1D_2}\right)^2\sigma^2$$

$$\sigma_3 = \underline{\frac{D_3}{D_1D_2}}$$

$$= \left(\frac{D_2^2\sin^2\theta + D_1^2 - 2D_1D_2\cos\theta + D_2^2\cos^2\theta}{D_1^2D_2^2}\right)\sigma^2$$

$$= \frac{D_2^2 + D_1^2 - 2D_1D_2\cos\theta}{D_1^2D_2^2} = \underline{\frac{D_3^2}{D_1^2D_2^2}}$$

6-15) from example 4-6, $\sigma_0^2 = 0.1$, $N^{-1} = \frac{1}{31.25}\begin{bmatrix} 3.5 & -14.5 \\ -14.5 & 69 \end{bmatrix}$

$V + B\Delta = f$, $\Sigma_{\Delta\Delta} = \sigma_0^2 Q_{\Delta\Delta}$, $Q_{\Delta\Delta} = N^{-1}$ $\Rightarrow$ $\Sigma_{\Delta\Delta} = \begin{bmatrix} 0.0112 & -0.0464 \\ -0.0464 & 0.2208 \end{bmatrix}$

$$\Sigma_{\Delta\Delta} = \begin{bmatrix} \sigma_{\hat a}^2 & \sigma_{\hat a\hat b} \\ \sigma_{\hat a\hat b} & \sigma_{\hat b}^2 \end{bmatrix} \qquad \sigma_{\hat a}^2 = 0.0112 \quad \sigma_{\hat a} = \underline{0.1058}$$

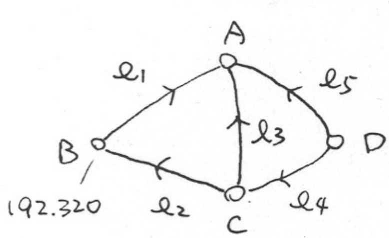
$$\sigma_{\hat b}^2 = 0.2208 \quad \sigma_{\hat b} = \underline{0.4699}$$

6-16) from example 4-9. $\Sigma_{\Delta\Delta} = \sigma_0^2 N^{-1}$. $\sigma_0 = 0.005$, $N = [0.503426]$

$$\sigma_x^2 = \Sigma_{\Delta\Delta} = 0.005^2 \cdot (0.503426)^{-1} = 4.966\times10^{-5},\quad \sigma_x = \underline{7.05\times10^{-3}}\,m$$

* using $N = [0.503722]$ is also ok.

6-19) problem 4-8 indirect. obs. 4-11 obs. only



l_1 12.386 m, $\sigma_1 = 18$
l_2 11.740 m, $\sigma_2 = 12$
l_3 24.101 m, $\sigma_3 = 20$
l_4 8.150 m, $\sigma_4 = 8$
l_5 32.296 m, $\sigma_5 = 22$

$n=5$
$n_0=3$
$\underline{r=2}$
$u=3$
$c=5$

parameter A.C.D

$\hat{l}_1 = A - 192.320$
$\hat{l}_2 = 192.320 - C$
$\hat{l}_3 = A - C$
$\hat{l}_4 = C - D$
$\hat{l}_5 = A - D$

