CE 506 Homework # 6 22-Oct-02

6-1) interior angles: $x_1, x_2, \dots x_n$. $\sigma_1 = \sigma_2 = \dots \sigma_n = 4''$

Sum of the angles: $y = x_1 + x_2 + \dots x_n$

$$\sigma_y^2 = (1)^2\sigma_1^2 + (1)^2\sigma_2^2 + \dots (1)^2\sigma_n^2$$

$n=5$ $\sigma_y^2 = 5 \cdot 4^2 = 80$ $\sigma_y = \sqrt{80} = \underline{8.94''}$

$n=10$ $\sigma_y^2 = 10 \cdot 4^2 = 160$ $\sigma_y = \sqrt{160} = \underline{12.65''}$

$n=20$ $\sigma_y^2 = 20 \cdot 4^2 = 320$ $\sigma_y = \sqrt{320} = \underline{17.89''}$

6-2) α, β are measured interior angles. $\sigma_\alpha = 4.5''$, $\sigma_\beta = 6.0''$

γ is computed angle

$$\gamma = 180^\circ - \alpha - \beta = \begin{bmatrix} -1 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + 180^\circ$$

$$\sigma_\gamma^2 = (-1)^2\sigma_\alpha^2 + (-1)^2\sigma_\beta^2 = 4.5^2 + 6.0^2 = 56.25 \qquad \sigma_\gamma = \sqrt{56.25} = \underline{7.5''}$$

6-4) $$\overline{x_w} = \frac{w_1x_1 + w_2x_2 + w_3x_3}{w_1 + w_2 + w_3} \qquad \Sigma w = w_1 + w_2 + w_3$$

$$\overline{x_w} = \frac{w_1}{\Sigma w}x_1 + \frac{w_2}{\Sigma w}x_2 + \frac{w_3}{\Sigma w}x_3 \qquad \sigma_1 = 0.02 \quad \sigma_2 = 0.03, \quad \sigma_3 = 0.06$$

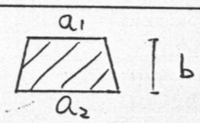
let $\sigma_0 = 0.06$. $w_1 = \frac{\sigma_0^2}{\sigma_1^2} = \frac{0.06^2}{0.02^2} = 9$, $w_2 = \frac{\sigma_0^2}{\sigma_2^2} = \frac{0.06^2}{0.03^2} = 4$, $w_3 = \frac{\sigma_0^2}{\sigma_3^2} = 1$

$$\overline{x_w} = \begin{bmatrix} \frac{w_1}{\Sigma w} & \frac{w_2}{\Sigma w} & \frac{w_3}{\Sigma w} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$Q_{ww} = \begin{bmatrix} \frac{w_1}{\Sigma w} & \frac{w_2}{\Sigma w} & \frac{w_3}{\Sigma w} \end{bmatrix} \begin{bmatrix} \frac{1}{w_1} & 0 & 0 \\ 0 & \frac{1}{w_2} & 0 \\ 0 & 0 & \frac{1}{w_3} \end{bmatrix} \begin{bmatrix} \frac{w_1}{\Sigma w} \\ \frac{w_2}{\Sigma w} \\ \frac{w_3}{\Sigma w} \end{bmatrix} = \begin{bmatrix} \frac{1}{\Sigma w} \end{bmatrix}.$$

$$\sigma_w^2 = \Sigma_{ww} = \sigma_0^2 Q_{ww} = 0.06^2 \cdot \left(\frac{1}{9+4+1}\right) = 2.57 \times 10^{-4} \qquad \sigma_w = \sqrt{2.57 \times 10^{-4}} = \underline{0.016\ m}$$

