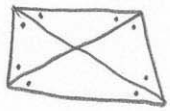
CE 506 Fall '02 Homework #1

assigned 22 aug – due 29 aug

3-1, 3-2, 3-3, 3-4, 3-5, 3-7, 3-12

3-1



note: four angles fix relative positions of all vertices, that fixes the shape.

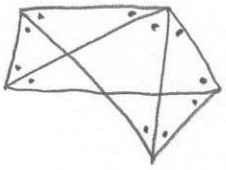
$n = 8$
$n_0 = 4$
$r = 4$

3-2 add 6 "side" or "edge" distance measurements, now model is shape & size

$n = 8 + 6 = 14$
$n_0 = 4 + 1 = 5$
$r = 9$

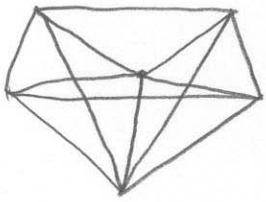
we only need 1 length to fix size of the figure

3-3 marked angles and all sides measured, model is shape & size
what are model elements?



n = 11 angles + 8 sides = 19
n_0 = 6 angles + 1 distance = 7
r = 12

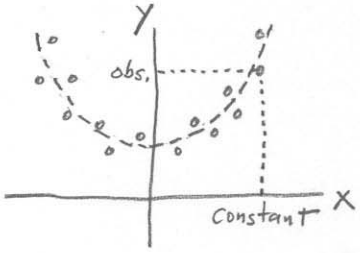
3-4 all sides measured, model is size and shape, what are model elements?



$n = 13$
$n_0 = 9$
$r = 4$

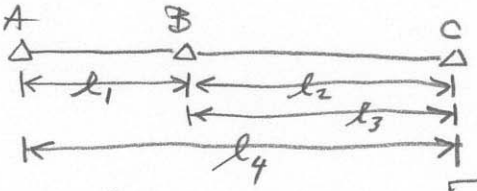
hint: build up the figure by triangles

3-5 14 y-coords observed, x-coords are constants, model is parabola
$y = ax^2 + bx + c$, what are model elements?



$n = 14$
$n_0 = 3$
$r = 11$

3-7



model is 1D distance network

$n = 4$, $n_0 = 2$, $r = 2$

$\ell_1 = 100.01$, $\ell_2 = 200.05$, $\ell_3 = 200.07$, $\ell_4 = 300.09$

write $r=2$ condition equations: $\boxed{\hat{\ell}_2 = \hat{\ell}_3 \;;\; \hat{\ell}_1 + \hat{\ell}_2 = \hat{\ell}_4}$; put in numbers for obs.

$\hat{\ell}_2 - \hat{\ell}_3 = 0$, $\ell_2 + v_2 - \ell_3 - v_3 = 0$, $200.05 + v_2 - 200.07 - v_3 = 0$

$\hat{\ell}_1 + \hat{\ell}_2 - \hat{\ell}_4 = 0$, $\ell_1 + v_1 + \ell_2 + v_2 - \ell_4 - v_4 = 0$, $100.01 + v_1 + 200.05 + v_2 - 300.09 - v_4 = 0$

$\boxed{\begin{aligned} v_2 - v_3 &= .02 \\ v_1 + v_2 - v_4 &= .03 \end{aligned}}$ keep $v_1, v_2 \Rightarrow$ solve for v_3, v_4 to eliminate $\boxed{\begin{aligned} v_3 &= v_2 - .02 \\ v_4 &= v_1 + v_2 - .03 \end{aligned}}$ now substitute into Φ

$\Phi = v_1^2 + v_2^2 + v_3^2 + v_4^2 = v_1^2 + v_2^2 + (v_2 - .02)^2 + (v_1 + v_2 - .03)^2 \longrightarrow \min$

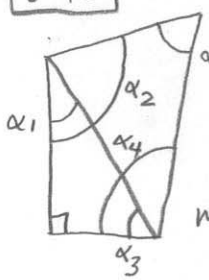
$$\left.\begin{aligned} \frac{\partial \Phi}{\partial v_1} &= 2v_1 + 2(v_1 + v_2 - .03) = 0 \\ \frac{\partial \Phi}{\partial v_2} &= 2v_2 + 2(v_2 - .02) + 2(v_1 + v_2 - .03) = 0 \end{aligned}\right\} \quad \begin{aligned} 2v_1 + v_2 &= .03 \\ v_1 + 3v_2 &= .05 \end{aligned} \;;\; \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} .03 \\ .05 \end{bmatrix}$$

normal equations, symmetric

solve normal equations by MATLAB $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} .008 \\ .014 \end{bmatrix}$; $v_3 = v_2 - .02 = -.006$; $v_4 = v_1 + v_2 - .03 = -.008$