$$Q_{\hat l\hat l} = BN^{-1}B^T$$
$$= Q - Q_{vv}$$

$\hat{l}_1 + \hat{l}_2 - \hat{l}_3 = 0$
$\hat{l}_3 - \hat{l}_5 + \hat{l}_4 = 0$

$Q_{\hat l\hat l} = Q - Q_{vv}$
$Q_{vv} = QA^TW_eAQ$

Compare two different method. $Q_{\hat l\hat l}$ should be the same.

```
% 506 Problem 6-19
% 4-8 Indirect observations

clear all
clc

% Model Parameters
n = 5;
no = 3;
r = n-no;
u = no;
c = r+u;

% Observations
Lo = zeros(n,1);
Lo(1) = 12.386;
Lo(2) = 11.740;
Lo(3) = 24.101;
Lo(4) = 8.150;
Lo(5) = 32.296;
L = Lo;

% initial value for Unknown Parameters
Xo = zeros(u,1);
Xo(1) = 204.706;   % parameter A
Xo(2) = 180.580;   % parameter C
Xo(3) = 172.430;   % parameter D
X = Xo;

% Cofactor & Weight Matrix
Q = diag([18 12 20 8 22]);;
W = inv(Q);

% initializing values
v = zeros(n,1);

    A = X(1);  % parameter A
    C = X(2);  % parameter C
    D = X(3);  % parameter D

    B = zeros(c,u);
    B = [-1 0 0;
          0 1 0;
         -1 1 0;
          0 -1 1;
         -1 0 1];

    % Compute a new F for each iteration
    F = zeros(c,1);
    F(1) = L(1)-A+192.320;
    F(2) = L(2)+C-192.320;
    F(3) = L(3)-A+C;
    F(4) = L(4)-C+D;
    F(5) = L(5)-A+D;

    % Least Squares Equations
    f = -F+v;
    N = B'*W*B;
    t = B'*W*f;

    % Updating the parameters and observations
    de = inv(N)*t;
    v = f-B*de;
    X = X+de;
    L = Lo+v;

    Qvv = Q-B*inv(N)*B';
    Qll = Q-Qvv;

fprintf('\nAdjusted Observations:\n')
for i=1:n
    fprintf('L%d\t= %12.7f\n',i,L(i));
end
fprintf('\nAdjusted Parameters:\n')
for i=1:u
    fprintf('X(%d)\t= %12.7f\n',i,X(i));
end

Qvv
Qll
```

```
Adjusted Observations:
L1      =   12.3830000
L2      =   11.7380000
L3      =   24.1210000
L4      =    8.1566667
L5      =   32.2776667

Adjusted Parameters:
X(1)    =  204.7030000
X(2)    =  180.5820000
X(3)    =  172.4253333
```

```
Qvv =

    7.7143    5.1429   -5.1429    1.3714   -3.7714
    5.1429    3.4286   -3.4286    0.9143   -2.5143
   -5.1429   -3.4286   11.4286    2.2857   -6.2857
    1.3714    0.9143    2.2857    1.5238   -4.1905
   -3.7714   -2.5143   -6.2857   -4.1905   11.5238
```

$Q_{\hat{e}\hat{e}}$ for indirect

```
Qll =

   10.2857   -5.1429    5.1429   -1.3714    3.7714
   -5.1429    8.5714    3.4286   -0.9143    2.5143
    5.1429    3.4286    8.5714   -2.2857    6.2857
   -1.3714   -0.9143   -2.2857    6.4762    4.1905
    3.7714    2.5143    6.2857    4.1905   10.4762
```

```
% 506 Problem 6-19
% 4-11 Observations Only

clear all
clc

% Model Parameters
n = 5;
no = 3;
r = n-no;
c = r;

% Observations
Lo = zeros(n,1);
Lo(1) = 12.386;
Lo(2) = 11.740;
Lo(3) = 24.101;
Lo(4) = 8.150;
Lo(5) = 32.296;
L = Lo;

% Cofactor & Weight Matrix
Q = diag([18 12 20 8 22]);
W = inv(Q);

% initializing values
v = zeros(n,1);

% Create a new A matrix for each iteration
  A = zeros(c,n);
  A = [1 1 -1 0 0;
       0 0  1 1 -1];

    % Compute a new F for each iteration
    F = zeros(c,1);
    F(1) = L(1)+L(2)-L(3);
    F(2) = L(3)-L(5)+L(4);

    % Least Squares Equations
    f = -F+A*v;
    We = inv(A*Q*A');
    v = Q*A'*We*f;
    L = Lo+v;

    % Standard deviation of estimated observation
    Qvv = Q*A'*We*A*Q;
    Qll = Q-Qvv;

 fprintf('\nAdjusted Observations:\n')
 for i=1:n
    fprintf('L%d\t= %12.7f\n',i,L(i));
 end
```

```
Adjusted Observations:
L1      =   12.3830000
L2      =   11.7380000
L3      =   24.1210000
L4      =    8.1566667
L5      =   32.2776667

We =

    0.0238    0.0095
    0.0095    0.0238

Qvv =

    7.7143    5.1429   -5.1429    1.3714   -3.7714
    5.1429    3.4286   -3.4286    0.9143   -2.5143
   -5.1429   -3.4286   11.4286    2.2857   -6.2857
    1.3714    0.9143    2.2857    1.5238   -4.1905
   -3.7714   -2.5143   -6.2857   -4.1905   11.5238
```

$Q_{\hat{e}\hat{e}}$ for observation only

```
Qll =

   10.2857   -5.1429    5.1429   -1.3714    3.7714
   -5.1429    8.5714    3.4286   -0.9143    2.5143
    5.1429    3.4286    8.5714   -2.2857    6.2857
   -1.3714   -0.9143   -2.2857    6.4762    4.1905
    3.7714    2.5143    6.2857    4.1905   10.4762
```