6-6) $\text{Area} = \left(\frac{a_1 + a_2}{2}\right) b$



$a_1: 319.44$, $\sigma_{a_1} = 0.030$

$a_2: 481.112$, $\sigma_{a_2} = 0.042$

$b: 502.307$, $\sigma_b = 0.020$

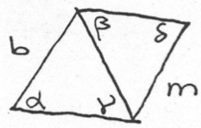
$$\Sigma_{AA} = \begin{bmatrix} \frac{\partial A}{\partial a_1} & \frac{\partial A}{\partial a_2} & \frac{\partial A}{\partial b} \end{bmatrix} \begin{bmatrix} \sigma_{a_1}^2 & 0 & 0 \\ 0 & \sigma_{a_2}^2 & 0 \\ 0 & 0 & \sigma_b^2 \end{bmatrix} \begin{bmatrix} \partial A/\partial a_1 \\ \partial A/\partial a_2 \\ \partial A/b \end{bmatrix}$$

$$\frac{\partial A}{\partial a_1} = \frac{b}{2}, \quad \frac{\partial A}{\partial a_2} = \frac{b}{2}, \quad \frac{\partial A}{\partial b} = \frac{a_1 + a_2}{2}$$

$$\Sigma_{AA} = \sigma_A^2 = [251.1535 \quad 251.1535 \quad 400.2630] \begin{bmatrix} 0.0009 & 0 & 0 \\ 0 & 0.0018 & 0 \\ 0 & 0 & 0.0004 \end{bmatrix} \begin{bmatrix} 251.1535 \\ 251.1535 \\ 400.2630 \end{bmatrix}$$

$\sigma_A^2 = 232.1242 \qquad \sigma_A = \sqrt{232.1242} = \underline{15.23\ m^2}, \qquad A = \underline{201054.9\ m^2}$

6-7)



$m = b\dfrac{\sin\alpha \sin\beta}{\sin\gamma \sin\delta} = \underline{1405.687m}$

$b = 2140.720m, \quad \sigma_b = .018m$

$\alpha = 52°12'40'', \quad \sigma_\alpha = 3.0'' = 1.45 \times 10^{-5}$ rad

$\beta = 51°37'15'', \quad \sigma_\beta = 3.0'' = 1.45 \times 10^{-5}$ rad

$\gamma = 73°55'11'' \quad \sigma_\gamma = 2.0'' = 9.70 \times 10^{-6}$ rad

$\delta = 79°04'22'' \quad \sigma_\delta = 2.0'' = 9.70 \times 10^{-6}$ rad

$$\Sigma_{mm} = \sigma_m^2 = \begin{bmatrix} \frac{\partial m}{\partial b} & \frac{\partial m}{\partial \alpha} & \frac{\partial m}{\partial \beta} & \frac{\partial m}{\partial \gamma} & \frac{\partial m}{\partial \delta} \end{bmatrix} \begin{bmatrix} \sigma_b^2 & & & & \\ & \sigma_\alpha^2 & & & \\ & & \sigma_\beta^2 & & \\ & & & \sigma_\gamma^2 & \\ & & & & \sigma_\delta^2 \end{bmatrix} \begin{bmatrix} \partial m/\partial b \\ \partial m/\alpha \\ \partial m/\beta \\ \partial m/\gamma \\ \partial m/\delta \end{bmatrix}$$

$\dfrac{\partial m}{\partial b} = \dfrac{\sin\alpha \sin\beta}{\sin\gamma \sin\delta} = 0.6566, \qquad \dfrac{\partial m}{\partial \alpha} = b\dfrac{\cos\alpha \sin\beta}{\sin\gamma \sin\delta} = 1089.926$

$\dfrac{\partial m}{\partial \beta} = b\dfrac{\sin\alpha \cos\beta}{\sin\gamma \sin\delta} = 1113.302, \qquad \dfrac{\partial m}{\partial \gamma} = b\dfrac{\sin\alpha \sin\beta \cos\gamma}{\sin\gamma^2 \sin\delta} \cdot (-1) = -405.207$

$\dfrac{\partial m}{\partial \delta} = b\dfrac{\sin\alpha \sin\beta \cos\delta}{\sin\gamma \sin\delta^2} \cdot (-1) = -271.385$

$\sigma_m^2 = 0.000676 \qquad \sigma_m = \sqrt{0.000676} = \underline{0.026m}$

6-8)

$X_1 = 2u_1 + 3u_2 + u_3$

$X_2 = u_1 + u_2 - u_3$

$\Sigma_{uu} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

$$\Sigma_{xx} = A\Sigma_{uu}A^t = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 1 \\ 1 & -1 \end{bmatrix} = \underline{\begin{bmatrix} 42 & 9 \\ 9 & 5 \end{bmatrix}}$$