$\hat{\ell}_1 = 100.01 + .008 = 100.018$
$\hat{\ell}_2 = 200.05 + .014 = 200.064$
$\hat{\ell}_3 = 200.07 - .006 = 200.064$
$\hat{\ell}_4 = 300.09 - .008 = 300.082$

3-12



$\alpha_1 = 40\text{-}00\text{-}00''$
$\alpha_2 = 100\text{-}00\text{-}30$
$\alpha_3 = 50\text{-}00\text{-}20$
$\alpha_4 = 120\text{-}00\text{-}00$
$\alpha_5 = 50\text{-}00\text{-}20$

$n = 5$, $n_0 = 3$, $r = 2$

$r=2$ condition equations

$\boxed{\begin{aligned} \hat{\alpha}_1 + \hat{\alpha}_3 &= 90^\circ \\ \hat{\alpha}_2 + \hat{\alpha}_4 + \hat{\alpha}_5 &= 270^\circ \end{aligned}}$

$\alpha_1 + v_1 + \alpha_3 + v_3 - 90^\circ = 0$

$\alpha_2 + v_2 + \alpha_4 + v_4 + \alpha_5 + v_5 - 270^\circ = 0$

model is shape of figure

$v_1 + v_3 = 90^\circ - (40\text{-}00\text{-}00) - (50\text{-}00\text{-}20)$,

$v_2 + v_4 + v_5 = 270^\circ - (100\text{-}00\text{-}30) - (120\text{-}00\text{-}00) - (50\text{-}00\text{-}20)$;

$\boxed{\begin{aligned} v_1 + v_3 &= -20'' \\ v_2 + v_4 + v_5 &= -50'' \end{aligned}}$

keep v_1, v_2, v_4 solve for v_3, v_5: $\boxed{\begin{aligned} v_3 &= -v_1 - 20'' \\ v_5 &= -v_2 - v_4 - 50'' \end{aligned}}$

$\Phi = v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 = v_1^2 + v_2^2 + (-v_1 - 20'')^2 + v_4^2 + (-v_2 - v_4 - 50'')^2 \longrightarrow \min$

$$\left.\begin{aligned} \partial\Phi/\partial v_1 &= 2v_1 - 2(-v_1 - 20'') = 0 \\ \partial\Phi/\partial v_2 &= 2v_2 - 2(-v_2 - v_4 - 50'') = 0 \\ \partial\Phi/\partial v_4 &= 2v_4 - 2(-v_2 - v_4 - 50'') = 0 \end{aligned}\right\} \quad \begin{aligned} 2v_1 &= -20'' \\ 2v_2 + v_4 &= -50'' \\ v_2 + 2v_4 &= -50'' \end{aligned} \quad \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_4 \end{bmatrix} = \begin{bmatrix} -20'' \\ -50'' \\ -50'' \end{bmatrix}$$

solve by MATLAB

$\begin{bmatrix} v_1 \\ v_2 \\ v_4 \end{bmatrix} = \begin{bmatrix} -10''.00 \\ -16''.67 \\ -16''.67 \end{bmatrix}$

$v_3 = -v_1 - 20'' = -10''$

$v_5 = -v_2 - v_4 - 50'' = -16''.67$

$\hat{\alpha}_1 = 40\text{-}00\text{-}00 - 10'' = 39^\circ\text{-}59'\text{-}50''$
$\hat{\alpha}_2 = 100\text{-}00\text{-}30 - 16''.67 = 100\text{-}00\text{-}13.3$
$\hat{\alpha}_3 = 50\text{-}00\text{-}20 - 10'' = 50\text{-}00\text{-}10$
$\hat{\alpha}_4 = 120\text{-}00\text{-}00 - 16''.67 = 119\text{-}59\text{-}43.3$
$\hat{\alpha}_5 = 50\text{-}00\text{-}20 - 16''.67 = 50\text{-}00\text{-}3.3$